# Behaviour of ultrasonic waves in porous rigid materials: anisotropic Biot-Attenborough theory

Haydar Aygün and Christopher Barlow

Maritime and Technology Faculty, Southampton Solent University, Southampton, SO14 0RD, UK; haydar-aygun@solent.ac.uk

Short Title: Ultrasonic waves in porous rigid materials
Keywords: ultrasound, wave propagation, bone, Biot's theory

**Abstract**:

The anisotropic pore structure and elasticity of cancellous bone cause wave speeds and attenuation in cancellous bone to vary with angle. Anisotropy has been introduced into Biot theory by using an empirical expression for the angle- and porosity-dependence of tortuosity. Predictions of a modified anisotropic Biot–Attenboorugh theory are compared with measurements of pulses centred on 100 kHz and 1 MHz transmitted through water-saturated porous samples. The samples are 13 times larger than the original bone samples. Despite the expected effects of scattering, which is neglected in the theory, at 100 kHz the predicted and measured transmitted waveforms are similar.

## Introduction

Understanding the propagation of acoustic waves through cancellous bone is an important pre-requisite to improving the diagnosis of osteoporosis by ultrasound. Bone essentially has two types of structure, both having the same mineralised collagen composition. Cortical bone has porosity less than 30% and may generally be considered to be solid; cancellous bone has porosity greater than 30% and consists of a complex open-celled porous network of rod- and plate-shaped elements termed trabeculae. The porosity of human cancellous bone ranges between 70% and 95%, the remaining volume being perfused with bone marrow. In the adult human vertebral body for example, both horizontal and vertical trabeculae range from 50–120 μm in thickness, and spaced at intervals of between 1200 – 5000 μm and 700 and 2000 μm respectively (Thomsen *et al* 2002). During childhood, more bone is added than is being taken away. During early adulthood, the amounts removed and added are the same. If however, more bone is removed than is being added, we have a condition called **osteoporosis** which literally means 'porous bone' and describes a period of largely asymptomatic bone loss leading to skeletal fragility and increased risk of fracture. It is caused by hormonal imbalance (oestrogen & testosterone) and long-term cortico-steroid use. It is also caused by low bone mass, as well as a weakened structure. One in three women and one in five men over the age of 50 will break a bone attributed to osteoporosis. It is second only to cardiovascular disease as a global healthcare problem (World Health Organisation).

Osteoporosis leads to nearly 9 million fractures annually worldwide (Johnell and Kanis 2006), and over 300,000 patients present with fragility fractures to hospitals in the UK each year (British Orthopaedic Association 2007). Direct medical costs from fragility fractures to the UK healthcare economy were estimated at £1.8 billion in 2000, with the potential to increase to £2.2 billion by 2025, and with most of these costs relating to hip fracture care (Burge et al, 2001).

To improve the prediction of fracture risk by ultrasound it is important to understand the propagation of acoustic waves through cancellous bone. The theory that is mostly used for investigation of acoustic wave propagation in cancellous bone is Biot theory. The theory predicts two compressional waves, often referred to as ‘fast’ and ‘slow’ respectively, when the waves propagating through the solid frame of bone and marrow are in-phase and out-of-phase respectively. The angular dependences of phase velocities for the fast and the slow waves in cancellous bone have been predicted (Hughes *et al* 1999), along with the anisotropic behaviour of acoustic wave propagation (Hughes *et al* 2007). Biot theory was specifically developed to describe acoustic wave propagation in fluid-saturated porous elastic media (Biot 1956a, 1956b); although originating for geophysical testing of porous rocks, it has been used extensively to describe the wave motion in cancellous bone. The Biot theory allows for an arbitrary microstructure, with separate motions considered for the solid elastic framework (bone) and the interspersed fluid (marrow), induced by the ultrasonic wave, and also includes energy loss due to viscous friction between solid (bone) and fluid (marrow). The Biot theory predicts a shear wave. McKelvie and Palmer (1991) were first to apply Biot theory to ultrasonic wave propagation in cancellous bone. Hosokawa and Otani (1997) first observed experimentally the two theoretically predicted compressional waves in cancellous bone at ultrasonic frequencies. The Biot model has since been used extensively to describe the wave motion in trabecular (cancellous) bone (Haire and Langton 1999, Fellah *et al* 2004, Sebaa *et al* 2006, Pulau *et al* 2008). Attenborough *et al*. (2005) presented tortuosities deduced from audio-frequency measurements in air-filled cancellous bone replicas and showed that there was strong anisotropy. The Biot theory has been further developed including semi-analytical approach that allows for transverse anisotropy in the frame elastic moduli, tortuosity and permeability for geophysical applications (Carcione 1996). A modified Biot-Attenborough (MBA) model has also been proposed for acoustic wave propagation in a non-rigid porous medium with circular cylindrical pores starting from a formulation for a rigid-framed porous material (Roh *et al* 2003, Attenborough 1982, Attenborough 1983). The MBA has been used to predict the dependences of velocity and attenuation on frequency and porosity in bovine cancellous bone (Lee *et al* 2003, Lee and Yoon 2006). The Biot model has also been modified to include the acoustic anisotropy of cancellous bone by introducing empirical angle-dependent parameters, and used to predict both the fast and slow wave velocities as a function of propagation angle with respect to the trabecular alignment of cancellous bone (Lee *et al* 2007).

