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HOW AND WHY MIGHT SECONDARY SCHOOL MATHEMATICS TEACHERS SITUATE REAL-WORLD EQUITY ISSUES IN THE CLASSROOM?

SUMAN GHOSH

<https://orcid.org/0000-0003-1559-457X>

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Abstract

This study focused on how and why secondary school mathematics teachers might situate real-world equity issues in their lessons. Real-world equity issues in the context of mathematics education are real-world issues which might be critically examined in the mathematics lesson and so encourage pupils to be democratic citizens who are critically literate through mathematics.

The first phase of the study involved data collection through semi-structured interviews with eight teachers to learn about teachers' mathematical beliefs. Four of the participants had mathematics degrees and the others had degrees in other disciplines. An adapted version of Ernest's model of mathematics-related belief systems was used as card sort prompts for the interview. The second phase of the study collected data from observing the participants teach a mathematics lesson in which they situated real-world equity issues. It also gathered data on the participants' reflections of the lesson. The data formed the basis of eight case studies.

In order to identify the different ways in which real-world equity issues can be situated in the mathematics classroom, data from the lesson observations and teachers' reflections were used to address the question: 'How might secondary school mathematics teachers situate real-world equity issues in their lessons?'. The data was analysed by identifying the areas of the curriculum addressed in the lesson, and Skovsmose's Milieus of Learning matrix was used as a framework to analyse the structure of the lesson.

Data from interviews and teachers' reflections were used to address the question: 'Why might secondary school mathematics teachers situate real-world equity issues in their lessons?'. Ernest's model of mathematics-related belief systems was used to analyse the interview data and identify what motivated teachers to situate real-world equity issues in their lessons in the way they did.

As the cohort comprised of mathematics teachers with mathematics degrees and degrees from other disciplines, the study also analysed the data between participants to determine if there is a difference between teachers from diverse academic

backgrounds in terms of their mathematical beliefs and practices.

By drawing on the analysis, the study arrived at conclusions to provide potential ways in which teachers from diverse beliefs and academic backgrounds might be able to situate real-world equity issues in the mathematics classroom.

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1. Introduction

The study aims to investigate how and why secondary school mathematics teachers might situate real-world equity issues (RWEI) in the classroom, but what exactly do I mean by RWEI in the context of secondary mathematics? Real-world equity issues in the context of mathematics education are real-world issues, which might be critically examined in the mathematics lesson and so encourage pupils to be democratic citizens who are critically literate through mathematics. For example, pupils could examine the role of mathematics in the on-going debate about climate change and so consider how misleading data could be used to support different arguments.

My initial interest in this area of study was prompted by my teaching on the Secondary Mathematics Postgraduate Certificate in Education (PGCE) Course at London South Bank University (LSBU) from 2009 to 2012. The course at LSBU had two assessed modules. One of these was the Equality, Inclusion and Citizenship (EIC) module. All students on the PGCE course studied the module. The areas covered include Race, Gender, Social Class, Inclusion and Education for Sustainability. For the most part students understood how the issues covered in the EIC sessions would be important to them as teachers. However they found it difficult to understand how issues of equality, inclusion and citizenship could ever be relevant in the secondary mathematics classrooms (Ghosh, 2012). Subsequently, in July 2014 I ran some workshops on situating RWEI in the mathematics classroom for 75 pre-service teachers. As part of the workshop I asked the pre-service teachers to reflect on their own experiences of the teaching and learning of mathematics they had recently observed in classrooms. For the most part they identified that the majority of lessons they had seen were teacher-led episodes followed by the pupils doing exercises from textbooks or pre-prepared worksheets. A few mentioned that they had observed some lessons of an investigative nature, including some with references to real life contexts. These were mainly observed near the end of the summer term after exams had taken place. Although working on exercise based tasks can be an important part of teaching and learning mathematics for pupils, within this sample there was certainly evidence of a dominance of the tradition of exercises over other approaches.

Boaler (1993) argues that there is a place for abstract mathematics, however this can often be seen as a remote body of knowledge to many pupils. Using real-world contexts presents mathematics as a subject which can be used to understand reality. This awareness of the relation of mathematics to the real-world is recognised as a factor that motivates and engages pupils, in particular girls.

The idea that mathematics education could address RWEI, and so possibly allow pupils to become critically aware citizens, has been addressed through the broader concept of Critical Mathematics Education (CME). CME challenges the assumption that mathematics is a neutral discipline isolated from social life and politics. It argues that a socio-political perspective is an element of teaching and learning mathematics in order for it to contribute to social justice and democracy (Ernest and Sriraman, 2016). I shall be referring to 'Critical Mathematics Education' throughout this study. However, as 'Critical Mathematics Education' covers a broad range of ideas (Ernest, 2010) I have focussed specifically on the idea of how real-world equity issues can be situated within the teaching of secondary mathematics. For many this definition of RWEI will seem similar to ideas in critical mathematics education such as ethnomathematics, mathematical literacy or the wider fields of critical mathematics education and social justice. All of these are, of course, related [and](#) although RWEI is certainly subsumed within critical mathematics education, it does not fit neatly within other related fields. For example mathematical literacy is defined as providing the skills and reasoning to make sense of a world full of data so allowing people to participate in everyday life (Venkat, 2014). However, RWEI would emphasise the idea that it is not just for 'making sense' through mathematical literacy but also to become *critically* literate through mathematics education. Further, many areas of critical mathematics education are difficult to define. Ethnomathematics, for example, has had a number of definitions proposed over the years. Throughout this thesis I have used different terms when referring to the wider concepts of critical mathematics education. This is partly as these are the terms used by authors cited in the thesis. Further I used terms such as critical thinking, critical mathematical practice or teaching mathematics equably somewhat interchangeably throughout the thesis as, although there are subtle differences, it was important that were synthesised in order to present them as common ideas in this study.

There are certain necessary 'conditions' associated with the practice of critical mathematics education in the classroom. Skovsmose (1996, p1269) states that critical mathematics education is 'concerned with the development of citizens who are able to take part in discussions and are able to make their own decisions'. Therefore democratic participation is necessary for critical mathematics education to take place in the classroom. In order for this to happen Bishop (2010) identifies that the mathematics classroom should be a place of choices for the pupils. He gives examples such as pupils selecting the problems to be solved, the approaches to be taken towards a solution and the criteria for judging the solution. Similarly, in describing her dream of 'mathematics in and through social justice', Nolan (2009, p215) states that 'teaching mathematics about, or through, social justice isn't just about poverty statistics and world population figures....it's also in the thoughts and actions of the teacher toward his/her students and in the thoughts and actions of students toward each other.' These 'conditions' are, indeed, essential aspects of critical mathematics education. However, there is limited evidence of the theory of critical mathematics being put into practice in the classroom (see Chapter 2, Literature Review). Therefore, I would argue that critical mathematics education needs a starting point for potential practitioners. Nolan (2009, p210) is somewhat critical of the teachers who want practical examples of social justice issues which they could readily apply to their lessons, or lessons which included poverty or world population data. She mentions, however, that 'theory without grounding in everyday practice can sit lifeless in prospective teachers' binders, which holds little transformative potential for mathematics teaching and learning'. Furthermore she acknowledges the importance of teachers integrating real-world issues into their lessons to educate pupils about injustice and inequality in their society. However, she feels that this is 'simply not enough'. I would agree with Nolan that this is 'not enough', but I would also argue that teachers lay the foundation for critical mathematics education by integrating real-world issues into their mathematics lesson in order to address issues of equity.

Therefore, although RWEI certainly includes elements of other areas of critical mathematics, there is no specific field of critical mathematics education which addresses the notion of teaching RWEI in the secondary mathematics classroom. There

are, however, specific examples of RWEI which could be used to educate future citizens to engage in dialogue using mathematics as a powerful tool. For example, in the context of critical mathematics education, Barwell (2013) explains that, by analysing data relating to climate change pupils can critically consider arguments relating to the topic in order to arrive at their own conclusion. However, as mathematics is often seen as a value-free subject (Frankenstein, 1983) it is rarely questioned or seen as a subject that could address issues relating to real-world equity issues. Bishop (2010) implies that there is little evidence of research in this area, as we do not know what is happening with values teaching or the extent to which teachers have control over values teaching.

In my study, I want to ascertain what mathematics teachers' opinions and practice are in relation to RWEI and its place in secondary mathematics and investigate how teachers situate RWEI in their lessons. Further, as mathematics teachers are now entering the profession from diverse academic backgrounds, I will also investigate if there is any difference between the opinions and practices of teachers from non-mathematical backgrounds and teachers from mathematical backgrounds.

In doing so, I aim to address the research question :

'How and why might Secondary School Mathematics teachers situate real-world equity issues in the classroom?'

In order to answer this question, I will observe a sample of teachers to understand 'how' they situate RWEI in their mathematics lesson. Thompson (1984) suggests that teachers' views and beliefs about mathematics influence their instructional practice; however, Clark and Peterson (1986) suggest that whereas teacher action is observable, the teachers' beliefs occur in their heads and so are unobservable. Therefore, as part of this study I will also be interviewing the teachers with regards to their mathematical beliefs to understand 'why' they have decided to situate RWEI in their lesson.

The data collected from the interviews and observations will be analysed using appropriate frameworks, which I discuss in the methodology section. As part of the introductory chapter I will discuss the fact that, within the field of mathematics education, many consider it important that critical issues are addressed in the mathematics classroom. The chapter will also discuss possible reasons as to why there is little evidence of this in classroom practice. Classroom practice has to be considered in the context of the mathematics curriculum and related political influences.

Therefore, as part of this introduction I will consider the question ‘What types of knowledge should a mathematics curriculum include?’ This will also include a broad historical review of the political landscape of the UK in relation to education policy and examine the priorities of the curriculum.

1.1 Mathematics and the curriculum

As I am considering how and why secondary school mathematics teachers situate real-world equity issues in the classroom it is important to consider the curriculum within the introduction of this study as the concept of the ‘curriculum’ cannot be detached from pupils’ experience of the mathematics classroom. The word ‘curriculum’ can be interpreted as the pupils’ experience of school both in and out of the classroom (Brown, 2014).

Different people and organisations will identify different priorities for the curriculum based on their ideologies. Ball (1990) and Ernest (1991) broadly describe these different ideologies using an adapted version of William’s analytical framework (1961). These include the utilitarian aims of the ‘industrial trainers’ as well as the concerns of ‘public educators’ with democracy and social equity. The ‘industrial trainers’ could be considered as those who expect the mathematics curriculum to be made up of traditional fact and skills. Epstein et al (2010) identify this as a need for mathematics in the global knowledge economy, something that governments are primarily concerned about. However, apart from this ‘practical’ aspect of the need for mathematics in the global knowledge economy, Epstein et al (2010) also identify other aspects of mathematics, which also reflect William’s (1961) analytical framework. One such aspect is the aesthetic value of mathematics, such as seeing patterns in nature and dealing with logical problems. Another area is that of social justice. One aspect of social justice in mathematics relates to access to mathematical knowledge and future careers. Social inequalities are reinforced when people are not able to access careers which require them to cope with mathematical thinking (Boaler, 1997; Cotton, 2001; Epstein et al, 2010). The other aspect of social justice is the mathematical thinking required to understand real-world issues, such as data driven arguments, so that people are not manipulated by convincing pseudo-scientific claims, political propaganda or ‘fake news’, to use a recently coined term. Rather than call these ‘real-world issues’ I have

called these 'real-world equity issues' as an understanding of these issues from a mathematical perspective are essential in order to address social inequalities. Historically, however, issues of social justice have not featured in the mathematics curriculum (Apple, 2000).

With regards to the curriculum and the political landscape, Bartlett and Burton (2007) explain that, for the fear of appearing undemocratic as fascist governments do, UK politicians in the mid-twentieth century were reluctant to implement a prescribed curriculum. Conservative Party wisdom in the 1970s and 1980s was that the 'real' knowledge was being replaced by an 'ideological curriculum', particularly in secondary comprehensives and progressive primary schools (Ball, 1993). Ball summarises the anxiety of the Conservative Party at the time by quoting an extract from Thatcher's speech at the 1987 Party Conference:

Too often our children don't get the education they need – the education they deserve. And in the inner cities – where youngsters must have a decent education if they are to have a better future - that opportunity is all too often snatched from them by hard-left educational authorities and extremist teachers. Children who need to be able to count and multiply are learning anti-racist mathematics - whatever that may be. Children who need to be able to express themselves in clear English are being taught political slogans. Children who need to be taught to respect traditional moral values are being taught that they have an inalienable right to be gay.

(Ball, 2013, p91)

However what is real knowledge if it is not an 'ideological curriculum' where children reflect on moral, political and ethical issues? I argue that an ideological curriculum is not one where an ideology is transmitted to the pupil but one which helps the pupil to think of their own ideologies. The 'ideological curriculum' returns to the focus of my study where within the mathematics national curriculum teachers are able to contextualise real-world equity issues.

Ball (1993, p197) identified that in education, under John Major and Kenneth Clarke, by replacing the 'theoretical' and 'innovative' by 'tradition', 'the losers in the policy-making arena were a coalition of educational 'modernisers'. A loosely constituted group made up of 'new progressive' educators, especially from the science and mathematics education communities, and 'progressive vocationalists' representing the

educational concerns of many of the UK's largest multi-national companies'. These concerns remain as a 2009 Confederation of British Industry report outlined several key areas of what it termed "employability skills" (Confederation of British Industry, 2009, p8) lacking in recent graduates. The following list demonstrates the manifestation of these skills, which are more likely to be developed through 'innovative', rather than 'traditional', learning:

- i. Self-Management - readiness to accept responsibility
- ii. Team-working - negotiating/persuading
- iii. Business Awareness - taking calculated risk
- iv. Problem Solving - applying creative thinking
- v. Communication - listening and questioning
- vi. Numeracy - estimating in practical contexts
- vii. Positive Attitude - openness to new ideas
- viii. Enterprise - innovative approach

Ball (1993) goes on to describe Kenneth Clarke's classroom which has desks set out in rows with silent children listening to the teacher at the front of the class, chalk dispensing knowledge. Clarke's vision, according to Ball (1993), was not dissimilar to that of education during Victorian times. As such, 'industrial trainers' heavily influenced the curriculum at the time. In 1997 Tony Blair became the first Labour prime minister in eighteen years, but David Blunkett, the Secretary of State for Education, also echoed the views of 'industrial trainers' as his aims seemed to focus on the ability of children to 'know their tables' and 'do basic sums' (Brown, 2014).

Under the ten year leadership of Tony Blair, the British economy would enjoy a record run of unbroken economic growth. He became Labour's longest-serving Prime Minister and led the Labour Party through three consecutive general election victories. Blair's commitment to education was made clear in the statement:

Ask me my three main priorities for government, and I tell you: education, education, education.

(The Guardian, 2001)

Amongst the education reforms introduced, under Blair, were a reduction in class sizes and more investment in education. A source of controversy was the introduction of 'academies', schools partly financed by outside sponsors, in order to replace failing schools.

There had been a total increase in government spending on education from £29 billion in 1997 to £60 billion in 2007. A statistical summary of Labour's 1997 election pledges for education established that, for the most part, Labour's pledges had been met.

In order to illustrate how they met their election pledges I will summarise Labour's education record between 1997 and 2007 (de Waal, 2009):

- During the academic year 1997/1998 spending on education rose from 4.9% of GDP to 5.6%.
- Spending per pupil rose from £2,910 to £5,430.
- The number of primary school teachers rose by 2.8%,
- The number of secondary school teachers rose by 14.5%
- The number of special education teachers rose by 17.7%.
- The number of pupils achieving 5 or more A*-C grades rose from 46.3% to 65.3%
- The number of A-level passes increased from 87.2% of all A-level entries, to 97.2%.
- The number of A-C grades at A-level rose from 55.7% to 73.9%.
- An improvement in the number of A grades awarded from 15.7% of entries in 1997 to 25.9% of entries in 2008.

There is clear evidence of a resounding success in terms of the investment in education and the associated results. However, much of this success was associated to a 'teaching to the test' culture. While recognising that test results had improved, Webb and

Vulliamy (2006, p153) expressed concerns that, “High stakes testing, which holds schools and teachers accountable for pupil attainment in literacy and numeracy, has narrowed the curriculum, diminished opportunities for teachers to develop the whole child, caused considerable stress for many children and changed the basis of teacher-pupil relationships. Teaching in ways that are not in the ‘best interests’ of children and contrary to their professional judgement in order to boost test results has compromised primary teacher professionalism’.

Little changed in 2010 with the new coalition government. Nick Gibb, the Conservative junior minister with the responsibility of drafting a new curriculum, advocated views similar to those of Blunkett and Clarke, in that all that was needed to stipulate the Primary Curriculum was facts, tables and written procedures (algorithms) (Brown, 2014).

Indeed in 2012 a key finding of Ofsted’s ‘Mathematics: Made to Measure’ report was:

While the best teaching developed pupils’ conceptual understanding alongside their fluent recall of knowledge, and confidence in problem solving, too much teaching concentrated on the acquisition of disparate skills that enabled pupils to pass tests and examinations but did not equip them for the next stage of education, work and life.
(Mathematics: Made to Measure, Ofsted Report Summary, 2012, p4)

Another aspect of the curriculum, which has always been addressed by successive governments, is the themes of citizenship and morals. Currently schools are legally obliged to provide a Spiritual, Moral Social and Cultural (SMSC) education. However, recent findings by the Royal Society for the encouragement of Arts, Manufactures and Commerce (RSA) report that despite the fact that’s schools have a legal commitment towards providing SMSC education this requirement to develop the broader human qualities of their pupils has become side-lined due to the pressures of delivering better exam results. Further, school provision for SMSC has a ‘scattergun approach’, which lacks any underpinning knowledge. This situation runs the risk of SMSC being ‘everywhere and nowhere’. Indeed, although the term SMSC first appeared in the 1988 Education Act, how each of these components are promoted in education remain an issue (Royal Society for the encouragement of Arts, Manufactures and Commerce, 2014, p16

1.2 Justification for this research

I started this section by identifying that different people and organisations have different views as to what types of knowledge the mathematics curriculum should include. It seems clear that the curriculum is heavily influenced by the nature of the examinations and the utilitarian aims of ‘industrial trainers’ who endorse a curriculum made up of traditional fact and skills. However, this section also identifies that there is a body of teachers and academics, like myself, who endorse the inclusion of concepts of democracy and social equality in the mathematics curriculum. Further, it reveals that education relating to morals and ethics is, for the most part, not promoted in the mathematics classroom. By researching the question ‘How and why might Secondary School Mathematics teachers situate real-world equity issues in the classroom?’ I want to identify the different ways in which it is possible to situate RWEI in the secondary mathematics classroom and why some teachers decide to do this. As mentioned in the previous section, a mathematics teacher’s practice is influenced by their mathematical beliefs, so I have identified the following questions as important in contributing to the main research question:

How and why might Secondary School Mathematics teachers situate real-world equity issues in the classroom?

- How might teachers situate RWEI in the secondary mathematics classroom?
- What are the mathematical beliefs of early career secondary mathematics teachers who are interested in teaching RWEI in their lessons?
- Are there differences in mathematical beliefs of early career teachers from diverse academic backgrounds?

As I have mentioned earlier, the first question is tied in with the second and third question as the different ways in which secondary mathematics teachers might situate RWEI in their classrooms will relate to their own mathematical beliefs. Therefore, the findings reported will be more informative when considering all three research questions.

2. Literature review

This chapter reviews the relevant literature in order to provide a theoretical background to the concepts within this study. Real-world equity issues in mathematics cannot be researched in isolation from other influences. Teachers teach mathematics lessons and so this influences how real-world issues are taught, or even not taught, in the classroom. The other influence is the curriculum children experience in the classroom, which in itself is likely to be influenced by the examinations pupils are prepared for (Brown, 2014).

The structure of the literature review reflects the relevant influences on RWEI in the classroom. The first section explores the wider field of mathematics in society with a specific focus on critical mathematics education. The second section explores the wide ranging beliefs held by mathematics teachers with regards to the subject. The third section explores how the curriculum and resources used in the secondary mathematics classroom can influence RWEI in classroom practice. The final section explores literature which has identified potential challenges in the field of teaching RWEI in the mathematics classroom.

2.1 Mathematics in Society

By considering RWEI in the secondary mathematics classroom I am working within the field of mathematics in society. This section reviews the literature relating to this field and considers the arguments put forward for and against teaching mathematics from a critical perspective in the context of social justice.

Historically mainstream mathematics education does not have a tradition of critically examining connections between mathematics as an area of study and its relation to unequal economic, political and cultural power (Apple, 2000). Indeed, while critical mathematics education is often theorised, this rarely transfers to the chalkface (Ernest, 2016). Although there is certainly potential for mathematics to address issues of equity, there is little research that examines mathematics teachers learning to teach for social justice. There is a significant amount of research relating to teachers in other disciplines learning to teach such issues (Bartell, 2013). This is reflected further in the

findings of Robbins et al (2003) where they concluded that trainee mathematics teachers, when compared to teachers of eleven other secondary subjects, had the least positive attitude towards education for global citizenship. In relation to particular issues, such as sustainability Barwell (2013, p6) explains that although ‘the role of mathematics in describing, predicting and communicating climate change does connect indirectly with many areas of research in mathematics education, so far there has not been any research that addresses how mathematics teaching might contribute to climate change education’.

There is evidence of a growth of interest in adopting diverse social practices into mathematics education, as illustrated in ‘The Social Turn in Mathematics Education Research’, Lerman (2000). These include a cross cultural view on mathematical practices (Bishop, 1998), a sociocultural basis for mathematics education (D’Ambrosio, 1985), critical mathematics education (Skovsmose, 1990) and sociology and mathematics education (Dowling, 1998). According to Atweh (2007), while these agendas often vary in their conclusions they share a common characteristic of rejecting the view that mathematics is a singular, objective and value-free discipline that is isolated from human interest. In terms of teaching mathematics, Atweh (2007) feels that these agendas challenge the dominant and traditional teaching practices in mainstream classes around the world.

Lerman (1990) states that the role of mathematics in society implies that mathematics is culture-laden or value-laden. This contradicts the mathematics we might experience in schools where a culture or value laden approach may be seen as political, and therefore an abuse of mathematics. However, people with power use mathematics to make decisions; so it is important that children are empowered to analyse and criticise mathematical information, so that they can pose questions and potentially make changes in society (Lerman, 1986). Similarly, Cotton (2001) identifies mathematics as a powerful tool which can be used to interpret and question our world. He gives the example of how we regularly use mathematics when we come across data in the media. However, to what extent is this data interpreted from a critical perspective? Cotton (2001) gives the example of unemployment data. He explains that one civil servant will use a particular mathematical model to present unemployment figures, whereas another civil servant could present different figures and a different picture of

unemployment using an alternative mathematical model. This type of RWEI could be situated in the mathematics classroom, where pupils discuss how the same information could be presented in different ways by manipulating data.

Although there may be many arguments put forward for teaching mathematics and the type of knowledge that should be in the mathematics curriculum, Lerman (2000) identifies the curriculum as broadly driven by the following views, 1) an authoritarian view, which instils ideas of an agreed set of moral values and ways of behaving; 2) a neo-liberal view, which prepares citizens in a democratic society for useful, wealth-producing lives; 3) a more old-liberal agenda which educates citizens so that they are able to fulfil their lives to the best of their abilities; 4) a more radical agenda which prepares people to critique and change the society around them. Venkat (2014) directs these views into two broader arguments that can be seen to lie at the centre of a debate. One argument is that mathematics teaching lays the foundations for the future learning of mathematics, such as post-compulsory mathematics. Another argument is that it provides the skills which allow people to engage in a world in which they are exposed to quantitative data on a daily basis. For Venkat (2014) the second argument leads to mathematically literate people who are equipped to engage with the quantitative claims and the mathematics in everyday life. In order for learners to be mathematically literate Venkat (2014) proposes that the mathematics teacher should provide opportunities in the classroom by drawing on issues in everyday life and adapting them into tasks for the mathematics classroom. In my opinion, both arguments are valid and not mutually exclusive. However, as mentioned earlier, the second argument might be sidelined because the pressure to perform well in exams may force schools to disregard broader moral, social and cultural issues in education (RSA - Schools with Soul Report, 2014). Therefore, in order to encourage more schools to engage with the mathematics of everyday life we need to be better informed as to how and why some teachers situate RWEI in their classroom.

The importance of situating RWEI in the mathematics classroom is emphasised by Frankenstein (1983) who argues that mathematical literacy is essential to work towards social change in an advanced technological society. Frankenstein approaches 'mathematical literacy' through the re-invention of Paulo Freire's (1968) critical education theory in the context of a mathematics curriculum. Frankenstein (1983)

gives an example of how mathematical illiteracy can lead people to believe that their declining standard of living is due to social welfare programmes when tax loopholes for rich taxpayers are not researched or criticised in the same way. Tutak et al (2011) explains how mathematical literacy is critical in order to reflect on the ways in which numbers dominate and liberate. This relates to Freire's works (Freire, 1968), which stress that critical literacy requires reading the word and the world simultaneously. In both cases rather than processing texts or numbers there should be, as far as possible, an interpretation of social issues affecting the learner. Similarly, Bartell (2013) explains that teaching mathematics for social justice provides pupils with the mathematics which is considered to be necessary to succeed in the current system, in addition to equipping them with the mathematical knowledge for confronting economic, political and social obstacles to their success.

Although there seems to be a clear argument for situating RWEI in the mathematics classroom my experiences of student mathematics teachers reveal that many of them are reluctant to take risks and encourage learning processes such as RWEI. In a small scale study, Ghosh (2012) reports that although there were positive statements from student mathematics teachers about situating issues such as sustainability in their lesson, many were unenthusiastic about such ideas:

Student A: Another political topic that should stay out of the mathematics classroom.
We have other priorities

Student B: Some of my students cannot write or read or do 8×2 in their head. I have other priorities.

Student C: Could use them in examples, but do not feel it's important, maybe more important in PSHE

However, Ghosh (2012) concluded that although these comments appear to reveal a negative approach to incorporating ideas such as sustainability into mathematics lessons, the comments contained hints to the contrary. Phrases such as 'we have other priorities' imply that the respondents were not averse to situating issues such as sustainability and global citizenship in their lesson but felt it would interfere with the mathematical content of the lesson. This supports Ernest's (1991a) assertion that, although a teacher's philosophical perspective will influence their teaching strategies

in the classroom, social context has a huge influence on this. Encouraging critical mathematics, in practice, is much more difficult if peer groups and the ethos of schools are not persuaded by this approach. Further pressures, such as teaching to the exam and following the curriculum, leave schools reluctant to take risks. When faced with these restrictions teachers are compelled to 'shift their pedagogical intentions and practices away from their espoused theories' (Ernest, 1991, p. 289).

Although mathematics is a subject which can allow pupils to structure and solve logical problems through democratic participation, Skovsmose (1990) identifies that the 'rituals' in the mathematics classroom take a different direction. Referring to this as the 'hidden curriculum' Skovsmose (1990,p114) gives the example of how the language used in teaching mathematics compels pupils to follow explicit and prescriptive statements, such as:

'solve the equation.....'

'find the length of.....'

'calculate the value of.....'

The language is instructive, and promotes the use of routine processes. It does not, in any way, encourage creative or investigative work in mathematics. Likewise, Bishop (2010, p236) proposes that teachers should be encouraging students to make choices by allowing them to select the problems to be solved and deliberating about the approaches to the solution. Hence, instead of using the language of the 'hidden curriculum', identified by Skovsmose (1990), teachers could use language such as 'Describe....' and 'Compare....', so encouraging democratic participation.

In relation to democratic participation and in considering the place of real life references in the mathematics classroom I shall refer to Skovsmose's Milieus of Learning matrix (Skovsmose, 2001) (Table 1). By combining different types of references to the nature of a lesson and the paradigm they belong to the matrix presents the six milieus of learning. The nature of the lesson refers to the context of the mathematical task and whether it is in the context of reality, semi-reality or pure mathematics, without any contextual reference. The two paradigms, 'Tradition of exercises' and 'Landscapes of investigation', represent contrasting classroom practices.

Skovsmose (2001) describes the exercise paradigm as exercises in textbooks, formulated by an authority and with a premise of one right answer. In contrast, the investigative approach can take many forms and is located in a 'landscape' which opens up opportunities to make investigations, thereby addressing competences such as interpretation and problem solving, so emphasising that mathematics is not a subject where concepts are taught and learnt.

	Tradition of exercises	Landscapes of investigation
References to pure mathematics	(1)	(2)
References to a semi-reality	(3)	(4)
Real-life references	(5)	(6)

Table 1 : Milieus of learning (Skovsmose, 2001, p126)

Although Skovsmose (2001) recognises that the matrix represents a strong simplification and that there are huge possibilities within the vertical line separating the exercises paradigm from the landscapes of investigations, the matrix serves to facilitate discussion about mathematics education. For example, Skovsmose (2001) mentions how Denmark could operate in the learning milieu (6) as there were no exams, where pupils can fail, until the 9th year. Hence classroom activities, certainly up to the 9th year, were not influenced by end of year exams. Conversely, in classrooms in England, Cotton (1998, cited in Skovsmose, 2001) describes how he observed that the mathematics lesson is divided into two parts with the teacher presenting some mathematical ideas and techniques in the first part and pupils working through selected exercises for the second part, therefore predominantly operating in the learning milieu (1). Similar findings were made by Swan (2005a, p206) in his study where 779 students identified the most frequent behaviour they experienced in the classroom. I have listed some of the most commonly identified, which once again suggest a predominance of the learning milieu (1) where lessons are lessons taught in the traditions of exercise in the context of pure mathematics.

“I listen while the teacher explains.”
“I copy down the method from the board or textbook.”
“I only do questions I am told to do.”
“I work on my own.”
“I try to follow all the steps of a lesson.”
“I do easy problems first to increase my confidence.”
“I copy out questions before doing them.”
“I practise the same method repeatedly on many questions.”

These behaviours suggest an environment in which the learner is not encouraged to discuss, debate or work together with others. The role of the teacher is not to facilitate, but to instruct. Indeed, school mathematics is a set of isolated procedures and techniques to learn by rote (Swan, 2005a). These also reflect the features of Paulo Freire’s (1968) banking system which I mention later in the next section.

Ideas of RWEI in the mathematics classroom would certainly feature predominantly in milieu (5) and (6) as they are based on real life references. With regards to the matrix (Table 1), Skovsmose (1990) recognises that it is important to support a mathematics education which moves between all the different milieus. Consolidation exercises, which would feature in learning milieu (1), are as important as investigative work based around real life examples. However, it seems that learning milieu (1) is predominant in English classrooms (Swan, 2005a; Cotton, 1998) and therefore there is a need to address other learning milieus such as (6), which are lessons of an investigative nature with reference to real life contexts.

Conversely, there are also counter arguments that propose that mathematics should be a subject that is value free and I will explore these in the following paragraphs. The positivist view of knowledge as neutral, value-free, and objective could be seen as directly opposing the aims of ‘critical mathematics education’ (Frankenstein, 1983). Many believe mathematics to be a value-free subject (Bishop, 2010; Tutak et al, 2011) partly as it is seen as a subject based on unquestionable rules and right and wrong answers. In order to challenge the idea that mathematics is based on unquestionable rules, Tutak et al (2011) discuss the definition of a trapezoid. One definition of a

trapezoid states that it is a quadrilateral, with exactly one pair of parallel sides. Another definition states that a trapezoid is a quadrilateral with at least one pair of parallel sides.

There are also arguments that real life examples can be ineffective. Dowling (1999) explains why examples of semi-reality contexts are more appropriate for the school context than examples of reality. He gives the example of tomato puree priced at 100g at 31p and 200g at 37p. It was his intention to buy 200g but this was sold out. The options open to him were to buy two 100g tubes at 62p or try another shop. However, as he valued his time over his money he decided not to try another shop but go for a third option of convincing the store manager sell him two 100g tubes for a total amount of 37p. The decisions made were made in real time without entering a mathematical discourse. Dowling (1999) explains that this type of discourse does not privilege mathematical discourse; hence real life scenarios might not be appropriate in the mathematics classroom. Dowling (1999) gives another example which implies a best buy decision that is not commonly used, particularly as supermarkets now report prices per gram on different brands and packet prices:

Here are two packets of washing powder. The small size contains 930g of powder. It costs 84p.

The large size contains 3.1 kg of powder. It costs £2.56.

(a) How many grams do you get for 1p in the small size?

(b) How many grams do you get for 1p in the large size? (Remember you must work in grams and pence.)

(c) Which size gives more for your money?

(SMP 11-16 Book G7; p2)

The washing powder example emphasises the structural and strategic differences between the supermarket and school mathematics. Whereas the school privileges a body of knowledge against which pupil performance is assessed, the supermarket privileges goods and services; so, whereas the school asks 'Which size gives more for your money?', the question in the supermarket is 'Which would you buy?'. (Dowling, 1999). Like the tomato puree situation, the second question introduces non-mathematical decisions, such as, 'Can I afford the large size container?', 'Do I have space

for it where I live? So Dowling (1999) emphasises that if we want to teach someone how to shop we should take them to the supermarket. Dowling's (1999) arguments are certainly valid but serve more as a warning when using real life examples in mathematics rather than an argument not to use them at all. Examples, like the two that Dowling (1999) is critical of, can be found in numerous mathematics textbooks. Rather than support the use of real-world equity issues, they provide poor examples from the 'real-world'. I will go through some examples in Section 2.3 Curriculum and Resources.

There are further examples of the negative effects of mathematics set in a cultural context, such as the implementation of ethnomathematics in South African schools during apartheid (Skovsmose and Vithal, 1997). Skovsmose and Vithal (1997, p146) explain how there are 'conflicts in classrooms related to the different backgrounds of students and how students value their own and others' backgrounds. In South Africa bringing students' backgrounds into the classroom could come to mean reproducing those inequalities in the classroom. The aspect of conflict must be elaborated in ethnomathematics, as it forces us to consider the question: does bringing cultural contexts into the curriculum reconcile or exacerbate differences and conflicts?'. Referring to the Skovsmose and Vithal (1997) study, Pais (2010, p213) emphasises how ethnomathematical ideas were employed in certain non-white South African schools during apartheid. This resulted in the implementation of a 'lighter mathematical curriculum' for these pupils. Hence these children were ultimately excluded from accessing the formal academic mathematics aimed at white pupils. Consequently, Rowlands and Carson (2002, p98) argue, '[t]here is every danger that mathematics as an academic discipline will become accessible only to the most privileged in society and the rest learn multicultural arithmetic within problem solving as a life skill or merely venture into geometric aesthetics'.

There are further arguments that situating critical mathematics education in the curriculum could replace academic mathematics (Rowlands and Carson ,2002). Indeed, Rowlands (2007) identifies mathematics as very different to subjects such as history and geography, as the beauty of mathematics is that its theoretical aspects can be embraced as tools to cultivate pupils' minds, independent of the harsh realities of everyday life. Mathematics allows the pupil to escape from the realities which are

addressed in many other subjects.

These are all valid arguments. Mathematics can be studied for its own sake with little reference to the real world and its issues. However, mathematics is a powerful tool and there are many opportunities within the subject for it to allow pupils to critically assess their world, and not utilising this aspect of mathematics will leave pupils mathematically illiterate in a world surrounded by numbers and data. Hence, Apple (2006) argues that whilst there is certainly an aesthetic aspect of mathematics in relation to the importance of imagination and intuition, there are compelling reasons as to why it can also be political. For example, the mathematics we might expect to see in location (1) of Skovsmose's Milieus of Learning (Table 1) could be exercises of the form of:

Expand:

<i>a. $4(x + 2)$</i>	<i>b. $3(q - 5)$</i>
<i>c. $7(2m + 1)$</i>	<i>d. $-2(y + 6)$</i>

GCSE Higher Mathematics Textbook (Pearson Education, 2015, p33)

This type of exercise is important as part of learning and any attempt to address this type of mathematics through references to reality or semi reality might seem contrived.

How then can a secondary mathematics teacher situate RWEI when teaching mathematics, without making it look like a contrived add on? With regard to research literature, the area of critical mathematics is well addressed in mathematics education. As well as a theoretical perspective, there are also publications that focus on how these ideas can be used in the classroom. The book 'Rethinking Mathematics' (Gutstein and Peterson, 2005) provides examples of how to teach mathematics in a way that relates to the pupils' lives and surroundings; it 'rethinks' classroom mathematics by threading social justice issues throughout the mathematics curriculum including ideas for cross-curricular mathematics. Another book, 'Teaching Secondary Mathematics as if the Planet Matters' (Coles et al, 2013), looks at wide range of global issues which can be explored within different areas of the mathematics curriculum. These types of examples contradict the idea that real life mathematics is not suitable for the classroom.

Coles (2013, p111) gives an example of a population growth question which addresses real-world equity issues in mathematics:

Population growth

Using the population estimate data (Table 2) students could estimate current global population and predict the likely population in 2050. Students could draw a graph of the data and notice that the data is linear. Students might ask questions about the data at this stage. They might extrapolate to predict future population estimates.

Table 2 : Population estimate data

Year	Population Estimate	Population
1750	640,000,000	0.64
1800	970,000,000	0.97
1850	1,300,000,000	1.30
1900	1,630,000,000	1.63

Source: U.S Census Bureau, <http://www.census.gov/population/international/data/idb/worldhis.php>

In fact this data was *chosen* within acceptable bounds of historical estimates (Table 3). Students can now think about the use of numbers and algebra at a meta level.

Table 3 : Population bounds

Year	Population lower bound*	Population upper bound*
1750	0.63	0.96
1800	0.81	1.13
1850	1.13	1.40
1900	1.55	1.76

* in billions

Not only does this example set out the possibilities of working with algebra and numbers within a real life context, but it also raises a critical awareness for the learner. It may lead to learners taking a critical approach to population data and how they could be manipulated. They could ask questions such as who would manipulate data and why. Further questions could be asked such as which countries are 'responsible' for increases in population, and whether over population or over consumption is the greater threat to the earth.

However, even with these ideas and with related research literature, a tradition of critical mathematics education seems absent from the classroom. Apple (2006) identifies that where there have been gains, the materials do not relate to the daily realities of teachers' and pupils' lives.

In conclusion, whereas there are clear arguments centred around mathematics and its role in addressing particular issues in society, there are identifiable reasons as to why there is little evidence of this in school mathematics. Further, opinions are divided as to whether mathematics is a value-free subject or a subject that can be used as a tool to raise children's critical awareness of their society.

2.2 Beliefs and Values

In exploring mathematics in society in the context of RWEI, I have discussed why I think addressing RWEI in the classroom is important and also looked at some of the arguments as to why this should not be the case. In doing so, I have identified that there are different beliefs about mathematics teaching. Therefore, as I mentioned in the introduction, as well as observing teachers' practice, I will also consider their beliefs. This section reviews literature relating to teachers' beliefs. Teachers' judgements and perceptions are influenced by their beliefs, and these affect their behaviour in the classroom (Pajares, 1992). An understanding of the beliefs of the teachers in my study will provide an insight into 'why' some teachers situate RWEI in their lesson. Historically, the idea of 'beliefs' is not generally addressed in educational research, as it does not present obvious opportunities for empirical investigation. Further, in order to understand beliefs we must appreciate their situated nature; in other words, certain beliefs may only apply to particular situations and so there may be contradictions in teachers' beliefs (Swan, 2006).

As I am considering values teaching in the context of mathematics, it is important to make an explicit link between values and beliefs. Bishop (2010) identifies that in order to deal with issues of democracy in mathematics education, there is clearly a requirement to engage with values. However, Bishop (2010) identifies this to be problematic, since, unlike teachers of humanities and arts subjects, where the

discussion and development of values seem relatively easy, most mathematics teachers would not consider teaching values when they are teaching mathematics. Indeed, teaching explicitly about values in the mathematics classroom is rare, as there is a widespread belief that mathematics is a value-free subject. Further, Bishop (2010) argues we do know what is happening in the classroom in relation to values teaching. Seah (2010, p243) asserts that 'if values represent the deeply held beliefs and they represent the core of what one considers to be important in one's life, then it might be more effective and more productive to influence change (in ways of teaching and of learning) by facilitating value inculcation and development amongst students and teachers through their existing beliefs'.

In researching teachers' beliefs, Swan (2006) developed a set of teachers' characterisations based on descriptions from Ernest (1991a) and Askew et al (1997). In explaining the differences between mathematics teachers, Ernest (1991a) suggests that while knowledge is an important factor, there needs to be an emphasis on teachers' beliefs. These beliefs concern the teachers' conception of the nature of mathematics. For example, if we experience one teacher's practice as situated in learning milieu (1) of Skovsmose's matrix (Table 1) (teaching through traditional exercises, mainly with reference to pure mathematics), and another teacher as encouraging problem solving, this might not be because of their knowledge but because of their beliefs. Ernest categorises these beliefs into three broad components:

1. View or conception of the nature of mathematics
2. Model or view of the nature of mathematics teaching
3. Model or view of the process of learning mathematics

These components form the basis of a teacher's beliefs, or philosophy, of mathematics education. Ernest (1991a) broadly classifies these philosophies as: Instrumentalist, where mathematics is seen as a set of facts and rules; Platonist, in which mathematics is discovered and not created; and, thirdly, a problem solving and enquiry based approach to mathematics. Askew's model (1997) characterises teachers' conceptions of the teaching and learning of mathematics through three factors:

1. Transmission
2. Discovery
3. Connectionist

These three factors are similar to the components which form the basis of Ernest's classification of teachers' beliefs. The 'transmission' approach views mathematics as a subject with 'rules and truths' which are communicated through teacher-led classrooms and individual practice. In the 'discovery' approach, the teacher takes the role of a facilitator and encourages pupils to learn through individual exploration. Finally, the 'connectionist' approach sees pupils working in collaboration, viewing mathematics as a network of ideas where the teacher has a role in challenging the pupils.