Previous work on the influence of anisotropic pore structure and elasticity in cancellous bone has been extended by developing an anisotropic Biot–Allard model allowing for angle dependent tortuosity and elasticity by Aygün *et al*. 2009. The extreme angle dependence of tortuosity corresponding to the parallel plate microstructure used by Hughes *et al.* 2007 has been replaced by angle dependent tortuosity values based on data for slow wave transmission through air-filled bone replicas. Audio-frequency data obtained at audio-frequencies in air-filled bone replicas are used to derive an empirical expression for the angle-and porosity-dependence of tortuosity.

Most recently, Aygün *et al*. (2010 and 2011) have transmitted ultrasonic signals through water saturated stereolithograpical bone replicas. Predictions of a modified anisotropic Biot-Allard model, which neglects scattering have been compared to measurements made at normal and oblique incidence in a water filled tank at 100 kHz and 1 MHz. Remarkably, it is found that the expected occurrence of scattering does not cause significant discrepancies between predictions and data at 100 kHz (which would be equivalent to 1.3 MHz in real bone), perhaps as a consequence of the fact that the samples behave as low pass filters. Scattering should be even more important at 1 MHz (equivalent to 13 MHz in real bone) where the fast and slow wavelengths are 3 mm and 1.5 mm respectively. Nevertheless the modified Biot-Allard theory is found to predict the observed simple relationship between incident and transmitted waveforms at 1 MHz. Another effect of the structural anisotropy will be variation of permeability with direction.

Aim of this work is to investigate the dependence of ultrasonic wave propagation upon the material and structural properties of cancellous bone. Stereolithograpical (see Figure 1) bone replicas have been used for the investigation of the influences of perforation and thinning in cancellous bone on the acoustical and mechanical properties of the bone structure.

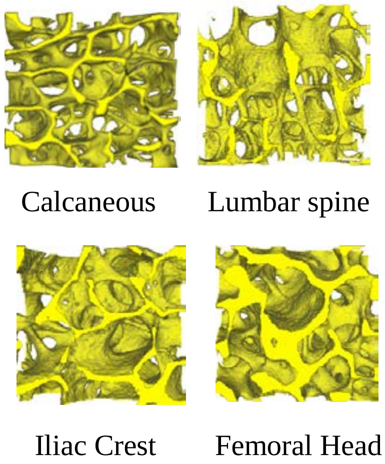


*Figure 1: Views of four stereolithographical (STL) bone replicas*

**Theory**

A porous rigid sample of length $L$ is subjected to an incident ultrasonic wave in fluid (water), $P^i$(see Figure 2). Part of incident wave is reflected back into the fluid, $P^r$, while other part is transmitted through the sample, $P^t$. Fellah *et al.* [8] have presented an analytical model in order to describe the viscous interaction between fluid and a porous elastic structure. The Fourier transform of the transmitted field is given by Fellah *et al.* [8] as:

$$P_3(x,\omega) = \tilde{T}(\omega)\exp\left(-j\omega\frac{(x-L_{TP})}{c_0}\right)\varphi(\omega), \qquad x \geq L_{TP} \tag{1}$$

where $\varphi(\omega)$ is the Fourier transform of the incident field ($P^i(t)$), $\tilde{T}(\omega)$ is the Fourier transform of the transmission kernel, $\omega$ is the angular frequency of motion, $c_0$ is the speed of sound in fluid, and $L_{TP}$ is the transmission path. A more detailed consideration of the transformed field can be found in the paper by Fellah *et al.* [8]. The transmission coefficient $T(\omega)$, which is the Fourier transform of $\tilde{T}$ .

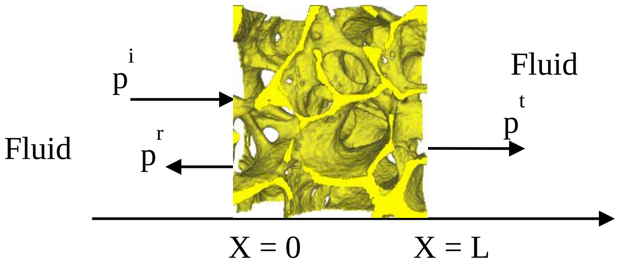


*Figure 2: Geometry of a porous rigid material*

Aygün *et al.* [15] have introduced a transverse anisotropy into Biot-Allard model by allowing angle-and-porosity dependent tortuosity, and angle-dependent elasticity. Tortuosity, defined as the ratio of the average length of the flow path through a porous medium sample to the thickness of the sample, is known to have an important influence on high frequency sound propagation in fluid saturated porous media. A heuristic form for porosity- and angle-dependent tortuosity is proposed by Aygün *et al.* [15] *as*:

$$\alpha_\infty = 1 - r\left(1 - \frac{1}{\phi}\right) + k\cos^2(\theta) \tag{2}$$

where $\phi$ is the porosity, $\theta$ is the variable between $0^o$ and $90^o$, $r$ and $k$ can be considered adjustable. A range of possible values of $r$ and $k$ have been found by comparing predictions of equation (2) for $\theta = 0^o$ and $90^o$ respectively with values deduced from air-filled replica [1] of known porosity. Values of $r$ and $k$ are found by solving the resulting simultaneous equations.

To predict transmission through an anisotropic poroelastic sample it is necessary to allow for elastic anisotropy also. The dependence of skeletal frame modulus (Young's modulus, $E_b$, Bulk Modulus, $K_b$, and rigidity modulus, $\mu_b$) in terms of bone volume fraction $(1 - \phi)$ and the Young's modulus of the solid material of the frame ($E_s$) are given by Williams [19]:

$$E_b = E_s(1-\phi)^n,$$

$$K_b = E_b/(1-2\upsilon_b),$$

$$\mu_b = E_b/(1+2\upsilon_b), \tag{3. a, b, c}$$

where $v_b$ is the Poisson's ratio of frame, and the exponent $n$ varies from 1 to 3 according to Gibson [20], depending on the angle ($\theta$) with respect to the dominant structural orientation according to $n = n_1 \sin^2(\theta) + n_2 \cos^2(\theta)$. Values of $n_1 = 1.23$ and $n_2 = 2.35$ are chosen by Lee *et al.* [5] to be consistent with the work of Williams [19].

**Measurements**

The experimental procedure used by Fellah *et al.* [5] has been followed to perform measurements in a water tank (see Figure 1). Two broadband Panametrics A 303S plane piezoelectric transducers having 1 cm diameter with 1 MHz central frequency have been used for experiments. 400 V pulses are provided by a 5058PR Panametrics pulser/receiver. Electronic interference is removed by 1000 acquisition averages. Once we start revolving the bone replica sample around itself, its thickness that ultrasonic wave transmitted through became equal to $L.\cos(\theta)$ where $\theta$ is the angle of propagation.