Further categories of students' conception of mathematics were developed by Crawford et al (1994, p355) for their 'conceptions of mathematics questionnaire' which was used in an ongoing study of student learning in a first year mathematics course. The study identified the qualitative variation of the conceptions, based on the question: 'Think about the mathematics you have done so far. What do you think mathematics is?'. The conceptions were:

- A. Mathematics is numbers, rules and formulae.
- B. Mathematics is numbers, rules and formulae which can be applied to solve problems.
- C. Mathematics is a complex logical system and way of thinking.
- D. Mathematics is a complex logical system which can be used to solve complex problems.
- E. Mathematics is a complex logical system which can be used to solve complex problems and provides insights used for understanding the world.

The conceptions above relate to the nature of mathematics rather than the process of learning mathematics.

The categories of beliefs identified or used by Askew, Ernest and Crawford clearly evidence that the beliefs in mathematics education are extensive (Handal, 2003), and in their extremes can be seen as two epistemological perspectives; the 'absolutist' view and the 'fallibilist' view. As teachers' beliefs are central to this study, it will be

important to identify how these two epistemological perspectives relate to classroom practice. Ernest (1991a) describes the absolutist view of mathematics as an incorrigible, objective and certain body of knowledge resting on foundations of deductive logic. The absolutist philosophy of mathematics includes the schools of Platonism, Logicism, Intuitionism and Formalism. According to the absolutist view of Mathematics, all mathematical knowledge consists of unchallengeable truths (Ernest, 1991a). Absolutist mathematics, therefore, is based on the idea of a fixed and certain knowledge that is absolute, objective and rational. Absolutism can take different forms, and Lerman (1990) gives examples such as the embedding of mathematics within logic and the construction of mathematics from the basic intuition of time and the natural numbers. For example, the beliefs espoused by Rowlands and Carson (2002), discussed in the previous section, in which they suggest mathematics is a purely theoretical subject, fit with the absolutist view of mathematics.

Alternative to the absolutist view is the fallibilist philosophy. In many ways, fallibilism can be seen as a reaction, or response, to the absolutist view. In describing fallibilism, (Hersh, 1979, cited in Ernest, 1991a, page 19) explains: 'It is reasonable to propose a new task for mathematical philosophy: not to seek indubitable truth but to give an account of mathematical knowledge as it really is – fallible, corrigible, tentative and evolving, as is every other kind of human knowledge'. As the absolutist philosophy of mathematics considers mathematics to exist independent of the history of mathematics, or indeed anything else, learning mathematics becomes a practice of memorising facts. According to an absolutist philosophy, mathematics is a body of knowledge. It is, therefore, a case of applying the correct methods in order to answer questions. A mathematics curriculum based on an absolutist philosophy would be content-based with a possible emphasis on memorising facts in order to be able to regurgitate them; this would be a curriculum predominantly taught using traditional exercises with reference to pure mathematics, so situated in learning milieu (1) (Traditions of exercise in a pure mathematics context) of Skovsmose's framework. An absolutist-like view may be communicated in school by giving students mainly routine mathematical tasks in which they are expected to apply a set of learnt procedures and arrive at a fixed answer. Failure to achieve the correct answer could be met with disapproval and criticism (Ernest, 2007).

Lerman (1983, p62) describes the teaching style of an absolutist as one where mathematics is essentially geared towards an esoteric view of the subject and becomes a 'steadily accumulated body of knowledge, linear or hierarchical, dependable, reliable and value-free'. He gives examples of where the teacher, when asked why a particular topic is being studied, is likely to reply:

'Because it's on the syllabus'

'You will see its intrinsic value later on when you understand its connection with the rest of mathematics, as I do';

'It is used in all sorts of ways, ask me later, and I'll give you an example';

'Because I say so'.

These replies imply that pupils learn a system of mathematics and at some point in the future, for example in employment, learn to apply the mathematics.

The points made here, such as the teacher acting as an authority of knowledge and the narrative nature of the teacher, are characteristics also identified by Freire (1968). Through a careful analysis of the student-teacher relationship, Freire (1968) identified the relationship to be essentially characterised by its richness of commentary and explanations. Freire gives the example of how '[f]our times four is sixteen' is a fact which the student records, memorises and repeats without understanding what 'four times four' really means. Freire (1968) refers to this as the 'banking system' where the 'narrator' (teacher) leads students to mechanically memorise content. The 'banking system' therefore turns students into 'containers' waiting to be filled by the 'depositor'. By being asked to store the information, they are 'trained' to store knowledge but are not 'taught' to be reflective, creative or critical.

Freire (1968, p73) identifies the following attitudes and practices as characteristics of the banking system:

- i. The teacher knows everything and the student knows nothing;
- ii. The teacher thinks and the students are thought about;
- iii. The teacher talks and the students listen;
- iv. The teacher disciplines and the students are disciplined;

- v. The teacher chooses and enforces the choice and the students comply;
- vi. The teacher acts and the students have the illusion of acting through the teacher;
- vii. The teacher chooses the program content, and the students (who are not consulted) adapt to it;
- viii. The teacher confuses the authority of knowledge with his own personal authority, which he sets in opposition to the freedom of the students;
- ix. The teacher is the subject of the learning process, while the pupils are mere objects

Mathematics, according to Lerman (1994), is a creative human endeavour which is growing, changing and fallible. As such, Lerman (1994) suggests that mathematics education should empower pupils to question and criticise. In order for this to happen, students should be given the independence to develop skills such as proposing their own ideas, developing their own methods and testing their hypotheses (Lerman, 1983). Lerman's suggested model of mathematics education reflects Freire's (1968) solution to the 'banking system' of education. Freire (1968) suggested that 'banking education' could be replaced with 'problem-posing' education. Whereas 'banking' education resists dialogue, treats students as objects of assistance and inhibits creativity, problem posing education regards dialogue as important in the act of cognition, making learners critical thinkers. Central to 'Problem posing' education is the change in the relationship between the student and the teacher. The teacher no longer teaches from a point of authority relating to a knowledge base, but rather becomes a participant in the dialogue with the students. The classroom can become a place where children are relaxed and mathematical conversations can take place. Taking a fallibilist approach, the teacher can encourage the students, and indeed other students, to test their answers and ideas with other examples and counter examples. The following task from the Standards Unit (Swan, 2005b) is an example of such an approach:

Is the following statement always, sometimes or never true?

$$xy = x+y$$

- if it is sometimes true, then write examples around the statement to show when it is true and when it is not true;
- if it is always true, then to give a variety of examples demonstrating that it is true, using large numbers, decimals, fractions and negative numbers if possible;
- if it is never true, then to say how we can be sure of this.

This is a stark contrast to the ‘disapproval and criticism’ of any failure to achieve an answer, which Ernest (2007, p7) identified with the absolutist approach.

However, Ernest (2007) emphasises that many mathematicians are attracted to mathematics for its absolutist features and, therefore, it would be wrong to see this approach as inappropriate. Indeed, a teacher could hold an absolutist view of mathematics but realise that it is necessary to adopt a fallibilist approach in the classroom. Skemp (1978), too, discusses the possible reasons as to why a teacher might adopt a ‘rules without reason’ approach. Skemp discusses mathematical understanding as ‘relational’ and ‘instrumental’. Similar to the absolutist and fallibilist approach, ‘relational understanding’ is knowing both what to do and why, whereas ‘instrumental’ understanding is ‘rules without reasons’. As Skemp identified that so many teachers teach instrumental mathematics he, playing devil’s advocate, discussed the advantages of an ‘instrumental’ approach to teaching mathematics. He explained that instrumental mathematics is usually easier to understand and the rewards can be more immediate. For example, where children lack confidence in the subject, instrumental mathematics can help restore self confidence more quickly. Further, less knowledge may be involved in achieving the answer more quickly instrumentally.

Another point to consider when we discuss teachers’ beliefs is the possible conflict between beliefs and practice. In Section 2.1 (Mathematics in Society), I discussed that, when facing certain restrictions, teachers are often compelled to shift their practice away from their beliefs (Ernest, 1991a). Similarly, Swan (2006) also reveals that there are inconsistencies between a teacher’s beliefs and practices. For example, although there were teachers in his study whose belief was that mathematics is a creative

subject where pupils can create their own concepts and methods, they reported that their practice only had occasional examples of this. For the most part, teachers felt that they were prevented from teaching in their preferred styles because of external factors such as the perceived need to cover the syllabus and a lack of resources. Skemp (1978) also highlights situational factors, such as the backwash effect of examinations and an over-burdened syllabus, leading to the absence of relational understanding. There is an underlying implication that approaches such as 'relational' and 'discovery' allow for more creativity and questioning in mathematics, and so teachers who have these beliefs are more likely to include critical mathematics into their lessons. The beliefs mentioned here are often perceived to be polarised approaches in the classroom but should not be seen as entirely separate (Skemp, 1978). Further, Thompson (1992) asserts that individual mathematics teachers may identify with several aspects of these beliefs to varying degrees.

As part of the main research question, 'How and why might Secondary School Mathematics teachers situate real-world equity issues in the classroom?', this study will explore how teachers with different beliefs implement real-world equity issues in the classroom.

2.3 Curriculum and Resources

According to Bishop (2004), one aspect of democratising mathematics education is giving learners more control over their own learning. He identifies that the reason for this not taking place is not the fault of the subject or the learner, but the fault of the curricula, and the way in which teachers are required to teach. In Section 1.2 (Mathematics and the Curriculum), I discussed how the curriculum does not encourage a critical approach to mathematics education. In Section 2.1 (Mathematics in Society), I explained how in resources, such as mathematics textbooks, content is often value free and does not encourage discussion or student led activities.

Bishop, at the 9th International Congress of Mathematics Education (ICME 9) in 2000, identified that 'current mathematics education situation in most countries is non-democratic, that the curriculum is a mechanism of governmental control rather than educational enlightenment, that the commercial textbook 'business' community

controls the materials available to teachers to an extent that teachers are slaves of the textbook rather than autonomous and enlightened users and that assessment is still primarily and predominantly a mechanism for selecting the mathematical elite only' (Keital and Vithal, 2008, p167). In Section 2.1 (Mathematics in Society), I referred to these as 'rituals' in the mathematics classroom identified by Skovsmose (1990), where students follow explicit and prescriptive statements used for teaching mathematics.

The prescriptive language reflects the structure of questions in exam papers set by the same examining bodies. This then becomes a closed cycle of resources controlled by textbook publishers and examining bodies. The mathematics curriculum, therefore, becomes a set of instructions and rules which inhibits critical thinking with pupils. The opportunities for conjecture, hypothesising, discourse and conclusion are absent. D'Ambrosio (2007) refers to such opportunities as 'Matheracy', a mathematical literacy, which he regrets is almost completely absent from many school systems. D'Ambrosio (2007, p.29) observes that 'even conceding that problem solving, modelling, and projects can be seen in some mathematics classrooms, the main importance is usually given to numeracy, or the manipulation of numbers and operations'.

Indeed, in the UK the mathematics curriculum has been monopolised by the Examining Bodies and this has been strongly criticised. John Dunford, general secretary of the Association of School and College Leaders, commented that '[t]eenagers are increasingly relying on a small number of textbooks scripted by exam boards to pass tests instead of using a wide variety of sources.....it helped maximise results and boost schools' positions in league tables at the expense of a proper understanding of key subjects' (Daily Telegraph, March 2010). Christine Gilbert, the then chief inspector of schools in 2008, also came out against 'teaching to the test', saying that "[r]outine exercises and preparation for tests impair the development of understanding as well as enjoyment of mathematics, particularly but not exclusively at Year 9". She added that high-stakes testing is also having an impact on the range of teaching at GCSE and A-level (TES, July 2008).

Further, the awarding bodies were being accused of 'insider dealing' by Mick Waters, the former director of curriculum at the Qualifications and Curriculum Authority, who

claimed that head examiners wrote textbooks giving pupils tips for answering questions which they would later mark, and that he witnessed senior exam board officials trying to persuade head teachers that their exams were the easiest, in an attempt to win business over their rivals (Independent, September 2010).

It seems that secondary mathematics curriculum in the UK has been cartelised by a few examination awarding bodies. By influencing the direction of the curriculum, based on the content of their exams, there is a pressure on schools to teach towards this content, which has very little in the way of RWEI. Indeed the new Edexcel-endorsed Pearson Mathematics GCSE textbook (Pearson Education, 2015, p344) highlights some questions as 'Real' but, for the most part, fails to address any RWEI. For example, in the Multiplicative Reasoning chapter there is the following question under Reasoning/Real:

A car travels 320km and uses 20 litre of petrol.

- a. Work out the average rate of petrol usage. State the units with your answer.
- b. Estimate the amount of petrol that would be used when the car has travelled 65 km.
- c. Discussion: Why does the question ask for 'average rate' rather than 'exact rate'.

This is definitely a valid mathematics question. However, the question could easily have added a 320km coach journey as a comparison of the fuel impact on the environment. For example,

A 46-seater coach travels 320km and uses 60 ltr of petrol.

Compare the fuel impact on the environment of the two journeys if

- a. There were 5 people on the car journey
- b. There was only the driver on the journey

Gates (2002, p225) gives further examples of how questions from a 'traditional approach' can be adapted to an 'equity approach', so helping teachers to develop ideas and tasks which address global issues in the mathematics classroom. One such example is a probability question in the context of the national lottery:

Traditional Approach:

Is the national lottery fair?

What numbers come up most and least often?

Do they come up as expected?

What are the best and worst combinations of numbers to choose?

Equity Approach:

Is the national lottery fair?

What proportion of disposable income do punters from different social classes spend on lottery tickets?

Who spends more?

Who wins more?

Where do the lottery funds go?

Who benefits from the lottery grants?

Has the election of the Labour government made any difference?

Both questions use the real life example of the lottery. However, the second question brings into question some of the equity issues relating to the National Lottery while still making the mathematics relevant.

In the new Pearson Edexcel GCSE book (Pearson Education, 2015, p61), there are some references to RWEI but these are missed opportunities, very rarely followed up. For example, in the introduction to the chapter 'Interpreting and Representing Data', the book mentions that "[s]tatistics are a major part of our everyday lives. They help us to compare our own characteristics with other people whether it is at work, rest or play. How does your salary compare to others? Do men earn more than women on average?' However, the chapter itself did not have any questions relating to these issues. Instead, it included quite traditional questions using data such as marks in a test and holiday destinations. Indeed, in the 700 page book, which covered the whole of the GCSE syllabus, there was only one question which could be said to address RWEI. This was a problem-solving exercise looking at the legal limit for air particulates for companies, so as not to cause too much air pollution. A newspaper accused the company of being over the legal limit and challenged two frequency polygons the company had published to show that they were below the limit. Pupils had to construct a statistical argument supporting the claims of the newspaper. This was an excellent example of how textbooks could address RWEI without compromising on the mathematical content.

However, these examples are few and far between.

Seah (2010) raises the question of our conception of the nature of mathematics and of school mathematics. If school mathematics were to follow the absolutist philosophy and perceive mathematics to be a body of hard facts and relationships, then, according to Seah (2010), it would be learnt in school as a subject describing the world in absolute and pan-cultural terms. He gives the example of rounding off numbers and decimals. Seah explains that we rarely come across mathematics lessons where the need of rounding off is taught in the context of different contexts or cultures. He gives the example of the use of change in Australia and Malaysia, where there are several factors behind the decision to round off cash to the nearest 5 cents, instead of 10 cents. Seah (2010) emphasises the importance of teachers to share with their students the different ways in which mathematical operations are conveyed in different cultures as this not only values equality, diversity and creativity but may also deepen a student's understanding of a concept by introducing alternative ways of working mathematically. It also has the advantage of valuing the different cultural knowledge and skills that ethnic minority pupils might bring into the class.

We would, at times, find it difficult to put forward rational arguments or appreciate the results of social studies without a basic understanding of mathematics. It seems ironic, therefore, that when teaching a subject which plays a key role in social analysis, we should be considering a pedagogy that does not empower students to question and reflect on issues relevant to their lives.

2.4 Challenges

We need to be aware of the challenges for mathematics teachers when planning and teaching mathematics lessons which address RWEI. Jacobsen and Mistele's (2010) study provides examples of issues and challenges in this area. They give examples where, in their attempts to connect mathematics lessons with social issues, pre-service teachers trivialise the social issue or, by focusing on the social issue, they use the mathematics without any mathematical instruction. This is further supported by Garii and Rule's (2009) analysis of student teachers' integration of social issues into mathematics and science. They concluded that student teachers needed additional

support and guidance from the faculty in order to allow them more opportunities to expand their knowledge and confidence to present interesting and relevant lessons that meet both academic and societal needs. These points need to be considered when situating RWEI within the secondary mathematics classroom, as there is a danger that the mathematical content of a lesson can be trivialised as the focus on RWEI takes over.

The way in which mathematics can be viewed is also a potential challenge. I have mentioned earlier that mathematics is often seen as a value-free subject (Frankenstein, 1983), and this view can be widespread in institutions and thus become detrimental to teachers wishing to situate RWEI in their lessons. For example, Anderson and Valero (2016) report on a school-based project, 'The Newspaper Flyer Workshop', in which pupils were given the task of working in small groups to create flyers which have mathematical content in order to engage people with one of the 54 articles in the United Nations 'Conventions on the Rights of the Child'. The context was a two week 'Human Rights' cross subject project in the school. However, the mathematics department had not been invited to be part of this project. The mathematics teacher felt that this marginalisation of mathematics sent the pupils a message that mathematics has no relation to other subjects or, indeed, the outside world.

Seaman (2003) discusses the fact that real world examples may also bring to light other, possibly more controversial, issues which some teachers might consider to be problematic. He gives the example of using data from a 1988 boxing match between Michael Spinks and Mike Tyson. Spinks was guaranteed \$13.5 million for the fight and Tyson received around \$20 million. Each spectator paid \$1,500 to sit ringside and TV viewers paid \$35 to view the match. Questions were related to the amount of money athletes were paid to perform (in this case) per second, and also to how much viewers were paying per second. However, Seaman (2003) discusses possible issues with this question: 'Would the lesson be more equitable if women's boxing was also considered?' 'Should this example be avoided because of Tyson's trouble with the law and his treatment of women?' 'Should athletes be role models for the youth?' Are these decisions that a mathematics teacher should have to take or should particular real world examples be avoided?

A further challenge is posed by potentially interfering factors. In England, it is difficult

to discuss classroom practice without reference to the curriculum and the related high stakes external assessment regime. In particular, there is a strong focus on Mathematics and English in the evaluation of schools (Brown, 2014). These factors influence the expectations and philosophy of a school, and there is a competitive pressure to teach to the test. As mentioned earlier in Section 2.1, these practical constraints make it difficult for teachers to work within the framework of their own pedagogical intentions (Ernest, 1991a).

2.5 Original contribution to knowledge

In Section 2 I have discussed the contributions of academics, such as Alan Bishop, Ole Skovsmose and Ubiratan D'Ambrosio, in the area of social issues in mathematics. There is much debate in relation to mathematical practice in mathematics education within the mathematics community and the mathematics education community, and I have discussed this in the literature review. However, as mentioned earlier, Bishop (2010, p232) suggests that dealing with values in mathematics education is problematic as 'we do not know what currently happens with values teaching in mathematics classroomsbut we have even less idea of how potentially controllable such values teaching is by teachers'. As an example, I mentioned earlier that so far there has not been any research that addresses how mathematics teaching might contribute to RWEI such as climate change education (Barwell, 2013). Similarly, Apple (2000) mentions that historically, issues of social justice have not featured in the mathematics curriculum. However, there have been a number of significant studies in the field of critical mathematics practice in the classroom, and it is important to consider these in the context of this study.

Gutstein's (2007) research is a practitioner research study of teaching and learning for social justice in his middle school mathematics classroom in Chicago. Beyond learning mathematics, he wanted pupils to read and write the world of mathematics, and so developed a series of 'real world' projects where pupils investigate social injustices using mathematics as a key analytical tool. He concluded that teaching mathematics for social justice in urban schools can make a difference to pupils' lives past the classroom. Further, pupils developed a sense of agency and the issues of justice resonated with their own values. Esmonde and Caswell's (2010) project involved a

research study group with teachers, university researchers and the school district staff working to teach mathematics equitably in an urban elementary school in Canada. The project presented three examples of classroom inquiry to demonstrate the principles of teaching social justice in the area of elementary mathematics. The projects were: the Water Project to focus on the idea that water is a human right, the Languages Project which was designed to integrate the theme of diversity, given that many languages are spoken throughout their school, Toronto and Canada; and the Number Book Project which aimed to build instruction on the mathematical knowledge of the communities and families. All three projects were successful in supporting teachers to develop an approach of social justice in their teaching. Stinson et al (2012) explore critical pedagogy within the mathematics classroom and provide narratives of two lessons. In the first lesson, the teacher had decided on the topic of racial profiling. She felt that this was a topic which the majority of her 'white students' had little exposure to, but it was a topic which could possibly have an effect on their lives. The second lesson used minimum wage data to explore mathematical functions as a means to develop models that might predict possible future wages. Gregson (2013) reports on a case study examining the practice of a full-time mathematics teacher and social activist in an American secondary school. In particular, the study details the teacher's relationship with mathematics education and social justice, and how this takes place in practice. The case study shows that although the teacher's practice reflected models of social justice teaching, the teacher and her pupils were not able to engage at the level of pedagogic questioning or complete as many social justice projects as expected. This was as a result of losing more class time to standardized testing and 'coverage' of content on tests. In Bartell's (2013) study, eight secondary mathematics teachers enrolled on her 15-week graduate course on learning to teach mathematics for social justice. The study was guided by an examination of teachers' developing and implementing of lessons for social justice and the possible compromises between mathematical goals and social justice goals. The analysis revealed tensions between these goals, with teachers focusing more on the social justice component. Anderson and Valero (2016) report on two teachers in Sweden collaborating to introduce elements of critical discourse into the mathematics classroom. This was done by introducing project blocks which addressed the curriculum but also allowed for some key changes in activities. The first project block, called 'Making your dreams come

true?’ was about taking out loans and the related interest. The second was ‘The Newspaper Flyer Workshop’, a project discussed earlier in Section 2.4 (see Challenges). The third was a project in which pupils had to engage with statistics in the context of climate change and sustainability to work out the average ecological footprint of a pupil at the school. In all three cases, there were questions that encouraged critical discourse and the projects were concluded through the use of pupils’ reflections.

The studies described are certainly significant but also emphasise the fact that there is limited research in the area of critical mathematics education taking place in the mathematics classroom. Further, in some of the studies discussed the context was such that the teacher had particular knowledge of social justice teaching that would not normally be the case for most mathematics teachers. For example, Gutstein’s (2007) study involves lessons being taught by the researcher, who already has substantial knowledge of this field, and Bartell (2013) reports on teachers who have enrolled on a course for teachers learning to teach for social justice. Anderson and Valero’s (2016) study had a different emphasis as it focused on critical discourse and conclusions about the project centred around pupils’ reflections. Also of significance is the fact that none of the studies mentioned were based in schools in England.

My study focuses on two aspects of critical mathematics education: ‘How and why might Secondary School Mathematics teachers situate real-world equity issues in the classroom?’. In doing so, it considers the mathematical beliefs of teachers, in order to identify why they might be situating RWEI in their lessons. Although some of the other studies mentioned have addressed how teachers have situated issues of social justice in their mathematics lesson, I have found less evidence of research linking this to teachers’ mathematical beliefs. By observing how and why secondary mathematics teachers situate RWEI in their teaching, my study will make an original contribution to knowledge in this area. The study is not simply a set of examples of RWEI lessons being taught in the mathematics classroom; rather, the research will identify, through the analysis of the eight case studies, the different ways in which mathematics teaching can address RWEI and the reason teachers in the study have chosen to take this pedagogical approach. Further, I would argue that the academic background of secondary mathematics teachers has become more diverse over the recent years. For example, the Initial Teacher Education programme Teach First does not require its

candidates to hold a degree in the subject they are teaching. My study will also contribute new knowledge to the field by comparing the perspectives and practices of mathematics teachers with degrees from other disciplines and those with mathematics degrees, in relation to situating RWEI in their lessons.

The conclusions drawn from this study will be of relevance to mathematics teachers and those involved in mathematics teacher education.

3. Methodology

My research question asks: 'How and why might Secondary School Mathematics teachers situate real-world equity issues in the classroom?'

The following research questions relate to the cohort that took part in the research:

- How might teachers situate RWEI in the secondary mathematics classroom?
- What are the mathematical beliefs of early career secondary mathematics teachers who are interested in teaching RWEI in their lessons?
- Are there differences in mathematical beliefs of early career teachers from diverse academic backgrounds?

This was a non-experimental study where I did not interfere with the situation or circumstance of the participants. Robson (2002) identifies a feature of the non-experimental strategy as one in which there is a sample selection of individuals from a known population. This study did not have any preconceived ideas about the participants' opinions and practices, but built the theory based on the emerging results of the research, which was conducted as a set of case studies. As such, I took an interpretive approach to the study. Basit (2010) describes the interpretive approach as one that relates to a non-traditional, but nevertheless rigorous, view of educational research focusing on smaller numbers and a detailed analysis of human behaviour and perceptions.

An exploratory approach to interviews, where participants were initially prompted by a card sort activity, minimised researcher intervention. Participants who had shown an interest in teaching a mathematics lesson which addressed RWEI were then observed teaching a lesson. My observations of the lessons took place in schools while the participant was teaching a timetabled lesson. The study involved the collection of qualitative data, which were later analysed in order to draw conclusions. My role as a tutor at the University requires me to observe lessons, take detailed notes and then give feedback to students. Therefore, a particular advantage was that, as a mathematics specialist, I have over seven years of experience in observing and evaluating mathematics lessons. Although the observations in the study were in a different context, my skills in this area were valuable for the study.

The reliability of qualitative research is often challenged by many who claim that with qualitative data, there is no possibility of the replication which is possible with quantitative data. Consequently, whereas claims can be made for quantitative data to be 'valid', when working with qualitative data it will be appropriate for the data to be 'credible', 'dependable' and 'trustworthy' (Cohen et al, 2007).

The following methods of data collection were used in the study:

- 8 semi-structured interviews of an exploratory nature, divided between teachers who have a mathematics-related background and those with non-mathematics related backgrounds. The purpose of the interviews was to determine the teachers' own experiences and pedagogical approaches relating to the place of RWEI in the secondary mathematics classroom. The interviews were prompted by a card-sort exercise, which outlined different mathematical related belief systems, such as authoritarian, progressive or socially aware (Ernest, 1991b).
- Observations of a small sample of teachers in their mathematics lessons in which they make reference to RWEI. I carried out 8 observations of mathematics lessons taught by teachers who have shown a positive approach to RWEI having a place in mathematics in secondary schools. I observed teacher practice during their lessons in secondary mathematics, where the teacher made reference to real life equity issues. In particular, I took notes on how appropriate equity issues were taught within the context of the mathematical content of the lesson. I asked for some written reflections from the teachers about the lesson they taught. A guide sheet for observations is included in Appendix 6 (p 220).

For the purposes of this study, teachers with a 'mathematics related background' were defined as participants who have a degree in solely mathematics or a joint degree in which the main part was mathematics.

3.1 Case study

This was case study research through the exploration of eight cases. I argue that the case study approach was a valid methodology as I was taking an interpretive approach to the eight cases, with semi structured interviews and observations. Although observations have traditionally been the method of conducting case studies, other methods, such as interviews are also appropriate (Basit, 2010). As such, my research is strengthened by using both observations and interviews.

Bassey (1999) explains that the case study is an interpretive exploration of a case which attempts to analyse and interpret the data collected. In doing so, it tries to make a coherent report which is meaningful and readable. The case study offers a portrayal of real people in real situations (Basit, 2010). However, Nisbet and Watt (1984) explain that the case study is not simply a narrated anecdote; rather, it has to gather evidence, or data, in a systematic manner. The case study is criticised for its small sample size, but Basit (2010) suggests that even a case study of a single individual is legitimate. Indeed, as is relevant in this study, the case study allows the researcher to explore aspects and gain a better understanding in a way which is not possible when relying on large numbers (Nisbet and Watt, 1984). Basit (2010) explains that the environment appropriate for case studies is one where the researcher can access the case to be studied. In this study, this was the secondary school where the participants were teaching.

Limitations of case studies are that they are open to observer bias and subjective and selective reporting. Case studies are criticised for their interpretive tradition of research (Cohen et al, 2007), and so there is a requirement to demonstrate credibility as there can be bias involved in the reporting from the observer. However, I did not enter the study with any preconceived ideas and so had no interest in reporting my findings with any personal bias. I was, however, aware that the case study is not simply a set of illustrative reports. In relation to this, Nisbet and Watt (1984) specifically advise researchers to avoid distorting the account of the case study to sensationalise the findings or misrepresent the case through selective reporting. More relevant to this study was the danger of reporting the cases in an anecdotal style, such as reporting the

lessons as a series of illustrations which lack any in-depth reflection. A further limitation is that results of case studies are not generalisable across a population (Nisbet and Watt, 1984), however Robson (2002) argues that this does not rule out analytic or theoretical generalisations which can help in understanding other similar cases and situations.

3.1.2 Card sort in research

Card sorts have been employed in research for gathering data, although there is limited advice relating to this within research literature (Saunders, 2012). Cataldo et al (1970) highlight that the limitation of a worded card sort is that it is disadvantageous to illiterate people or people who are not familiar with the language. In looking at the merits of the technique, Cataldo et al (1970) suggest that card sorts allow respondents to select their own pace and review their choices. With regards to the credibility of using card sorts as a stimulus, Cataldo et al (1970) explain that credibility is highly questionable if a stimulus is misunderstood. However, it can help to eliminate ambiguity when presented as a stimulus, as long as the response categories are clearly and unambiguously printed on the cards, allowing respondents as much time as required to make decisions and respond accordingly.

Rugg et al (1992) say that sorting techniques have the advantage of being systematic and easy to use both for the respondent and questioner. With regards to conducting the session Rugg et al (1997) suggest that respondents look at all the cards before they consider sorting them. Practical considerations dictate that a table top be kept clear in case the sorting requires space. The session should be recorded on a tape player and Polaroid type cameras are recommended. One issue identified by Rugg et al (1997) is that sorting implies binary clear cut decisions, whereas reality is more complicated as categories may grade into one another. However, in this study, the card sort was a prompt for the interview and participants were able to further explain their choices and discuss any categories they felt were unclear or, indeed, discard any cards they felt did not reflect their belief at all. Further, if participants felt there were any categories which were not covered by the card statements, then they were free to add their own.

3.2 Interviews

My ontological stance views peoples' knowledge and interactions as meaningful and in order to generate data it is important to talk and listen to people as this is a way in which to access their attitudes, beliefs and philosophies. Dowling and Brown (2010) explain that interviews allow the researcher to explore complex issues in detail and allow clarification, probes and prompts facilitated by the personal engagement of the researcher. These features were essential to my study and therefore outweighed the advantages of a questionnaire, such as being able to use a larger sample size within a limited time frame.

Although there are a number of types of interview (Cohen et al, 2007), I broadly considered three main types (Burton et al, 2014; Basit, 2010; Robson, 2002): structured, semi- structured and unstructured. The correctness of choice of interview type relates to 'fitness for purpose' rather than particular advantages and disadvantages of each type. Dowling and Brown (2010) identify structured and unstructured interviews as products of contrasting approaches to social research. Whereas the positivist approach to social research can be interpreted as a search for social facts, for which a closed structured approach with pre-determined questions can be appropriate, the interpretivist approach can be seen as a search for meaning where there is no pre-determined format to the interview. Thomas (2009) describes this format as one where the interviewer approaches the interview open-mindedly, with the interviewee leading the discussion around the research objectives. The unstructured interpretivist approach was certainly more appropriate for the study; however, as I was following up participant responses with supplementary questions, the semi-structured approach seemed better suited to the needs of this study. This allowed for supplementary questions to be asked, depending on the response of the participant, in such a way that there was no need to ask the same questions of each participant. Also, although all questions addressed the research question, there was no requirement for equivalence (Basit, 2010).

An important point which was considered in the interview setting was the power dynamics. Power relationships are significant, as Cohen et al (2007) suggest that the

situation is not a simple act of data collection, but rather a dynamic of social, and possibly political, views between people. Although both interviewer and interviewee can be in positions of power in an interview setting, it is suggested that as it is the interviewer who generates the questions and designs the course of the interview and it is the interviewee who is under scrutiny, the power typically sits with the interviewer (Kvale, 1996, cited in Cohen et al, 2007, p152). However, Neal (1995) suggests that feelings of powerlessness can manifest themselves in an interviewer in a situation where the interviewee, for instance a highly regarded academic or a university Vice-chancellor, holds a position of power (Neal, 1995, cited in Cohen, 2007, p152) .

In this study, two predominant power dynamics had to be considered. The first was the power relationship between the participant and me. The participants in the study had completed the qualifying course for teaching and were working in schools as teachers; however, they had all been on the course on which I teach and some had been my tutees. I had to consider whether participants would be aware of my philosophy of mathematics education and if this would influence their answers in the interview. On the course, my contact with the participants was during university taught sessions and visits to their schools to see them teaching and to give them feedback. The focus of the taught sessions is mathematical pedagogical subject knowledge, and the limited time allocated to the university teaching days does not allow for time to refer to critical mathematics education or mathematical beliefs in an explicit way. During school visits I had observed the participants teaching and my pedagogical beliefs could have influenced my feedback. For example, I had observed a student teacher, who was not part of the study, on 20th March 2015, the day of the solar eclipse in the UK. The pupils had come into the classroom just after the eclipse and had been talking about it. The lesson on similar triangles, had been a perfect opportunity to refer to the eclipse that had occurred that day, but this had been overlooked in the lesson. I had mentioned this missed opportunity in the feedback, so exposing my own beliefs of relating mathematics to our surroundings. I also had given feedback in another case, suggesting that elements of collaborative learning would have been appropriate during the lesson. My own philosophy of mathematics education was implicit in the feedback I had given to participants so they certainly had an awareness of my beliefs. However, it is unlikely that an awareness of my beliefs influenced the participants' answers, as the

participants in the study had completed the course and been awarded their PGCE, as such I had no influence over their qualifications or career progression. Further, in November 2012 I had piloted the interview with two mathematics teachers (see Pilot Study 3.4) and there was clear evidence that they had very strong views with regards to their mathematical beliefs, which did not necessarily reflect my own beliefs. An important factor was that I was doing this study and therefore it was obvious that I had an interest in RWEI. I was careful to point out to participants that there were no right or wrong opinions with regard to RWEI in the mathematics classroom. However, while it is not possible to mitigate the effects of power relationships and eliminate researcher influence, it may be possible to create conditions that allow for an authentic dialogue in which participants can articulate their beliefs. Hence, in order to minimise researcher intervention, the study took an exploratory semi structured approach to interviews.

The second power dynamic was the potential conflict between the pedagogical beliefs of the school and that of the participant. Although anonymity was emphasised to the participants throughout the study, there was a possibility that they would be concerned about the perceived risks of having their views recorded. In my experience, I have had many teachers mention to me how they would like to try a teaching approach, but certain restrictions in the school, whether they be the schemes of work or a specific way in which the pupils must be seated, do not allow for this. Whereas teachers may be comfortable mentioning these frustrations as part of an informal conversation, it brings in a different dynamic if their views and beliefs are recorded. Credibility can be compromised by further aspects such as question sequence, recording of the interview and rapport between interviewers and interviewee (Oppenheim, 1992). However, I argue that the qualitative data from open-ended interviews, in which participants are able to establish their unique beliefs, outweighs the benefits of credibility that is gained in structured interviews.

Handal (2003, p47) identifies that 'the range of teachers' mathematical beliefs is vast since a list would include all teachers' thoughts on personal efficacy, computer, calculators, assessment, group work, perceptions of school culture, particular instructional strategies, textbooks, students' characteristics and attributional theory, among others.' I have discussed different models for examining teachers' beliefs in

Section 2.2 (Beliefs and Values). I discussed how Ernest categorises beliefs into three broad components (view or conception of the nature of mathematics, model or view of the of the nature of mathematics teaching, and model or view of the process of learning mathematics). Ernest (1991a) uses these components to develop an overview and a comparison of different groups and five overarching ideologies. The beliefs of the five different social groups (Industrial Trainer, Technological Pragmatist, Old Humanist, Progressive Educator and Public Educator) are compared across the following ideologies:

Political Ideology, View of Mathematics, Moral Values, Theory of Society, Theory of the Child, Theory of Ability, Mathematical Aims, Theory of Learning, Theory of Teaching Mathematics, Theory of Resources, Theory of Assessment in Mathematics, Theory of Social Diversity.

Ernest recognises that this model is somewhat superficial, though it serves an orientating function. Ernest presents a simplified version of this model in his paper 'Mathematics Teacher Education and Quality' (Ernest, 1991b, p61). An assumption of the model (Table 4) is that a deep-seated ideology, comprising of intellectual and ethical theories, lies at the heart of any teacher's belief system (Ernest, 1991b). In this model, Ernest outlined a more refined grouping of five different mathematics related belief systems which he infers can be found amongst teachers. The model can be used as an empirical research instrument for assessing teachers' beliefs and as a stimulus for reflection (Ernest, 1991b).

Table 4 : Table of teachers' beliefs

TEACHER AIMS AND BELIEFS	RANGE OF TEACHERS' MATHEMATICS-RELATED BELIEF-SYSTEMS				
	1 AUTHORITARIAN	2 UTILITARIAN	3 MATHS-CENTRED	4 PROGRESSIVE	5 SOCIALY AWARE
THEORY OF MATHEMATICS	A collection of facts and rules	Unquestioned body of useful knowledge	Structured body of pure knowledge	A process view: a personalized activity	A socially constructed practice
AIMS OF MATHEMATICS EDUCATION	Back-to-Basics: numeracy and strict social training (repressive)	Useful maths to appropriate level certification vital (industry-centred)	Transmit body of pure mathematical knowledge (maths-centred)	Creativity, self- realization via mathematics (child-centred)	Critical awareness and democratic citizenship via maths (emancipatory)
THEORY OF LEARNING MATHEMATICS	Hard work, effort practice and rote the only way	Skill acquisition and practice, practical experience	Understanding and application key to progress	Activity, play exploration all central	Questioning and negotiating meaning essential
THEORY OF TEACHING MATHEMATICS	Authoritarian transmission drill, no frills	Skill instructor motivate through work-relevance	Explain, motivate pass on structure of knowledge	Facilitate personal exploration protect from failure	Discussion, conflict questioning content and pedagogy.
THEORY OF ASSESSMENT IN MATHS	Formal testing of basics, prevent cheating	External tests and certification skill profiling	External examinations based on hierarchy	Teacher led informal assessment avoid failure	Negotiated and non-competitive assessments

(Ernest, 1991b, p61).

However according to Ernest (1991b), the model is intended to exemplify a development of quality from position 1 to 5, so associating 'quality' with a constructivist approach.

In considering Ernest's model I acknowledge that he recognises that it makes many assumptions and will be seen as too simplistic. However, the model is theoretically well grounded and offers appropriate prompts for more detailed discussions in an interview about mathematical beliefs. Whereas Ernest has introduced his position of 'quality' into the model, I adapted the model by eliminating the progressive nature of the original table (Table 4) and rearranging the model as in Table 5 so that participants were not influenced by the implication of 'quality'.

The statements were then transferred onto individual cards in order that they can be used to prompt discussions throughout the interview (Figure 1).

Table 5 : Adapted version of 'Range of teachers' beliefs

THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS	THEORY OF ASSESSMENT IN MATHEMATICS
A collection of facts and rules	Back-to basics – numeracy	Practice and rote.	Transmission and drill, no frills	Formal testing of basics (prevent cheating)
Unquestioned body of useful knowledge	Useful maths to appropriate level - certification vital (industry-centred)	Skill acquisition and practice, practical experience	Motivate through work relevance	External tests and certification, skill profiling
Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate pass on structure of knowledge	External examinations based on hierarchy
A process view: a personalised activity	Creativity and self-realization via mathematics	Activity, play and exploration are central	Facilitate personal exploration (protect from failure)	Teacher led informal assessment (avoid failure)
A socially constructed practice	Critical awareness and democratic citizenship via maths	Questioning and negotiating meaning essential	Discussion, conflict questioning content and pedagogy	Negotiated and non-competitive assessments

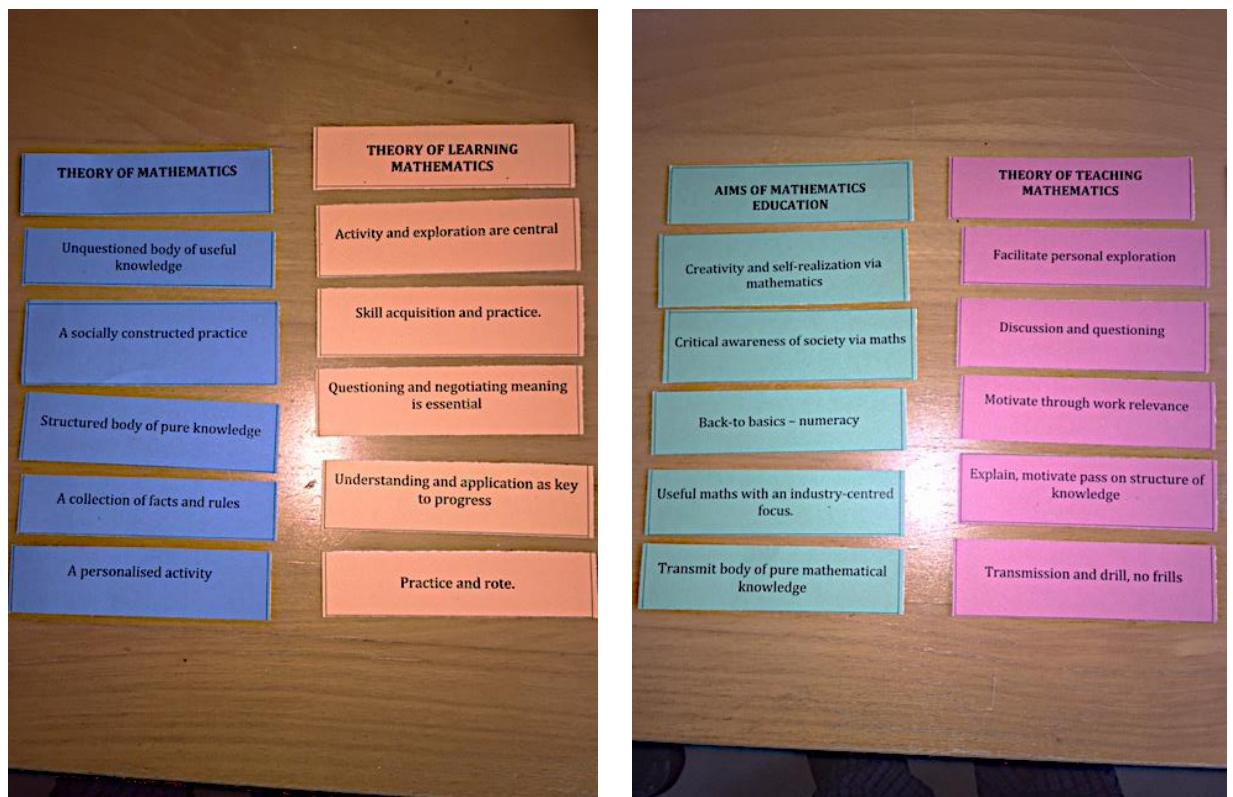


Figure 1: Card sort

The cards were set out in piles, under their title and they were organised randomly. The cards relating to 'Theory of Assessment in Mathematics' was removed for the final card sort. This was as a result of feedback from the pilot study (see Pilot Study 3.4).