When a wave impinges on a STL bone replica, part of the wave is reflected back. The part of the wave penetrating into the sample undergoes mode conversion into fast and slow components which are transmitted through the STL bone replica. The measurements have been made parallel to the trabeculae direction. The stereolithographical bone replicas used in the measurements are in the form of 57 mm cubes. The incident (reference) signals generated by 100 kHz and 1 MHz transducers and transmitted through fluid (water) are shown in Figure 4a and Figure 5a, and their spectra are shown in Figure 4b and Figure 5b, respectively. Input parameters for samples tested are given in Table 1.

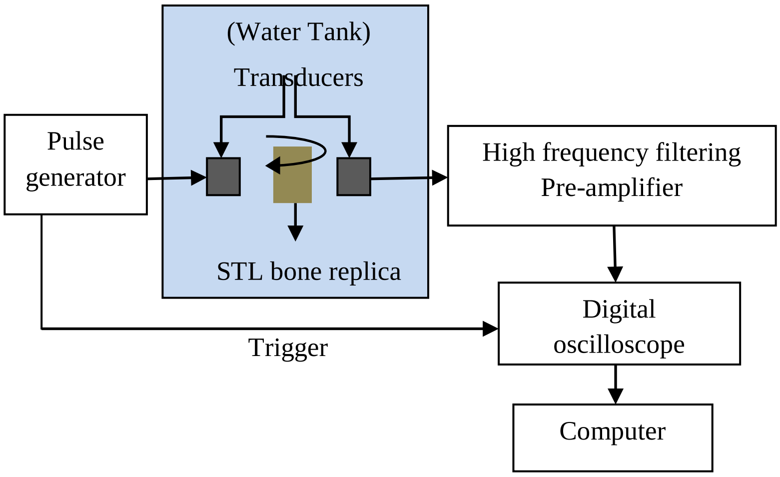


*Figure 3: Experimental setup for ultrasonic measurements.*

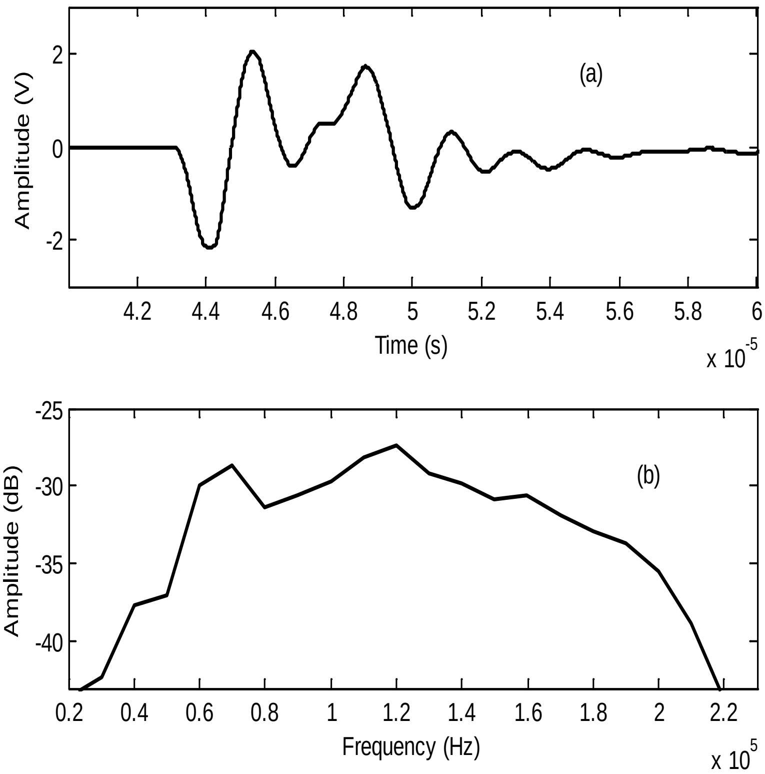


*Figure 4: a-) Inicident signal versus time, b-) its spectrum versus frequency at 100 kHz.*