Dowling and Brown (2010) advise that the use of prompts involve suggesting possible responses and thus interrupt the spontaneity of the interview; however, this does mean not that they should be avoided and, for particular types of interviews, they may be crucial. They explain that interviewees can find abstract questions difficult to comprehend, particularly if these have not been given any prior thought. They suggest that 'it is essential to provide the interviewee with initial stimuli – questions or activities – with which they can easily engage' (Dowling and Brown, 2010, p81). For this reason, I felt that the use of prompts was appropriate as part of the interview. Further, I assumed that as some of the participants have degrees in disciplines other than mathematics it might have been more difficult for them to articulate their mathematical beliefs in response to an open question. Therefore, I presented the statements (listed in Ernest's model) as a set of cards (Figure 1); the cards were

classified according to Ernest's categories. The cards in each category (Theory of Mathematics, Aims of Mathematics Education, Theory of Learning Mathematics, Theory of Teaching Mathematics and Theory of Assessment in Mathematics) were a different colour. For example, all the Theory of Learning Mathematics cards were orange, so these would be easier to identify for both the participants and myself. Participants had to choose the cards which reflected their belief system. Participants could choose more than one card from each category and place them in order of importance, or they did not have to choose any cards and could discuss why they felt they did not agree with any of the statements on the cards. It was important that all prompts were used in a consistent way with all interviewees (Robson, 2002). The interview was to be prompted by the responses from the card sort activity and by asking participants the reasons behind their decisions; however, in order to achieve more in-depth responses, I supplemented further questions into the interview where appropriate.

3.2.2 Credibility and dependability of interviews

Earlier in this section I discussed how 'validity' is more appropriate when working with quantitative data. Therefore, in this section I will explain how I worked towards a level of 'credibility', 'dependability' and 'trustworthiness' when working with qualitative data in the context of the interviews.

The interview process is a subjective experience for both the interviewer and the interviewee. Hence Dowling and Brown (2010) mention interview bias as a possible limitation of interviews. This can be both in the way the questions are asked or, indeed, the way the questions are answered. I have mentioned before that as a mathematician, and a mathematics educator, I have my own mathematical beliefs. However, I approached the interviews with an open mind as to what type of data might be generated from them. Cohen et al (2007) identify particular issues of credibility in relation to interviews. Two of these, the opinions of the interviewer and their tendency to seek answers that support preconceived notions, have been addressed later under 'Ethical Considerations'. The other points identified are misperceptions on the part of the interviewer in relation to what the respondent is saying, and misunderstandings on the part of the respondent about what is being asked. As I was doing semi-

structured interviews, I had have the scope to avoid bias by asking interviewees to explain some of their responses or, if appropriate, give an example of something they said, as suggested by Basit (2010). As for the possibility of misunderstandings of what was being asked, much of my semi-structured interview was conversation and prompt-based as opposed to a set of closed questions. Indeed, in the pilot study (see Pilot Study 3.4), my questions with two interviewees were usually for the purpose of verifying answers or asking for examples.

Oppenheim (1992) identifies further causes of bias in interviewing. These include the possibility of bias sampling, poor prompting or poor rapport between interviewer and interviewee. I have addressed issues of sampling and prompts in earlier sections of this thesis. As for a poor rapport, this is unlikely, as I know the interviewees. Indeed, my concern about any possible bias in this area was related to the fact that I was once a tutor to the participants. However, I had experience of interviewing tutees in research that I had undertaken (Ghosh, 2012) prior to this study, in which I found teachers to be forthcoming about their views. In the interviews for this study, I approached similar issues from the viewpoint that it is the debates surrounding these issues that are important in mathematics education and that there is no 'wrong' or 'right' view.

I considered the constraints of bias within this interview process and introduced factors to minimise potential bias. However, I am aware that there is no escaping the possibility of subjectivity and bias in interviews and, indeed, in the analysis of interviews. Nisbet and Watt (1984) suggest the use of appendices as an effective way to achieve 'verifiability', as the reader can then scrutinise the raw data, therefore the transcription of the interviews (Appendix 5, p.181) as well as pictures of the participants' card (Appendix 4, p.172) sort are included in the appendices.

3.3. Observations

As part of the study I needed to gather data on the type of RWEI lessons the participants were teaching. Although data could have been collected by asking the participants for their lesson plans and a discussion about the lesson, an observation of the participant teaching a RWEI lesson proved to be a suitable source of data in a naturally occurring,

uncontrived setting. Teachers may not be accurate when reporting on their own practice, as they may be influenced by the objectives of the study or may conform to the norm of the school (Basit, 2010). Robson (2011) highlights that in interviews and questionnaires, there may be discrepancies between what people say and the reality of what they actually think or do. Therefore, depending on the situation, the data gathered from observations is likely to be more valid and authentic than data from other methods, such as questionnaires. (Cohen et al, 2007).

My observations of the lessons took place in school while the participant was teaching a timetabled lesson. The study collected qualitative data, which was then analysed in order to draw conclusions. As mentioned earlier in this section, my role as a tutor at the University requires me to observe mathematics lessons, take detailed notes and then give feedback to students. Although the observations in the study were in a different context, my lesson observation skills were valuable in this study.

The observations informed me of the following:

- How appropriate RWEI were taught within the context of the mathematical content of the lesson.
- How the lesson taught related to the participant's beliefs using Ernest's framework.

Although observations are useful for reporting data in natural settings, there are possible problems related to small samples, difficulties relating to reporting and obstacles to getting access when required (Cohen et al, 2007).

The research question and the project design were factors to consider when deciding on the type of observation to conduct. Flick (2008) identifies that the researcher will need to consider if the observation is a participant or non-participant observation, if it is unstructured or structured, if it is an overt or covert observation and if it is in natural or artificial settings.

I assumed the role of a non-participatory observer (Flick, 2007), as I was not involved in the lesson. However, Basit (2010) specifies that in order to make meaningful inferences from the observation, the non-participant observer needs to be familiar with the context and the group to be observed; these conditions were met as my

observations were in classrooms, a context I was and am familiar with. The observations were recorded as field notes. A high level of administrative discipline is required when working with field notes (Dowling and Brown, 2010); in particular, I made notes on how appropriate equity issues are taught within the context of the mathematical content of the lesson. A guide sheet for observations is included in Appendix 6 (p.220).

Observation Guide

- What is the mathematical aim of the lesson?
- What aspect of RWEI is being addressed in the lesson?
- To what extent is RWEI situated in the lesson?
- How relevant is the RWEI to the mathematical content of the lesson?

The head of each page had contextual information about the lesson, such as the number of pupils and the length of the lesson. Field notes were taken throughout the lesson and a synopsis of the lesson was written after the lesson. I had requested participants to give me soft copies of lesson resources, such as powerpoint presentations and printed worksheets, at the start of the lesson. I also asked them to send me hard copies of these after the lesson, along with an evaluation of the lesson. The evaluation was unstructured, but asked them to focus on the RWEI aspect and the mathematical objectives of the lesson. Cohen et al (2007) suggest that notes are recorded as soon as possible after the observation in case information is forgotten. My observation notes were taken during the lesson and the participants' evaluations were added to the notes as soon as they were sent to me.

With regards to interviews and observations, there are logistical implications relating to gaining access when required (Bailey, 1994, cited in Cohen, 2007, p397). I did not envisage any issues with access to the school with regard to permission, but there were potential issues as to when the observations would take place. At the end of every interview, observation slots were mutually agreed to accommodate schedules of all parties.

3.3.1 Credibility and dependability of observations

For data to be 'trustworthy', Cohen et al (2007) suggest, there needs to be a stability of observations, where the same observations and conclusions would have been made if the observations had been done at a different time. One particular lesson addressing RWEI at a given time in a mathematics classroom is different from another RWEI lesson at another time; hence, my observations were also different in each of the lessons I witnessed, which is unavoidable in real life classroom contexts. Further, in a school scenario, classes respond in different ways in different lessons due to a number of factors, and this would also impact on the 'trustworthiness' of any study which includes an observation involving children in schools.

However, as mentioned earlier, there are advantages to using observations and these outweigh the limitations mentioned. Indeed, 'researchers using observations insist that this method of data collection is better than any other as they are able to observe behaviour directly and gather data *in situ*' (Basit, 2010, p120). Further, the inclusion of participants' evaluations (Appendix 7, p.226) and lesson plans (Appendix 8, p.231) in the appendices allow the reader to scrutinise the raw data and so verify the credibility of the analysis (Nisbet and Watt, 1984).

Another consideration in this study is the Hawthorne effect (Dowling and Brown, 2010, p46), as the presence of the researcher may influence the behaviour of the participants in the lesson observations. However, my not observing from a judgmental position, I anticipated, would minimise any such problems. Further, Basit (2010) argues that the presence of an observer in the classroom is not unusual in educational contexts. Mentors and university tutors observe student teachers, Ofsted inspectors regularly observe teachers in schools, and observations of colleagues by peers and school management is common, or even compulsory, in many schools.

I have discussed power relationships in depth already and the same argument applies to observation data as to interview data.

3.4 Pilot Study

The teachers in the pilot study were from two different schools. Ian was a Physics graduate teaching in an East London school and Jenny was a History graduate teaching in a school in North London. For the purposes of this study the names have been anonymised,

Name	Experience	Date Interviewed
Ian	2 years teaching	15 .11.2012
Jenny	2 years teaching	26.11.2012

My initial questions were drafted and then tested on Ian. However, at this initial stage Ian was a critical friend. I had explained the study to him and mentioned the fact that he was my first interviewee. As such, I was interested in his input into the structure of the interview and wanted him to be an active contributor to any modifications to the interview design.

I recorded the interview on a digital recorder. However, as the purpose of this initial interview was to ensure that the questions were appropriate to aims of this study, I also took written notes to record any changes that needed to be made to the interview process. Following the changes identified through the initial interview, I used the refined process to interview Jenny as the second participant in the pilot study.

With Ian, I started by explaining the card sort activity and then gave him the cards. I had initially planned to give Ian some time to sort out the cards and then start the process of the interview with the chosen cards as prompts. However, as Ian started sorting out the cards, he discussed his reasoning, thus effectively articulating his thought process for the study. He explained that it was easier to talk about the process of choosing the cards rather than summarise his decisions later. Further, my original intention was that the participant choose the cards which best reflected their beliefs. Ian was happy to talk about cards he had not chosen and why, so giving additional insight into his mathematical beliefs. Therefore, the interviews were run in this way, with the participant conveying their mathematical beliefs during the card sort.

Both Ian and Jenny commented that some of the statements on the cards were vague and open to interpretation. Further, the card sort took considerable time as there were so many statements and they required considerable reflection on the part of the participant. Consequently, I adapted the card sort further, removing or rewording the statements which were identified as vague. I also removed the 'Theory of Assessment in Mathematics' column which, on reflection, seemed less relevant to the research questions. Another adaptation was giving the participants the option of adding their own cards.

3.5 Analysis of data

The data collected needs to be analysed based on a framework relevant to qualitative data analysis, particularly case studies. Miles and Huberman (1994) approach case study analysis as three concurrent activities:

Data reduction

Data display

Conclusion drawing and verification

As I was working with eight case studies it was appropriate to analyse the data using this model.

Data reduction

Data can become overwhelming, particularly in the case of qualitative data. As such, Miles and Huberman (1994) suggest that the process of data reduction runs through the study and has to be planned even before the data is collected. I used purposive sampling for my study, and followed the suggestion of Miles and Huberman to use a summary sheet (mentioned in the previous section as an Observation Guide) to record my observation notes. My interviews were recorded and transcribed and relevant evidence was transferred to a two-column evidence table (Appendix 9, page 249) which I discuss in the next section.

Data display

Miles and Huberman (1994) suggest that information is better organised as matrices, charts or networks. This also helps with data reduction. Nisbet and Watt (1984) suggest this format of 'separating conclusions from evidence' in order to distinguish factual information from inferred information. Dowling and Brown (2010) refer to this approach of separating lengthy extracts, such as interview data, from evidence as elaborated description. Elaborated description emphasises the relationship of theory and data, so bringing the theoretical and empirical aspects closer together and allowing the researcher to establish a credibility of their interpretations. I displayed the data on a two column table, with the evidence from the data in one column and the relevant analysis in the second column as a basis for providing elaborated description (Appendix 9, page 249). It is impractical to quote all the evidence in the table. For example, using the transcription of the whole interview in the table would be tedious for the reader. However, this does require the data to be credible; otherwise, the conclusions are questionable. Therefore when displaying data, in order to establish credibility, I took into account particular points raised by Nisbet and Watt (1984) as 'dangers to avoid' in writing a case study. Nisbett and Watt (1984) warn against journalistic writing, selective reporting, and anecdotal style and blandness. Of these, selective writing was the most relevant to this study. This occurs when the researcher chooses evidence to support their conclusion. However, as I had no pre-conceived ideas about this study, there was no reason for me to selectively report data.

My first data display table focused on the research question: 'What are the mathematical beliefs of early career secondary mathematics teachers who are interested in teaching RWEI in their lessons?' For this, I used interview data from the eight case studies for evidence and, for each participant, analysed the data using Ernest's 'Range of teacher's mathematics related belief-systems' (Table 4) as a framework. Based on the participants' response to arranging and discussing the card sort, I was able to use the framework to identify where the participants' beliefs tended towards under each of the categories. For example, under the 'Aims of mathematics education' a participant's card sort and discussion prioritised a 'Mathematics Centred' approach. I used their interview and card sort data as evidence on the left column of

my table and, using Ernest's framework, completed the analysis in the right hand column. In the final analysis I also included a summary table for each teacher (Tables 6 to 13), which was a colour coded version of the adapted 'mathematics related belief-systems' table (Table 5) highlighting the participants' choice in order of hierarchy. This helps to understand the participants' beliefs in the form of a visual summary.

My second data display table focused on the research question: 'How did the sample of teachers situate RWEI in their classrooms?'. The curriculum area and the RWEI addressed in the lesson were the indicators in this data analysis. I used a range of data for evidence:

1. The National Curriculum was used to identify the curriculum area the mathematics was addressing. It was also important to identify this, as some argue that critical mathematics education in the curriculum could replace academic mathematics (Rowlands and Carson, 2002).
2. Post-lesson reflections in which teachers discussed how the lesson was motivated both in terms of mathematics content and RWEI. They also discussed aspects of RWEI they planned to situate into future lessons.
3. The lesson plan further indicated which areas of the curriculum were addressed and how the RWEI was situated into mathematics lessons.
4. My lesson observation notes described the lesson; this included reference to how the teachers motivated the lesson and the lesson content, both in terms of curriculum content and RWEI.

The main body of the analysis used these summary tables as a source of data. For each participant, I presented a detailed analysis in order to answer the two questions:

1. How did the participant situate RWEI in their mathematics lesson?
2. Why did the participant position RWEI in their mathematics lesson?

Conclusion drawing

In drawing conclusions from the data, I focused on my research questions:

- How might teachers situate RWEI in the secondary mathematics classroom?
- What are the mathematical beliefs of early career secondary mathematics teachers who are interested in teaching RWEI in their lessons?
- Are there differences in the mathematical beliefs of early career teachers from diverse academic backgrounds?

My conclusions for the first question were drawn from the analysis of the question: 'How did the sample of teachers situate RWEI in their classrooms?'. This worked towards the conclusion to the question: 'How might teachers situate RWEI in the secondary mathematics classroom?'.

However, these conclusions were based on the analysis of data from individual case studies, whereas a second level of analysis, comparing the data across the eight case studies, gave a more complete picture of how teachers might situate RWEI in the secondary mathematics classroom.

My conclusions for the second question were drawn from the analysis of the question: 'What are the mathematical beliefs of early career secondary mathematics teachers who are interested in teaching RWEI in their lessons?'. Once again, these conclusions were based on the analysis of data from each case study and a second level of analysis compared data from all eight case studies. Comparing data across all eight case cases allowed me to compare mathematical beliefs of teachers from diverse academic backgrounds, so providing more depth to the study.

My final conclusion was drawn from both data sets to work towards answering the question: 'Are there differences in the mathematical beliefs of early career teachers from diverse academic backgrounds?'. I investigated this in order to understand if there was such a difference, and if there was any further evidence of any differences in the lessons they had taught.

4. Ethics

To comply with ethical regulations, I consulted the British Education Research guidelines and submitted my application for ethical review to the London South Bank University Research and Ethics Committee (Appendix 1, p.155). My application (UREC 1423) was approved on 11th September, 2014.

I secured voluntary informed consent from all the participants, who were also informed that they could withdraw from the research, without suffering any consequences, if they felt uncomfortable at any stage. Participants understood the process of the research, including their role in the research and how the research would be used.

The aims of the study, the proposed locations of the interviews and of data storage were summarised in a participant information sheet given to each participant (Appendix 2) and participants were required to sign a consent form (Appendix 2, p.164). As interviews were held in schools and the study involved observing participants teaching, I required permission from headteachers. I sent them letters seeking permission to conduct the study, detailing the aims of the study and the role of the participants (Appendix 3, p.170).

In the case of interviews, where the names and identities of the participants are known, the emphasis of confidentiality and anonymity is particularly important (Basit, 2010). All participants in the study were assured confidentiality and anonymity, all interviews were conducted in respectful and courteous manner and the participants' views were valued. Names of schools and participants were anonymised. I have discussed these issues further in Section 3.2 (Interviews). The interviews were recorded on a digital recording device. The digital recording device and hard copies of the observation notes are kept in a locked storage unit. Digital recordings were copied to my computer, and copies of observations as well as transcriptions of the interviews were typed and stored on the same computer. The computer is password protected. Data was backed up on secure 256 Bit encrypted USB drives and later transferred to a password secured cloud storage. Recordings on the original recorder will be erased and all data will be

destroyed one year after the viva.

5. Analysis

In Section 2.1 (see Literature Review) I discussed that there was a widespread belief that mathematics is a value-free subject, as a result of which values teaching in the mathematics classroom is rare and we are not currently aware of what happens with values teaching (Bishop, 2010). Further, there is little research or evidence of how teachers might situate issues of equity in the mathematics lesson (Apple, 2000; Bartell, 2013). When I explored reasons for this, I identified constraints teachers face in situating RWEI in the mathematics classroom, such as having to teach to the curriculum in an assessment based regime (Brown, 2014). This analysis aims to address these points through the research question: ‘How and why might secondary mathematics teachers situate real-world equity issues in the classroom?’.

The analysis will identify the ways in which the participants situated RWEI into their mathematics lesson and the possible reasons as to why they decided to take this pedagogical approach. I will analyse each participant’s data and then do an overall analysis comparing the data related to all eight participants.

I will approach the analysis by examining the research question in two parts:

1. How might secondary mathematics teachers situate real-world equity issues in the classroom?

The aim of this part of my research question is to explore ways in which a sample of eight teachers have tried to overcome the constraints identified, in order to situate RWEI in the mathematics classroom. Rather than explore theoretical models of how to overcome constraints, or consider textbook examples of social justice in mathematics education, the study articulates practical examples of how secondary mathematics teachers have overcome the constraints of situating RWEI in their lesson. With little evidence or research related to this (Apple, 2000; Bartell, 2013), it becomes difficult for teachers who want to teach in this way to look for practical examples.

The data used to address this question is from:

1. National Curriculum

The Secondary Mathematics National Curriculum is used to identify areas of the curriculum addressed in the lesson.