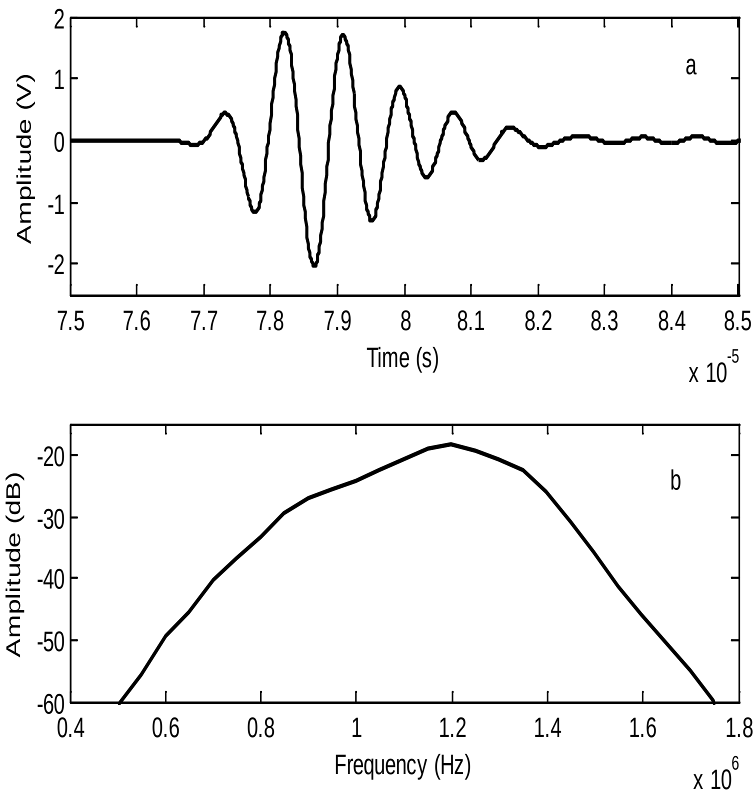


*Figure 5: a-) Incident signal versus time, and b-) Its spectrum versus frequency at 1 MHz.*

*Table 1: Input parameters for Stereolithograpical (STL) bone replicas [2].*

| **Parameters** | **Iliac Crest, ICF** | **Femoral Head, FRA** | **Lumbar spine, LS2** | **Calcaneus, CAB** |
|---|---|---|---|---|
| Density, $\rho_s$, kg/m$^3$ | 1233.4 | 1227 | 1206.6 | 1171 |
| Young's modulus, $E_s$, GPa | 6.04 | 6.04 | 6.04 | 6.04 |
| Poisson's ratio of solid, $v_s$ | 0.30 | 0.30 | 0.30 | 0.30 |
| Poisson's ratio of frame, $v_b$ | 0.36 | 0.40 | 0.38 | 0.34 |
| Porosity, $\phi$ | 0.8386 | 0.7426 | 0.9173 | 0.8822 |
| Permeability, $k_0$, m$^3$ | $845 \times 10^{-9}$ | $845 \times 10^{-9}$ | $845 \times 10^{-9}$ | $845 \times 10^{-9}$ |
| Form factor, $c$ | 1.2 | 1.2 | 1.2 | 1.2 |
| Form factor, $c'$ | c/2 | c/2 | c/2 | c/2 |
| Viscous characteristic length, $L$, µm | 100 | 60 | 220 | 150 |
| $r$ | 0.888 | 0.591 | 0.521 | 0.816 |
| $k$ | 0.468 | 0.684 | 0.143 | 0.574 |