2. Post-lesson reflection

In the post-lesson reflection, teachers discussed how the lesson was motivated both in terms of mathematics content and RWEI. They also discussed aspects of RWEI they plan to situate into future lessons.

3. Observer's Notes

My own lesson observation notes describe the lesson. This includes reference to how the teachers motivate the lesson and its content both in terms of curriculum content and RWEI.

4. Lesson Plan

The teachers' Lesson Plans also form part of the analysis as it is likely to make reference to the two indicators. This data will further indicate which areas of the curriculum are addressed and how RWEI is situated in mathematics lessons.

2. Why did the teacher decide to situate real-world equity issues in the classroom?

The other part of my research question asks why teachers decided to situate RWEI into their lessons. By analysing the data, I want to identify what motivated teachers to take this approach. Teachers' behaviour in the classroom is influenced by their beliefs (Pajares, 1992). The interview data and the card sort data, identifying teacher's beliefs, will be important in identifying why a teacher decided to situate RWEI in their lesson. For example, teachers who identified with aspects such as 'facilitate personal exploration' and 'activity and exploration are central' will tend to be more confident teachers who are more open to working within milieu six (investigative learning in real life contexts) of Skovsmose's milieus of learning (Table 1), referred to in the Literature Review. However, there could be conflict between a teacher's belief and their practice, so beliefs might not always reflect practice (Skemp, 1976; Ernest, 1991a; Swan, 2006).

In Section 3.2.2 (Credibility and dependability of interviews) I have mentioned the constraints of bias and subjectivity when analysing interview data. As such, there is an

element of speculation when using such data for the analysis of the question ‘Why did the teacher decide to situate real-world equity issues in the classroom?’. This level of speculation is somewhat minimised by the fact that the interviews have been approached with an open mind as to the outcome of the analysis, as detailed in Section 3.2.2 (Credibility and reliability of interviews). Further, an important feature of the analysis is that it does not simply look at data for individual participants but looks for patterns and possibilities across the cases.

A teacher’s academic background is also relevant to this question. This is reflected in the findings of Robbins et al, (2003) where they concluded that trainee mathematics teachers had the least positive attitude towards education for global citizenship when compared with trainee teachers of eleven other secondary subjects. Similarly, Bishop (2010) identifies that unlike teachers of humanities and arts subjects, for whom the discussion and development of values seems relatively easy, most mathematics teachers would not consider teaching values when they are teaching mathematics. This study comprises of teachers with mathematics degrees and non-mathematics degrees such as history, politics and the classics. Therefore, the study also analyses the data to determine if there is a difference in mathematical beliefs and practice, between teachers from these diverse academic backgrounds. In doing so, it aims to establish if teachers with non-mathematics degrees are more likely to engage with RWEI than teachers with mathematics degrees. For example, a teacher with a geography degree is likely to have more knowledge about climate change than a teacher with a mathematics degree, and so might be more confident about situating issues of climate change into a mathematical context. The study will also comparatively analyse the data of all participants to determine if there is a difference in mathematical beliefs and practice, among teachers from diverse academic backgrounds.

The data used to address this question is:

1. Interview data

Language from the interviews was used as identifiers, using belief phrases from Ernest’s outline of teacher’s belief systems. These indicators were then used in the analysis to indicate teachers’ aims and beliefs within the following framework of:

authoritarian, utilitarian, progressive and socially aware, as discussed in Section 3 (in Methodology).

2. Card Sort data

The teacher's arrangements of the prompt cards were also used to verify the interview evidence (see Appendix 4, p.172)

3. Academic background

The teacher's subject specialism for their degree is a factor in the analysis.

4. Post-lesson Reflection

In the post-lesson feedback, teachers discussed how the lesson was motivated both in terms of mathematics content and RWEI. They also discussed aspects of RWEI they planned to situate into future lessons.

5.1 Analysis of participant data

This section analyses the participant data by investigating how and why individual participants situated RWEI in their mathematics lesson. In Section 3.5 (Analysis of data) I discussed that the approach of elaborate description was used as a basis for data reduction, data display and data analysis. This was presented on two columned evidence tables (Appendix 9, p249). The data for this section is sourced from the data displayed on the tables in Appendix 9.

Each of the eight cases start with a brief overview of the school context. In order to identify how the participant situated RWEI in their lesson I have started by discussing the lesson objectives and identifying the areas of the National Curriculum addressed in the lesson. The lessons are analysed by drawing from data, such as lesson plans, observation notes and post-lesson reflections. In order to analyse why participants situated RWEI in their lesson I have used data from the participant's card sort and the related interviews as well as post-lesson reflections. I have started each case with a visual summary of their beliefs by colour coding their card sort choices using the adapted version of Ernest's model of Mathematics-Related Belief Systems.

1. Aron

Aron was a mathematics graduate and taught the subject in a large mixed gender secondary comprehensive school situated in North London. The vast majority of pupils were from minority ethnic backgrounds with no dominant ethnic group. More than half the pupils spoke English as an additional language. The proportion of students who were known to be eligible for the pupil premium, children looked after by the local authority and children of service families, was nearly double the national average. In the last Ofsted inspection, the school was graded as 'Good'.

(School's Ofsted Report, 2015)

Aron had an interest in real-world equity issues in mathematics. He had recently taught a lesson on mortgages and felt that it was important that pupils were aware of aspects of finance through mathematics education.

(Interview, 7.5.15)

How did Aron position RWEI in his mathematics lesson?

Aron taught a one hour lesson to a Year 7 class of about 30 pupils.

Aron's objective was that: pupils should know how to calculate compound interest, and realise how compound interest can cause a sum of money to grow vastly in a short space of time. In the context of compound interest, Aron also wanted to address how mathematics can be used as a tool for understanding the real-world.

(Lesson plan, 18.5.15, Lesson observation and Post-lesson reflection, 21.5.15)

The lesson covered the following areas of the National Curriculum:

Working mathematically: Solve problems

- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

Number

- define percentage as ‘number of parts per hundred’, interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

Ratio, Proportion and rates of change

- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 7)

Ratio, proportion and rates of change

- set up, solve and interpret the answers in growth and decay problems, including compound interest {and work with general iterative processes}.

(Mathematics Programmes of Study: Key stage 4, National Curriculum in England, 2014, DfE, Page 9)

Aron taught the lesson by setting four non-contextual percentage questions on the board for pupils to try on their own. This was followed by Aron briefly explaining: ‘we will use percentages to gain skills and avoid making bad decisions’. A video clip for ‘Kwik Cash’, a Payday loan company, was shown to the pupils and this was followed by a brief discussion about loans. Much of what followed focused on the mathematical methods to calculate compound interest. However, this was followed by a worksheet on calculating loan repayments from fictional loan companies. Pupils had to work out repayments for a £1000 loan based on different interest rates and payback timescales. The second part also introduced the idea of some companies charging an initial fee for taking out the loan. As well as working out the repayment amounts, pupils had to decide which was the best loan and which was the worst. Within the Skovsmose’s

milieus of learning framework (Table 1), this would be classified as traditional exercises with references to semi-reality.

A real case study followed, which involved using the loan company Wonga as an example, encouraging pupil discussions about a scenario in which one could borrow £400 for 1 month at 292% APR and calculations of how much one would have to pay back at the end of the month. Aron also had a discussion with the class about payday loan companies and if these practices were ethical. This part of the lesson was a shift in the milieus of learning and made references to real life experiences, moving away from the traditions of exercise towards, although not entirely inhabiting, an investigative landscape.

(Observation notes, 18.5.15)

Aron felt that the RWEI was successfully situated into the lesson and engaged the pupils:

'By motivating the pupils with the real world problem of borrowing/saving money at a particular interest rate, I was able to engage them in solving problems which lay at the edge of many of their abilities'

(Post-lesson reflection, 21.5.15)

The lesson covered the appropriate areas of the National Curriculum so there was no compromise of the academic mathematics, which is a concern raised by critics of social justice in mathematics education (Rowlands and Carson, 2002 ; Pais, 2010). Further, this was a Year 7 class who were working on some material from the Key stage 4 curriculum, so situating RWEI into the lesson did not result in a 'lighter mathematical curriculum' (Literature Review, Page 20).

This lesson provides an example of how RWEI can be situated in the mathematics lesson and taught without compromising the RWEI or the academic mathematics. Aron noted that this structure enabled him to situate RWEI in his lesson when there is an opportunity to do so:

'It is my opinion that the pupils were more focused on this content than they would have been otherwise. Additionally their comments to me after the lesson was how much they enjoyed it, especially in talking about the mathematics in a qualitative way, what mathematical conditions make a loan 'bad' or 'good'.

Where opportunities arise in the future, I will look to make lessons more real world focused and I wish the curriculum was built around these ideas’.
(Post-lesson reflection, 21.5.15)

Why did Aron position RWEI in his mathematics lesson?

Table 6 : Summary of Aron's beliefs

Colour coded in hierarchical order (Uncoloured entries means belief has been dismissed):

High  Low

	THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS
Authoritarian	A collection of rules and facts	Back-to-basics numeracy	Practice and rote	Transmission and drill, no frills
Utilitarian	Unquestioned body of useful knowledge	Useful maths with an industry centered focus	Skill, acquisition and practice	Motivate through work relevance
Maths - Centered	Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate, pass on structure of knowledge
Progressive	A personalised activity	Creativity and self realization via mathematics	Activity and exploration are central	Facilitate personal exploration
Socially aware	A socially constructed practice	Critical awareness of society via maths	Questioning and negotiating meaning is essential	Discussion and questioning

Positioning Aron’s beliefs in Ernest’s framework demonstrates that he has a wide range of beliefs, prioritising both ‘authoritarian beliefs’ and ‘socially aware’ beliefs within the framework. With regard to ‘aims of mathematics education’, Aron identifies ‘critical awareness of society via mathematics’ as a low priority in his beliefs:

‘Critical awareness of society via mathematics – I don’t know if that is what I would do in my mathematics education. I think that other subjects should speak numeracy, should speak number and that is where that would form, I don’t think there is time unless you gave mathematics teachers far more time in class to be able to do this’
(Interview, 7.5.15)

In this context Aron gave the example of how 'meaningless statistics' could be addressed in many humanities subjects:

'Statistics, so people are constantly throwing around meaningless statistics and saying 'we've done a survey and suddenly it destroys you to eat red meat after 7.30pm on a Wednesday' but they asked six people. And understanding statistics in other subjects because statistics are used so flippantly and people are so blind to the fallacies of statistical thinking. Statistics could be embedded into the humanities.'
(Interview, 7.5.15)

He also identified the exam-focused curriculum as a possible barrier to addressing critical issues in the mathematics classroom:

'I think the way the school is structured is too compartmentalised.....kids really suffer from that because they walk into Year 7 and they get 'ok, as a mathematics department our entire goal is to get you to pass your GCSE at Year 11'- the race has begun, you got to get your C and it's a race from day one which is always there and kids aren't prepared for things that are outside the curriculum, because there's no external motivation. Whether or not it's taught by a mathematics teacher or it could be someone trained in mathematics. Any school I come across do it as a one-off novelty as opposed to something that's necessary. Time isn't made, there is an illusion of a lack of time and I think there's other ways we can teach kids to make very fast progress. but it's not GCSE focused as, obviously, the Jo Boaler books talk about.'
(Interview, 7.5.15)

Although Aron felt that critical mathematics should be taught across the school and not necessarily in mathematics, he clearly identified a difference between mathematical literacy and critical mathematics, and was passionate about mathematical literacy being addressed in his lessons:

'I want kids to know which mortgage is best for them, I want kids to know how many litres of paint they need to paint their house and figure out that kind of stuff and that's not necessarily the focus of education as it currently stand[s]..... I think that it is a priority that kids are mathematically literate. Pupils aren't taught about taxes, pupils aren't taught about retirement or interest rates. There's no class for that and the incentives aren't there for the mathematics departments to teach that, but there should be because we are sending pupils out into the world, not teaching them how to figure out 'if I got a 1200% APR what am I going to pay next month?' and that's ridiculous. And a lot of pupils in the demographics of this school will be going for a pay day loan or

pawning something or taking a job and not releasing how little they will be paid.'
(Interview, 7.5.15)

Aron made an interesting distinction between critical mathematics and mathematical literacy. Further, some of Aron's beliefs fell within the 'authoritarian' and 'utilitarian' categories, areas which are not traditionally associated with teachers addressing RWEI in their class. However, there were clear reasons as to 'why' Aron situated RWEI in his lesson. He placed great importance on mathematical literacy and, based on the demographics of the school, felt it was important that pupils were taught to be mathematically literate about issues which were directly relevant to their lives. Aron was clear that teaching the lesson had a positive impact on him and the pupils:

'Planning the lesson and making sure my focus was on both conveying the real world truths as well as the mathematics was a far more pleasant experience for me, as I am often troubled by how little school can prepare pupils to effectively tackle real world problems. This lesson gave the pupils a tool they can use to start thinking about the benefits of saving versus the problems of credit.'
(Post-lesson reflection, 21.5.15)

Aron's case is particularly interesting, as it provides an example of a teacher who situates RWEI in his lesson, but identifies 'critical awareness via mathematics' as low in his priorities and believes in a 'back to basics' approach to teaching mathematics. Aron felt that mathematics teachers were not given enough time in class to address critical issues and other subjects should be addressing this under 'numeracy' in their subject. However, Aron did make a distinction between critical awareness via mathematics and mathematical literacy. Although I have mentioned in the introduction that mathematical literacy is not the same as RWEI, there can be examples of mathematical literacy which also address RWEI. Aron's lesson is a good example of this. It also demonstrates that a teacher, for whatever reason, might not feel strongly that there should be a critical awareness of society via mathematics, but still feel it is important to situate some RWEI in their lesson.

2. Jason:

Jason was a history graduate and taught in a larger than average academy school, which I shall call School H. The proportions of students from minority ethnic groups or who speak English as an additional language, of disabled students and of those with special educational needs were above the national average. The number of students eligible for free school meals was considerably above the national average. In their last Ofsted inspection, the school was graded as 'Good'. The Ofsted report in 2016 commented that teachers promote the school's distinguishing feature, called the "School H' Way', by encouraging pupils to 'aspire, achieve, care, and endeavour' and through improving pupils' welfare and academic development.

(School's Ofsted Report, 2016)

In his interview, Jason explained that he thought critical awareness via mathematics was important and that RWEI lessons could raise pupils' aspirations. He taught these lessons every half term.

(Interview, 27.4.15)

How did Jason position RWEI in his mathematics lesson?

Jason's objective was to apply probability to the real world by looking at the interrelationship between mathematics, government, medicine and science. He situated the RWEI of decisions that the NHS have to make in a lesson in which addressed experimental probability, estimating for different sample sizes and drawing tree diagrams. This was in the context of investigating 'auto recessive' conditions such as beta-thalassemia. It was a 100 minute lesson with a Year 9 class of about 25 pupils. The RWEI ran throughout the lesson.

(Lesson Plan 20.5.15, Lesson Observation 20.5.15)

The context addressed the following areas of the National Curriculum:

Working mathematically: Solve problems

- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

Probability

- use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

(Mathematics Programmes of Study: Key stage 4, National Curriculum in England, 2014, DfE, Page 10)

The lesson took place in 2015, soon after the general election. Jason started the lesson by summarising the outcome of the recent election and expressing the view that it took a lot of people by surprise that the Conservatives had won. This was followed by a slide with pictures of hospital wards and tabloid headlines about the National Health Service (NHS), such as 'Now hospitals run out of beds' and 'New NHS scandal as 37,000 jobs go.' He explained that the government are looking at how to manage the NHS better and the pupils have been tasked as the best mathematicians available to help them.

Jason put five 'auto recessive' conditions on the board. Working in pairs, pupils first choose a condition and then engaged with three tasks which Jason had set up on their computers.

This required pupils to work out expected samples of different populations having the disease and also draw a tree diagram to work out the probability of the condition getting passed on, based on combinations of one or both parents being carriers.

There were also 'Big Questions' of an ethical nature.

Big Questions:

Using what you have found out about your chosen condition

1. Should the government test everyone for these conditions?
2. If both parents are carriers, should they have children?

Use information and probability you have found to support your decision.

This also brought about a discussion that some people might have resources to carry out private tests, whereas others could only be tested if the government thought it was financially viable.

(Lesson Plan 20.5.15, Lesson Observation 20.5.15)

Although the theme of auto recessive conditions ran through the lesson, it was in the context of the probability question. As such, the lesson addressed the National Curriculum areas mentioned and the content was also appropriate to answer external assessment questions, so addressing concerns about trivialising the mathematical content in the lesson (Mistele and Jacobsen, 2009; Rowlands and Carson, 2002). Another criticism about situating critical mathematics education in a lesson is that results in a 'lighter curriculum' (Pais, 2010); however, although this was a Key Stage 3 class, the mathematics concepts taught were all from the Key Stage 4 curriculum.

In his reflection of the lesson, Jason was positive about how the children had benefitted from the lesson:

'The lesson covered the interrelationship between mathematics, government, medicine and science. We looked at different auto-recessive conditions. The end of the PowerPoint [presentation] would have covered them evaluating this learning to work out whether the government should test everyone for whether they are carrier, and whether parents who are carriers should have children. Overall, I was surprised how much they got from it. At the start, they really bought into the idea and were excited by what they had to do. The depth of the task did mean that some of them lost focus towards the end, but the same can always be expected in a 100 minute lesson. What I thought was particularly beneficial was how the relevance would increase their aspirations. This is something that I will be able to refer back to in future lessons. I think the most suitable balance would be having one lesson a half term on a topic like this'.

(Post- lesson reflection, 2.6.15)

The statement gives us some insight into the equity issues in the lesson, not only through the subject and the related questions, but also through how Jason felt it would 'increase' the aspirations of the pupils.

Jason saw the benefit of situating RWEI in his teaching if it took place at half-term intervals. He mentioned two future lessons: one on area, surface area, volume and elevation views and how these could be tied in with architectural design; and another lesson on frequency tables and graphs in the context of representation of political data. Although the lesson that had been observed and a lesson on political data would be statistics based, it is important to note that Jason's example of architectural design was addressing RWEI in geometry.

(Post- lesson reflection, 2.6.15)

In relation to Skovsmose's milieus of learning (Table 1), the lesson made reference to real life. The pupils investigated different medical conditions using tree diagrams; as the results had to be interpreted by the pupils, the lesson situates itself in the 'landscapes of investigation'.

This lesson, and the further examples Jason mentioned, demonstrate that RWEI can be situated into the mathematics classroom at certain times of the year when the pressures of the curriculum may not be as demanding. For Jason, this would be every half term and would probably be after the end of term or the end of half term assessment in most schools.

(Post- lesson reflection, 2.6.15)

Why did Jason position RWEI in his mathematics lesson?

Colour coded in hierarchical order (Uncoloured entries means belief has been dismissed):

Table 7 : Summary of Jason's beliefs

High  Low

	THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS
Authoritarian	A collection of rules and facts	Back-to-basics numeracy	Practice and rote	Transmission and drill, no frills
Utilitarian	Unquestioned body of useful knowledge	Useful maths with an industry centered focus	Skill, acquisition and practice	Motivate through work relevance
Maths - Centered	Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate, pass on structure of knowledge
Progressive	A personalised activity	Creativity and self realization via mathematics	Activity and exploration are central	Facilitate personal exploration
Socially aware	A socially constructed practice	Critical awareness of society via maths	Questioning and negotiating meaning is essential	Discussion and questioning

Positioning Jason's beliefs in Ernest's framework shows that his priority is firmly situated in a 'mathematics centred' approach. Indeed, apart from placing mathematics as a 'socially constructed practice' as a priority for the theory of mathematics, Jason placed all the other areas of the 'mathematics centred' approach as a priority in his hierarchy. Jason also placed 'authoritarian' and 'utilitarian' beliefs low down in his hierarchy. Along with this, he identified 'activity and exploration are central', which is a progressive approach, as lower down in his hierarchy.

Jason explained the reason for this:

'Facilitate personal exploration. Constructivism is a mistake by the mathematics educational body. You want pupils to be active and their brains to be making connections and thinking, because that's what's going to help them learn, and as a profession we made a mistake by thinking that the best way to do that was to give them

a task and let them get on with it, and in fact what is more important and more effective is to give them some of the information and ask them the right questions and definitely don't involve discovering the rule for yourself. You have the rule and now you can apply it to the problem'.

'Activity and exploration essential' I would put towards the bottom. As I said in the last one, creativity and self realization is important but I think a lot of the mistake with mathematics education is to think that they need to happen in a constructivist and indirect way and I don't think that's an effective way to teach what is essentially a large body of knowledge.'

(Interview, 27.4.15)

However even though Jason prioritised the aim of mathematics education as 'transmit a body of pure mathematical knowledge', while he also placed considerable emphasis on 'critical awareness of society via mathematics'.

'I think critical awareness is more important. We live in a world where the newspapers lead us into different ways of thinking and we need to be really careful when they do that, so pupils can be really aware of statistics and percentages and how that kind of data is gathered and used; that will make them more capable of making their own decisions when they are older'.

(Interview, 27.4.15)

These beliefs provide an insight into 'why' Jason situates RWEI in his lesson. Although he has 'mathematics centred' beliefs, he also realises the importance of mathematics as a tool for critical awareness of the real world. Further, in his reflection of the lesson, Jason mentioned how important the aspirations of the pupils were to him:

'What I thought was particularly beneficial was how the relevance would increase their aspirations'.

(Post- lesson reflection, 2.6.15)

I have written more about this in the previous section on 'how' Jason situated RWEI in his lesson. However, the importance placed on aspirations is particularly relevant as it reflects the school's philosophy to encourage pupils to 'aspire, achieve, care, and endeavour' by improving pupils' welfare and academic development. This philosophy seems central to the school's belief system, as it was highlighted in an Ofsted report (2016) as a distinguishing feature of the school, which is promoted by everyone on the staff.

Although Jason has prioritised his belief system within Ernest's framework, I have also discussed that certain beliefs may only apply to particular situations (Swan 2006). In Jason's case, although he was clear about his prominent beliefs, he also acknowledged that these beliefs may not always be reflected in practice because of particular circumstances. For example, within the framework, 'transmission drill and no frills' is a low priority for Jason. However, Jason explains that:

'Transmission drill and no frills, it's interesting. Maybe I am rejecting this out of hand but some of my lessons are very much transmission drill and no frill. I'm not going to lie about that, but it's not necessarily the way I would want them to be, but I don't think that's what mathematics should be. A lot of my lessons are more like this: explain, motivate and pass on structure of knowledge'.

'My opinion is different for different pupils. I said mathematics is about making them more intelligent, but as a teacher you do have that role of...for instance a lot of my Year 11s right now have aspirations that they want to go onto college. Some of them have football coach type aspirations, some have picked a school where they can go to do plumbing and they need to be able to get their C in mathematics in order to do that. And my responsibility is not to make them understand the mathematics in that situation; my responsibility is to make sure they have the opportunity to be able to go to that college – so it is a hoop jump'.

(Interview, 27.4.15)

However, as these beliefs only apply to certain situations, the framework which summarises the predominant beliefs can be used as a more reliable reflection of his beliefs when analysing 'why' Jason situates RWEI into his lessons.

In conclusion, although Jason's beliefs are predominantly 'mathematics-centred', he considers 'critical awareness of society via mathematics' to be important and this is certainly one reason that he situated RWEI into some of his lessons. His 'mathematics centred' approach explains why these types of lessons might be regular, but far between, taking place every half term. Another reason for situating RWEI in his lessons is that Jason had been teaching in the school for three years and seemed to be in agreement with the school ethos with regard to improving pupils' aspirations through their academic development.

3. Fabia

Fabia was a classics graduate who taught mathematics and some Latin in an all girls Roman Catholic Convent School in inner London, which was smaller than average sized secondary schools. A high proportion of the pupils were from minority ethnic backgrounds, and this proportion was well above the national average. A small proportion of pupils were at an early stage of learning English, and Black Africans were the largest ethnic group. Approximately two thirds of students were eligible for the pupil premium, and this is a proportion much higher than the national average. The school was graded as 'Good' in their last Ofsted inspection.

(School's Ofsted Report, 2016)

At the time of the interview Fabia was also studying for a Masters qualification and had attended a module, 'Understanding Mathematics Education', which included a session on 'Social Justice in Mathematics Education'. In her interview, Fabia had expressed the opinion that pupils should be able to *'use their mind and the skill that they gain from mathematics to assess and to live well in the real world'*.

(Interview, 30.4.15)

How did Fabia position RWEI in her mathematics lesson?

Fabia's lesson took place soon after the 2015 General Election. She explained that the objective of the lesson was:

'[T]o allow pupils the chance to engage with the election data, and show them the data in different ways, like the percentage of vote share compared to the number of seats, and then show them how the way in which we count the votes plays a part in this.'

(Post-lesson reflection, 22.5.15)

The 100 minute lesson was , taught to a Year 9 lower achieving set consisting set of 15 pupils., was a double lesson of 1 hour and 40 minutes.

(Lesson Plan, 21.5.15; Lesson Observation, 21.5.15)

RWEI was central to the lesson and the context addressed the following areas of the National Curriculum:

Working mathematically: Solve problems

- develop their use of formal mathematical knowledge to interpret and solve problems.

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

Number

- define percentage as ‘number of parts per hundred’, interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- interpret fractions and percentages as operators

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

The objectives of the lesson, as stated in the above quote, were communicated to the class at the start of the lesson. No explicit mathematics objectives were communicated to the class. The first part of the lesson discussed the UK voting system and, as such, there was little reference directly to any mathematics. The lesson started with a brief discussion about ‘constituencies’ followed by a discussion of the pupils’ constituency and the school’s constituency, and a comparison of the constituency maps of London in 2010 and in 2015. During the discussion, I observed that most pupils were not aware of the constituency in which they lived.

The second part of the lesson analysed the results in the schools’ constituency and asked some related questions. The questions were mathematical, such as ‘Can you work out what Labour’s majority was?’, and ‘How many votes did the Liberal Democrat candidate need to beat the green party candidate?’. Although there was no critical discussion of the results, the exercise did make pupils aware of the election results in the school’s constituency, and it was obvious from the classroom discussions that most pupils had previously not been aware of this. The next task did introduce a more critical discussion, as pupils had to calculate how many people actually voted in the UK, given that the electorate was 46,425,386 and 66.1% of the electorate actually voted. Discussions centred on why people might not vote and who should be allowed to vote.

This was followed by a bar chart showing the number of seats each party had in the new parliament and then a summary of this information on a table. Fabia then listed the raw number of votes for each party and had a discussion about the fact that more votes did not necessarily have more seats in Parliament. Most of the class felt this to be unfair, it did not seem right to them that elections were not decided by the number of votes a political party has, but the number of 'seats' based on constituencies.

Pupils then had to work out the percentage of the total votes each party got, and then put these in order. On doing this, they found that the UK Independence Party, which only had one seat and were one of the lowest ranking parties, would now be the third largest political party in the UK . In a class, which reflected the demographics of the school (mainly Black Africans with a small number of recent immigrants who were at an early stage of learning English), the prominence of the UK Independence party when considering raw votes led many pupils to review their ideas about the voting system. The system of counting raw votes, which they originally thought was 'fair', was not a system they now agreed with.

Fabia then followed this with a practical activity taken from a book 'Human Rights in the Curriculum – Mathematics' (2004). Fabia explained five different voting systems to the class and they had to discuss which was 'fair'. Pupils worked in five groups, each working with one of the voting systems. They were given 11 filled ballot papers from fictitious voters and had to decide which candidate would win under their voting system. The ballot had been 'rigged' so that each candidate was elected at least once. Figure 2 is an example of one of the voting systems and Figure 3 is one of the ballot paper:

Borda Points with Geometric Weighting
 Electors vote for candidates using preferences (the first choice is shown by a '1', the second choice by a '2', and so on).

- Points are awarded to candidates, according to the electors' preferences, as shown below.
- If there are four candidates:
 8 points are awarded for each first preference
 4 points are awarded for each second preference
 2 points are awarded for each third preference
 1 point is awarded for each fourth preference
- The candidate with the largest points total is elected.

Note: For n candidates, there will be 2^{n-1} points for each first preference, 2^{n-2} points for each second preference and so on down to 1 point (2^0) for each n^{th} preference.

Figure 2 : Voting System

Ballot Paper
 Place a 1 in the box next to your first preference, a 2 in the box next to your second preference, and so on.

Candidate	
Ahmed (Progressive Party)	2
Boateng (Radical Alliance)	3
Cortez (Independent)	1
Donald (Peoples' Forum)	4

Figure 3 : Ballot paper

'Human Rights in the Curriculum – Mathematics' (Wright, 2004 , p16)

The activity further emphasised that different voting systems could elect different candidates, which was something the pupils had started to consider after looking at the UK elections. I observed that some pupils found the mathematics challenging, but were nevertheless engaged with the task.

(Lesson Plan, 21.5.15; Lesson Observation, 21.5.15)

In her evaluation of the lesson, Fabia commented:

'[P]upils did learn a lot in the lesson, and were more critical in the way they can think about voting as a result of this lesson'.

(Post-lesson reflection, 22.5.15)

As mentioned earlier, although the lesson did address areas of the National Curriculum, there was no explicit reference to mathematics when Fabia went through the objectives of the lesson with the pupils. Critics of critical mathematics education could argue that this sidesteps mathematics to prioritise social issues (Rowlands and Carson , 2002; Mistele and Jacobsen, 2009; Pais, 2010). However, this was a lower set Year 9 class with students who have difficulties with the subject, and Fabia felt that by learning the mathematics in a real context that meant something to them, the pupils were more involved with the mathematics:

'[I]t was great to see how engaged pupils were. I didn't really have any expectations of how pupils would respond, or engage with the material, and I was pleasantly surprised by how pupils worked in the lesson....it was really interesting to see pupils who usually really struggle in Mathematics getting settled and approaching activities more quickly

*and confidently. This may have been because pupils didn't see the work as 'Mathematics'.
(Post-lesson reflection, 22.5.15)*

With regard to the mathematical content, Fabia felt that whilst the lesson gave her a chance to see the different strengths and areas for improvement in her pupils, it also gave the pupils

*'opportunities to do mathematics that stretched them. For example, ordering percentages and decimals and organising and collating data.'
(Post-lesson reflection, 22.5.15)*

Fabia's lesson could be positioned in Skovsmose's milieus of learning (Table 1) as a lesson with reference to real life contexts. The task itself required pupils to interpret data from a critical perspective and so situates itself in the 'landscapes of investigation'. Fabia identified that lessons like this did not feature much in her school because of an 'exam focus'. However, she felt it was important to teach these lessons when the opportunity arose. With regard to future lessons, she mentioned that she wanted to look at the High Speed 2 Railway and its impact on the community and the school. She felt that there would be mathematics related work in terms of location and a lot of data analysis around, for example, the impact of pollution.

(Post-lesson reflection, 22.5.15)

Why did Fabia position RWEI in her mathematics lesson?

Table 8 : Summary of Fabia's beliefs

Colour coded in hierarchical order (Uncoloured entries means belief has been dismissed):

High  Low

	THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS
Authoritarian	A collection of rules and facts	Back-to-basics numeracy	Practice and rote	Transmission and drill, no frills
Utilitarian	Unquestioned body of useful knowledge	Useful mathematics with an industry centred focus	Skill, acquisition and practice	Motivate through work relevance
Mathematics - Centred	Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate, pass on structure of knowledge
Progressive	A personalised activity	Creativity and self realization via mathematics	Activity and exploration are central	Facilitate personal exploration
Socially aware	A socially constructed practice	Critical awareness of society via mathematics	Questioning and negotiating meaning is essential	Discussion and questioning

Having positioned Fabia's beliefs in Ernest's framework, it is clear that Fabia beliefs are firmly situated in a 'socially aware and 'progressive' approach. She explained that mathematics should be a subject which relates to the real world:

'If we are talking about the broad overall aims of mathematics education, this should be more about the discovery of mathematics and how that helps them live in the real world. That does not mean that they have to do mathematics that is directly associated with an industry, but that they can apply it to being a scientist or being a statistician, but also they can use their mind and the skill that they gain from mathematics to assess and to live well in the real world'.

(Interview, 30.4.15)

Although all the beliefs in 'theory of mathematics' have been highlighted as a priority, Fabia had simply given all the beliefs in this category the same importance:

'I feel it is a collection of facts and rules. It is socially constructed and should be taught in a way that they realise that there is pure knowledge in there and it can be personalized. But I think it [mathematics] is a specific way of working and using your brain and the process. Mathematics is about processing and how you get to [the] answer and how you tackle a problem, and for me that is a really central part of mathematics.'

(Interview, 30.4.15)

Although Fabia's beliefs were largely of a progressive and socially aware nature, she was not completely dismissive of some 'authoritarian' approaches, although these featured as lower priority choices in her hierarchy of beliefs.

"Practice and rote, I do think that rote learning does have a place in most subjects. Certainly, I do think it helps in speed and fluency in Mathematics, but in terms of learning mathematics I would put it there – maybe third'.

(Interview, 30.4.15)

She also discussed possible reasons as to why pupils were not able to engage in exploratory learning:

'Activity and exploration, I think they are important for learning, but not used as much in schools, especially in secondary, because they tend to take more time and tend to be less exam-focused, so you don't give pupils the chance to do more activities or explore.'

(Interview, 30.4.15)

In conclusion, Fabia situated RWEI into her mathematics lessons because she had strong beliefs about mathematics being a subject which could help people live in the real world. She also had concerns about her pupils and their knowledge of the real world, and felt there were issues that could be addressed in the mathematics classroom:

'Pupils' prior knowledge of politics in the UK was not strong, and so I had to adapt to make sure that pupils understood all the key terms. In particular, they found it difficult to grasp what a constituency and a seat were'.

(Interview, 30.4.15)

4. Minervia:

Minervia was a mathematics graduate and taught in a large mixed gender comprehensive school situated in East London. The proportions of pupils from ethnic minority groups and of those for whom English was an additional language were significantly higher than average. The proportions of students known to be eligible for the pupil premium (which is additional government funding provided for students eligible for free school meals), of children looked after by the local authority and of children of service families, were well above the national average. In their last Ofsted inspection the school was graded as 'Good'.

(School's Ofsted Report, 2013)

Minervia had an interest in real-world equity issues in mathematics. In her interview, she said she believed mathematics was a creative subject and pupils should be aware of the society around them through mathematics. She regularly looked for applications of mathematics in her lessons, and had recently taught a lesson on compound interest, although it was not addressed as an equity issue, in the context of which banks would be best to invest with.

(Interview, 14.4.16)

How did Minervia position RWEI in her mathematics lesson?

The lesson duration was one hour and the RWEI was addressed in the last 10 minutes, with the rest of the lesson dedicated to developing pupils' knowledge of scale factors, bearing and number in the context of flight plans. Near the end of the lesson, there was a 10 minute discussion about the environmental impact of flights. The context was used to cover the following areas of the National Curriculum:

Working mathematically: Solve problems

- develop their use of formal mathematical knowledge to interpret and solve problems.

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

Ratio, proportion and rates of change

- Using scale factors, scale drawings and maps.

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 7)

Geometry and measures

- Interpret and use bearings

(Mathematics Programmes of Study: Key stage 4, National Curriculum in England, 2014, DfE, Page 9)

Ratio, proportion and rates of change

- Convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts.

(Mathematics Programmes of Study: Key stage 4, National Curriculum in England, 2014, DfE, Page 9)

A particular challenge when situating RWEI in lessons is having to take into account the curriculum and external assessments. The mathematical content of the lesson addressed the three areas of the Mathematics in the National Curriculum. The content taught was also appropriate to answer external assessment questions such as the following GCSE Mathematics question (Figure 4):

15 Here is a scale drawing of a park.

15 (a) What is North-West of the Bandstand?
Circle your answer. [1 mark]

Lake Toilets Café Playground

15 (b) Measure and write down the 3-figure bearing of the Playground from the Lake. [2 marks]

Answer _____°

15 (c) A Tower is
North of the Toilets
and
on a bearing of 220° from the Café.
Mark the position of the Tower on the scale drawing. [2 marks]

Figure 4 : GCSE Question (Assessment and Qualifications Alliance, 2016)

The lesson demonstrated how RWEI can be situated in the mathematics classroom as a short add-on, rather than as a theme that runs through the lesson. Although a small proportion of the lesson time was dedicated to addressing the social issue, both my observation notes and Minervia's reflection of the lesson evidence that the discussion did achieve the aim of having some pupils think about the environmental impact of flights.

An extract from the post-lesson reflection illustrates this, as well as Minervia's desire to spend some more time on the RWEI:

'The pupils were interested in the discussion about the environmental impact of flights.'

They had not really thought about this before. I think it would have been better to leave more time for the discussion. For example, discussing which countries have more flights or how many people on each flight, so do some flights impact more on the environment than others?'

(Post-Lesson reflection, 10.6.16)

A criticism could be made that addressing a RWEI in the final part of the lesson can be seen as an 'add on' and trivialises the issue, not allowing time for pupils for further exploration (Mistele and Jacobsen, 2009). I would argue that with the challenges that teachers face with regard to introducing social issues into their lesson, even an issue addressed for a short part of the lesson is still a valid example of situating RWEI, so long as pupils engage with this. Indeed, one could argue that featuring an RWEI as an add on to the lesson addresses some of the arguments which claim that situating critical mathematics education in the curriculum could replace academic mathematics (Rowlands and Carson, 2002) or result in a 'lighter mathematical curriculum' (Pais, 2010).

The lesson could be positioned in Skovsmose's milieus of learning (Table 1) as a lesson with reference to real life contexts. The task itself did not require the pupils to engage critically with the data until near the end of the lesson. As Skovsmose states, the vertical line separating the exercise paradigm from landscapes of investigation represents several possibilities, and often there is no obvious classification; therefore, the critical element of the lesson does situate itself in the 'landscapes of investigation' milieu, but perhaps with an inclination towards the traditions of exercise, due to the other parts of the lesson.

In her reflection after the lesson, Minervia felt that situating RWEI in this manner meant that she is able to do it regularly in lessons:

'We regularly look at applications of the topics that we cover. For example, we were learning about compound percentage change last week, and we looked at different banks and their saving plans to decide which bank we would rather invest our money at. Soon, we will be looking at encrypting/decoding messages using algebra'.

(Post-Lesson reflection, 10.6.16)

While not addressing RWEI, this does demonstrate that real-world issues can be situated in mathematics lessons on a regular basis. Although this lesson addressed an

RWEI, Minervia's other two examples would suggest that a further challenge is to consider how equity issues might be addressed if there is an opportunity to do so when teaching within real life contexts.

Why did Minervia position RWEI in her mathematics lesson?

Table 9 : Summary of Minervia's beliefs

Colour coded in hierarchical order (Uncoloured entries means belief has been dismissed):

High  Low



	THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS
Authoritarian	A collection of rules and facts	Back-to-basics numeracy	Practice and rote	Transmission and drill, no frills
Utilitarian	Unquestioned body of useful knowledge	Useful mathematics with an industry centred focus	Skill, acquisition and practice	Motivate through work relevance
Mathematics - Centred	Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate, pass on structure of knowledge
Progressive	A personalised activity	Creativity and self realization via mathematics	Activity and exploration are central	Facilitate personal exploration
Socially aware	A socially constructed practice	Critical awareness of society via mathematics	Questioning and negotiating meaning is essential	Discussion and questioning

Having positioned Minervia's beliefs in the framework, the summary shows that her beliefs tend more towards the 'progressive and 'socially aware' with some emphasis on 'mathematics centred' beliefs in the Theory of Learning Mathematics and the Theory of Teaching Mathematics.

The structure of the lesson reflected Minervia's mathematical beliefs, as she also prioritised mathematics-centred beliefs with regard to the theory of learning and teaching mathematics, although she strongly believed that there should be a critical

awareness of society via mathematics. Minervia gave a high priority to ‘creativity and self realisation via mathematics’, suggesting she would teach lessons in milieu 6 (investigative lessons with real life references) of Skovsmose’s milieus of learning.

There was further evidence of this in the interview when Minervia, in the context of discussing ‘personal exploration’ from Ernest’s framework, mentioned a recent TED talk by an American mathematics teacher which she thought was ‘really good’:

Yes, personal exploration. Yes. There was this TED Talk actually from a math teacher, an American math teacher. He was saying how the books, the mathematics books, were really not well written, in a sense that every problem would be deconstructed into many steps, so that the pupils will be able to solve the problem; but if faced with the problem in real-life, they would never be able to come up with the solution. He was saying all about how we should teach the students to look at a mathematics problem and solve it by finding the steps for themselves. I thought that was really good.

(Interview, 14.4.16)

Minervia liked the fact that the speaker encouraged a problem-solving approach to teaching with real life references. On further investigation, I realised that Minervia was referring to the TED talk ‘Mathematics Class needs a makeover’. The speaker, Dan Myer, is an American mathematics teacher who highlights that often the most effective strategies for engaging pupils in mathematics make relevant connections to the real world, although he acknowledges that all teaching strategies have their limitations. The fact that Minervia watches these type of talks and is inspired by them gives another insight into ‘why’ she situates real life (and real-life equity) issues in her lesson.

We could conclude from this that the reason Minervia situates real life issues and real-life equity issues into her lessons is that, for the most part, she has a ‘progressive’ approach to mathematics education and clearly identifies a ‘critical awareness of society via mathematics’ approach as a priority in her belief system. Further, her positive reference to Dan Myer’s TED talk suggests that she believes that mathematics education should include investigative tasks with references to real-life or semi-reality.

5. Edwin

Edwin was a mathematics graduate and taught in a larger than average secondary girls' school in North London. Over half of the pupils were eligible for free school meals, or were in care. This is well above the national average. There were high proportions of pupils who speak English as an additional language and those from minority ethnic backgrounds. The proportions of those who had special educational needs and needed extra help with their education were above average. In their last Ofsted inspection, the school was graded as 'Good'.