**References:**


1. Attenborough K., Qin Q., Fagan M. J., Shin H.-C., and Langton C. M., 2005 Measurements of tortuosity in stereolithographical bone replicas using audio-frequency pulses. *J. Acous. Soc Am.* **118** 2779-2782.
2. Aygün H., Attenborough K., Lauriks W., and Langton C. M., 2010, "Ultrasonic wave propagation in Stereo-lithographical bone replicas." *J. Acoust. Soc. Am*. 127 (6), 3781 – 3789.
3. Aygün H., Attenborough K., Lauriks W., Rubini P.A., and Langton C. M., 2011, "Wave propagation in stereo-lithographical (STL) bone replicas at oblique incidence." *Applied Acoustics*. 72 (7) 458 – 463.
4. Aygün H., and Barlow C., "Ultrasonic wave propagation in porous rigid ceramics at oblique incidence" Submitted to Applied Acoustics for publication (3 January 2014)
5. Aygün H., Attenborough K., Postema M., Lauriks W., and Langton C. M., 2009, Predictions of angle dependent tortuosity and elasticity effects on sound propagation in cancellous bone. *J. Acoust. Soc. Am.* 126 (6), 3286-3290.
6. Johnell O., and Kanis J.A., 2006 An estimate of the worldwide prevalence and disability associated with osteoporotic fractures. Osteoporosis International 17: 1726–33.
7. British Orthopaedic Association, 2007 *The care of patients with fragility fracture.*
8. Burge R T, Worley D, and Johansen A, 2001 The cost of osteoporotic fractures in the UK: projections for 2000–2020. *J. Medical Economics* 4: 51–52.
9. Hughes E. R., Leighton T. G., White P. R., and Petley G. W., 2007 Investigation of an anisotropic tortuosity in a Biot model of ultrasonic propagation in cancellous bone. *J. Acoust. Soc. Am.* **121** 568–574.
10. Lee K. I., and Choi M. J., 2007, Phase velocity and normalized broadband ultrasonic attenuation in Polyacetal cuboid bone-mimicking phantoms. *J. Acoust. Soc. Am.* 121 (6) EL263-EL269.
11. Sebaa N., Fellah Z., Fellah M., Ogam E., Wirgin A., Mitri F., Depollier C., and Lauriks W., 2006 Ultrasonic characterisation of human cancellous bone using the Biot theory: Inverse problem. *J. Acoust. Soc. Am.* **120** 1816-1824.
12. Lee K. I., and Yoon S. W., 2006 Comparison of acoustic characteristics predicted by Biot's theory and the modified Biot-Attenborough model in cancellous bone. *J. Biomech.* **39** 364-368.
13. Fellah Z. E. A., Chapelon J. Y., Berger S., Lauriks W., and Depollier C. 2004 Ultrasonic wave propagation in human cancellous bone: Application of Biot theory. *J. Acoust. Soc. Am.* **116** 61–73.
14. Lee K. I., Roh H.-S., and Yoon S. W., 2003 Acoustic wave propagation in bovine cancellous bone: Application of the Modified Biot–Attenborough model. *J. Acoust. Soc. Am.* **114** 2284-2293.
15. Hughes E. R., Leighton T. G., Petley G. W., and White P. R., 1999 Ultrasonic propagation in cancellous bone: A new stratified model. *Ultrasound Med. Biol.* **25** 811–821.
16. Hosokawa A., and Otani T., 1997 Ultrasonic wave propagation in bovine cancellous bone. *J. Acoust. Soc. Am.***101** 558-562.
17. McKelvie M. L., and Palmer S. B., 1991 The interaction of ultrasound with cancellous bone. *Phys. Med. Biol.* **36** 1331-1340.
18. Biot M. A., 1956 Theory of propagation of elastic waves in a fluid saturated porous solid, I. Low frequency range. *J. Acoust. Soc. Am.* **28** 168–1178.
19. Biot M. A., 1956 Theory of propagation of elastic waves in a fluid saturated porous solid, II. High frequency range. *J. Acoust. Soc. Am.* **28** 179–191.

20. Williams J., L., 1992 “Ultrasonic wave propagation in cancellous and cortical bone: predictions of some experimental results by Biot’s Theory,” *J. Acoust. Soc. Am*. **92**, 1106–1112.
21. Gibson L. J., 1985 The mechanical behavior of cancellous bone. *J. Biomech*. **18** 317-328.
22. Tucci JR. 2006, “Importance of early diagnosis and treatment of osteoporosis to prevent fractures.” *Am J Manag Care.* May;12(7 Suppl):S181-90.
23. King A.B., Saag K.G., Burge R.T., Pisu M., Goel N., 2005, “Fracture Reduction Affects Medicare Economics (FRAME): impact of increased osteoporosis diagnosis and treatment.” *Osteoporos Int*. Dec;16 (12):1545-57.