(School's Ofsted Report, 2013)

How did Edwin position RWEI in his mathematics lesson?

This was a one-hour lesson with a Year 9 class. The lesson was in the form of a short investigation. Pupils were given the following scenario :

A soft drinks company has recently introduced a new type of can.

The new cans have the same volume as the original cans - 330ml.

After experimenting with various options, they decided that new cans would be taller and more slender than the original ones – the marketing department thought this gave them a distinctive modern look!

Task: Making the can look good is a great idea, but what about the cost of materials needed to make it? In this activity, you are going to explore the following question:
A cylinder can holds 330ml. What combination of radius and height will minimize the amount of material needed?

The lesson was from the Core Mathematics website (<http://www.core-mathematics.org/resources/algebra-graphs/max-cylinder/>). In addition to the set investigation, Edwin also mentioned that more than 7 billion cans are sold in the UK every year and that there is an environmental impact because of this.

(Lesson Plan, 28.4.16; Lesson Observation, 28.4.16)

The lesson addressed the following areas of the mathematics national curriculum:

Working mathematically: Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non- routine problems

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

Number

- use standard units of mass, length, time, money and other measures, including with decimal quantities
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately.
- interpret mathematical relationships both algebraically and graphically

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 6)

Geometry and measures

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 8)

Algebra

- identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically (and turning points by completing the square)

(Mathematics Programmes of Study: Key stage 4, National Curriculum in England, 2014, DfE, Page 6)

After introducing the problem to the class, Edwin had a discussion with the class about how to find the surface area and volume of a cylinder. This was teacher-led with pupil input. Following this up with some examples of how to find the surface area of cylinders with different radii, Edwin displayed a spreadsheet on the board. He then discussed how formulae could be inputted onto spreadsheets. Pupils worked in pairs on laptops to produce their own spreadsheets which found the surface area of a cylinder, given different inputs for the radius.

Most of the lesson involved pupils working on the task. Near the end of the lesson, Edwin used a graph plotting package (Desmos) to plot the function of the radius and surface area, deriving the formula for this with a discussion with the pupils. Finally Edwin returned to the question about the combination of radius and height which minimizes the aluminium needed.

Although a RWEI theme ran through the lesson, the lesson addressed a number of areas from the Mathematics National Curriculum. Indeed, there was an area of the Key Stage 4 mathematics curriculum covered in the lesson, although this was a Key Stage 3 class. The lesson also covered areas from the Problem Solving section of the curriculum. Therefore, there was no question of the mathematics content being compromised because of situating RWEI in the lesson.

(Lesson Plan, 28.4.16; Lesson Observation, 28.4.16)

Edwin's lesson demonstrated how problem solving tasks can be used effectively to situate RWEI into the mathematics classroom. However, Edwin did not situate RWEI regularly in his lessons, as he found it difficult to do so. He explained the reason by identifying some of the limitations of the curriculum:

'I would like to, but I am under pressure to raise standards in my classes and that is measured by their performance in exams. The curriculum is designed to be taught by topics and not through individual problems. This makes it difficult to teach the subject like this. I think with the current system, it would be impossible to teach every lesson through a real-world equity issue, but I would like to raise them if it was appropriate.'
(Post-lesson reflection, 24.5.16)

As such Edwin found it difficult to think of RWEI for future mathematics lessons:

'I find it very difficult to think of real-world equity issues that I would actually address in my mathematics lesson; however, if I came across one that I deemed good and appropriate, I would be very happy to teach it.'
(Post-lesson reflection, 24.5.16)

As someone who finds it difficult to situate RWEI in his lesson, Edwin used an investigation from the core mathematics website and made a minor adaptation by adding in the context of the environmental impact of the materials used for the cans. The lesson could be positioned in Skovsmose's mileus of learning as an investigation in a semi-real context. This provides an example of how problem-solving tasks or investigations, which might not address RWEI, could be adapted into a critical mathematics context.

An argument could be made that, apart from a brief reference at the start of the lesson, Edwin could have introduced further real-world contexts into the lesson. For example, cans of Pepsi are different in size from those of some energy drinks, and although their volume could be different it might still be useful to compare them with regard to package wastage. However, even without the real world reference, the lesson was still valid as an RWEI lesson, since the ideas discussed are certainly transferable to the real world. Edwin perhaps did not do this because of time issues or (reflecting on his interview) because he had not thought about it.

Why did Edwin position RWEI in his mathematics lesson?

Table 10 : Summary of Edwin's beliefs

Colour coded in hierarchical order (Uncoloured entries means belief has been dismissed):

High  Low

	THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS
Authoritarian	A collection of rules and facts	Back-to-basics numeracy	Practice and rote	Transmission and drill, no frills
Utilitarian	Unquestioned body of useful knowledge	Useful mathematics with an industry centred focus	Skill, acquisition and practice	Motivate through work relevance
Mathematics - Centred	Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate, pass on structure of knowledge
Progressive	A personalised activity	Creativity and self realization via mathematics	Activity and exploration are central	Facilitate personal exploration
Socially aware	A socially constructed practice	Critical awareness of society via mathematics	Questioning and negotiating meaning is essential	Discussion and questioning

Added: Discussion and questioning answers

Considering Edwin's Beliefs within Ernest's framework reveals that he has a wide range of beliefs, although he does prioritise the 'authoritarian' approach, followed by positioning 'utilitarian' and 'mathematics centred' beliefs higher than 'progressive' and 'socially aware'. With regard to the theory of mathematics he felt that the

'unquestioned body of useful knowledge could be true, [or] could not be true depending on how you look at it'.

(Interview, 10.2.16)

With regard to the aims of mathematics, Edwin prioritised 'back to basics numeracy', explaining that

'It's very different to when I started teaching, but I would put back to basics numeracy right at the top, because I think before I get on to any of these, they need to have this level of numeracy. You can't really do anything without this, so yes, put that right at the top'. (Interview, 10.2.16)

Edwin explained that his beliefs regarding 'back to basics numeracy' had changed since he started teaching because:

'I was in the top set mathematics for most of my life. I didn't really experience people of Year 9, Year 10 not being able to do basic skills and you're thinking about, "Well, they're going out into the world and they're not – well, some of the stuff they can't do is really basic." I think if I was to stop my year at the middle set class now - their math education now - they would still be able to function well in society. But the ones without the numeracy, they're the ones who I'm really worried about'. "
(Interview, 10.2.16)

Although Edwin felt that 'creativity and self-realisation via mathematics' was not an aim, he did place equal emphasis on 'useful with an industry centred focus', 'transmit a body of pure mathematical knowledge' and 'critical awareness of society via mathematics'.

'Creativity in self-realization about mathematics. I don't think the aims of mathematics education in this country is creativity. As much as I believe you can have that, I don't think that's necessarily an aim. I'd put that at the bottom.

I would really tie all of these three together - transmit body and pure mind useful. I would tie the industry-focused one as well with the transmit body for mathematics knowledge and the critical awareness of society via mathematics. Because I think I would try to fit in as much as I can all of three of these in my lessons. I couldn't really say that for the creativity, but when I see an opportunity, I would try and fit these in'. (Interview, 10.2.16)

So, although Edwin prioritised 'back to basics numeracy' in his beliefs, he would also situate 'critical awareness of society via mathematics' when the opportunity arose in his lesson. However, for the most part, Edwin's beliefs were definitely 'authoritarian', as was evident in his beliefs about discussions in mathematics:

'Discussion, I'm going to put down below, because - we talked about in theory of mathematics, we talked about when you can have mathematical conversations and definitely in my classes, the students can work together to discuss their answers. I think discussion probably works a lot better - it's a little easier in other subjects'.

(Interview, 10.2.16)

Therefore, with regard to the theory of mathematics, Edwin felt that 'discussion and questioning' were only valid if the class were discussing and questioning their answers. Although Edwin was dismissive of some of the 'socially aware' and 'progressive' beliefs, he agreed with all the beliefs with regard to the theory of learning mathematics, prioritising the 'socially aware' belief of 'questioning and negotiating meaning', while giving importance to 'practice and rote':

'Theory of learning mathematics. Activity and exploration central, questioning and negotiating, meaning essential, practice and rote. That's probably been the easiest one to agree with all, all five of them. Activity and exploration central, questioning and negotiating.

I don't necessarily see all of these as mutually exclusive. For example, the practice and rote one, I think gets an unfair representation. In China, they're very key on making sure everyone [does] practise and rote. I think through that as well, like, depending on what the questions are, there can be exploration and you're going to have to negotiate meaning when you're practising'.

(Interview, 10.2.16)

Edwin was an example of a teacher who had 'authoritarian' beliefs, but still felt it was important to situate RWEI when he could 'fit it in'. This addresses some of the criticisms of situating social justice issues in mathematics, as the mathematics is not being compromised. Indeed, the other criticism could be that the social issues could be trivialised (Mistele and Jacobsen, 2009). However Edwin was, himself, quite critical of trivialising issues and introducing real life mathematics simply for the sake of it:

'Well I think, a lot of times people talk about industry-centred or real-life mathematics. The mathematics involved isn't very interesting. They're not really learning anything. You've got two options. You're either doing a normal question and you just changed A and B to the name of a company or something'.

(Interview, 10.2.16)

In conclusion, Edwin's beliefs tended towards an 'authoritarian' approach. However, although he placed 'creativity' and 'discussion and questioning' lower down in his hierarchy of beliefs, the reason he situated RWEI into his lesson was that he still gave an importance to 'critical awareness of society via mathematics'. Further, Edwin mentioned that he found it difficult to think of how RWEI could be addressed in his mathematics lesson, but would teach it if he thought it was appropriate. In this case, Edwin had identified an investigation where an RWEI could be addressed with very little adaptation. Therefore, Edwin taught this lesson because he identified an appropriate RWEI opportunity in the context of a mathematics investigation. The lesson is an example that teachers, who do not prioritise a socially aware approach in their beliefs, can still situate RWEI in the lesson if the opportunity arises.

6. Tao

Tao was a mathematics teacher in a larger than average secondary school in North London. The proportion of students who speak English as an additional language is almost five times the national average and the proportion of students from minority ethnic backgrounds is over three times the national average. The students are from a wide variety of ethnic heritages and nationalities. The percentage of students for whom the school receives additional funding (the pupil premium) was almost double the national average.

(School's Ofsted Report, 2013)

Tao was a Bio-Chemistry and Natural Sciences graduate who taught mathematics at the school. He had recently taught his Year 10 class a lesson involving climate change. *(Interview, 7.4.15)*

How did Tao position RWEI in his mathematics lesson?

The context of Tao's lesson was to critically interpret information about proportions in the context of time and motion studies. Using pie charts for the representation of real-life proportional information and by reflecting on and discussing the ethics of time and motion studies, the lesson covered the following areas of the mathematics national curriculum:

Statistics

- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 9)

Tao started the lesson by asking the pupils to practise drawing and interpreting pie charts. This was done using data about pupils' favourite subjects and different makes of cars owned by people who were surveyed. He then set them a worksheet on drawing pie charts and bar charts using classroom data.

After 10 minutes, the pupils were stopped and shown a table of a time and motion study from IBM in the 60s and Tao explained what a time and motion study was. He then displayed data from a time and motion study from the 60s which analysed people's chair activity:

Chair Activity

Get up from chair	0.039 minutes
Sit down in chair	0.033 minutes
Turn in swivel chair	0.009 minutes

Based on this he asked the class to answer the following questions:

- i) Why might someone want to know this information?
- ii) If you get up from your chair 25 times in a day, how long in minutes do you spend getting up?
- iii) If you work for 8 hours, what fraction of the time do you spend getting out of your chair?
- iv) How might someone collect this information now? How might they have collected it in 1960?

This was followed by a short clip from the BBC 1 One Show (<https://www.youtube.com/watch?v=QsF0kGYgOI8>) explaining what time and motion studies were and why they were so controversial. The video clip explained that

unions were against the idea, whereas management was in favour of it. Tao started a discussion about whether time and motion studies were equitable, and revealed that as the pupils had been working today he had used three stopwatches to record the time one of the groups of pupils spent on writing, discussing and chatting off task. He put up the data and pupils were asked to draw a pie chart to represent the findings and also asked if they could guess which group he had focussed on for the study.

(Lesson Plan, 22.5.15; Lesson Observation, 22.5.15)

In relation to Skovsmose's milieus of learning (Table 1) the lesson made reference to real-world mathematics. The mathematics itself was taught in the traditions of exercise as it did not require the pupils to engage with the mathematics in an interpretive or investigative way.

Tao commented that:

Students contributed well to discussion about the IBM time and motion study.

'Students were shocked by the data I recorded on the time they spent working. This was reflected in their comments on the post-it notes. Although most pupils' feedback was that they had learnt how to draw and interpret pie charts, many commented on how it was 'creepy' or 'weird' to be watched while working.'

(Post-lesson reflection, 24.5.15)

Tao stated that he situates RWEI when the opportunity presents itself but also mentioned that he found it difficult to think of relevant examples.

(Interview, 7.4.15)

Why did Tao position RWEI in his mathematics lesson?

Table 11 : Summary of Tao's beliefs

Colour coded in hierarchical order (Uncoloured entries means belief has been dismissed):

High  Low

	THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS
Authoritarian	A collection of rules and facts	Back-to-basics numeracy	Practice and rote	Transmission and drill, no frills
Utilitarian	Unquestioned body of useful knowledge	Useful mathematics with an industry centred focus	Skill, acquisition and practice	Motivate through work relevance
Mathematics - Centred	Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate, pass on structure of knowledge
Progressive	A personalised activity	Creativity and self realization via mathematics	Activity and exploration are central	Facilitate personal exploration
Socially aware	A socially constructed practice	Critical awareness of society via mathematics	Questioning and negotiating meaning is essential	Discussion and questioning

Positioning Tao's beliefs in Ernest's framework shows that his priority is firmly situated in 'authoritarian' and 'utilitarian' approaches. However, there were also mathematics-centred beliefs such as 'understanding and application is key to progress', and 'socially aware' beliefs such as 'critical awareness of society via mathematics', which he also thought were important.

I have discussed how some teachers could have contradictory beliefs (Ernest, 1991a; Swan 2006). In Tao's case there is a contrast between how he should teach in the classroom and how he wants to teach the classroom. He explained that some of his prioritised beliefs were a reflection of this practice, whereas others were what he would like to do. For example, referring to 'back to basics numeracy', he explains:

I'd say that. In terms of the amount of time that I spend doing Mathematics teaching, I'd say most of the time that's what I'm doing. In terms of what I'd really like to be doing, I'd say that is 'creativity and self-realization via mathematics' and I think that comes in more the older the students get, and if you are giving them more some investigative tasks. That's what I'd like to be doing. This critical awareness of society via mathematics, I think when I use that it tends to be as context for probably transmitting mathematical knowledge'.

(Interview, 7.4.15)

Similarly, with reference to the theory of mathematics being 'a collection of facts and rules', Tao explained that this theory was high in his beliefs and he also regarded it as how pupils see mathematics:

"Collection of facts and rules'. I'm just going to put that right at the top because I think that's how students tend to see it. When they're first coming across Mathematics. 'Unquestioned body of useful knowledge', and again I think that's how students will quite often see it. In terms of how I see it, now it's harder. 'Structured body of pure knowledge', I think that's closer to how I see it, mathematics as a whole. It's kind of this way, this entity of information is nice'.

(Interview, 7.4.15)

However, in his reflection of the lesson Tao had mentioned that although he situates RWEI when the opportunity presents itself, he found it difficult to think of relevant examples. There was an insight into this when he discussed his own experiences of mathematics:

'This critical awareness of society via mathematics and this creativity and then in terms of useful mathematics with an industry centred focus... I'm not sure that I spent much time [laughs] actually doing that, because I never learnt Mathematics that way and I've never seen it taught that way'.

(Interview, 7.4.15)

In conclusion, Tao's beliefs seem to be predominantly centred around the 'authoritarian' and 'utilitarian' aims. However, he identified these beliefs to be prominent in his practice; they are influenced by how he believes the pupils see mathematics. Tao identified that he does situate real life critical issues in his lessons when the opportunity arises, but the fact that he did not learn mathematics in this way or, indeed, had not seen mathematics taught this way, suggests that he might have difficulties, or not have the confidence, with regard to situating RWEI in the classroom.

These factors explain why Tao's lesson, although based on a real world situation, made little reference to RWEI until near the end of the lesson.

7. Santana

Santana had a degree in Politics and International Relations and taught in a larger than average academy school. The proportions of students from minority ethnic groups, of those who speak English as an additional language, of disabled students, and of those who have special educational needs were above the national average. The number of students eligible for free school meals was considerably above the national average. In their last Ofsted inspection the school was graded as 'Good'.

(School's Ofsted Report, 2016)

Santana had recently taught a lesson on simple and compound interest and bank interest rates to her Year 8 class, in addition to discussing if it was fair that everyone should receive the initial child trust fund subscription from the government. This was in the form of a voucher for at least £250 for children born after 2002.

(Interview, 4.5.16)

How did Santana position RWEI in her mathematics lesson?

The objective of Santana's lesson was for pupils to understand government public spending figures and represent the data on pie charts. This was in a higher set Year 8 class with about 30 pupils. The lesson covered the following areas of the national Curriculum:

Working mathematically: Solve problems

- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

Statistics

- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 9)

The lesson started by asking pupils to draw a pie chart using the given raw data of musical instruments people played.

'I wanted the students to understand why we use Pie Charts and how to construct them. There was a focus on government public spending, with questions to get the students to think about their own opinions on where they think public spending should be allocated'. (Post-lesson reflection, 7.7.16)

In the discussion with the class Santana went through how to do this, demonstrating how to calculate the angles and then draw these on a pie chart. She then showed the pupils a 2 minute video from BBC Newsbeat (<https://www.youtube.com/watch?v=ANkLfBx4VkJ>), which went through 'how the government spends your money'. This involved working with the figure of £648,000,000,000 the government raised from taxes and information on how it was spent. At the end it was revealed that there was more spending than income, so the government had to borrow money and also pay £53 billion in interest on their previous loan.

Santana then put up the government spending figures for 2014-15 and asked pupils to make comparisons such as 'How much more was spent on health compared to education?'. This was followed by Santana asking the pupils: 'If you were Prime Minister, what would you want to change about this? What would you keep the same?'. From my observation, after some initial prompting, there was an interesting discussion between the pupils with various justifications as to why different areas should receive more or less spending. For example:

Teacher: *How much more was spent on health compared to education?*

(No response)

Teacher: *Can you make any comparisons?*

Pupil A: *They spend more on education than they do on defence.*

Teacher: *Don't you think that's important?*

Pupil A: *But they don't teach us anything!*

Teacher: *Then maybe that's why it needs more money.*

Pupil A: *We need to spend more on defence, otherwise there is more chance of our buildings getting knocked down by terrorists.*

Pupil B: *I think the environment is more important.*

Pupil C: *There is too much spent on Recreation, Culture and Religion.*

Pupil D: *That could have been because of the Olympics. It promotes health and other countries get together.*

Pupils were then shown the seven top areas of public spending, these included social security, health, education and defence. The pupils were asked to draw a pie chart of this data and compare spending in different areas. They were then given a task of deciding on the question: 'What public services would you spend on if you were Prime Minister?'. They were told they have been given a budget of £1000 and had to fill a table of spending for public services and then draw the relevant pie chart.

(Lesson Plan, 21.6.16; Lesson Observation 21.6.16)

With reference to Skovsmose's milieus of learning (Table 1), the lesson was taught in a real world context. The task itself required pupils to interpret data from a critical perspective and so situated itself in the 'landscapes of investigation'.

Reflecting on the lesson, Santana felt:

'Mostly, I feel the mathematics objectives were covered. It was interesting, the students found it difficult to articulate and think about their views on the equity issue, but with prompting and time they started to get into discussions. It was useful giving them time to discuss with partners where they perhaps felt more comfortable. I enjoyed the lesson and I think it is great to add another dimension to mathematics lessons, especially at KS3. Very useful when especially teaching statistic components to really bring to life where mathematics is used in the real world as well.'

(Post-lesson reflection, 7.7.16)

With regards to future lessons where she could situate RWEI, she mentioned:

'Looking at different countries' GDP and wealth or poverty figures and drawing scatter graphs comparing different things like GDP against literacy levels or average living age.'
(Post-lesson reflection, 7.7.16)

Why did Santana position RWEI in her mathematics lesson?

Table 12 : Summary of Santana's beliefs

Colour coded in hierarchical order (Uncoloured entries means belief has been dismissed):

High  Low



	THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS
Authoritarian	A collection of rules and facts	Back-to-basics numeracy	Practice and rote	Transmission and drill, no frills
Utilitarian	Unquestioned body of useful knowledge	Useful mathematics with an industry centred focus	Skill, acquisition and practice	Motivate through work relevance
Mathematics - Centred	Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate, pass on structure of knowledge
Progressive	A personalised activity	Creativity and self realization via mathematics	Activity and exploration are central	Facilitate personal exploration
Socially aware	A socially constructed practice	Critical awareness of society via mathematics	Questioning and negotiating meaning is essential	Discussion and questioning

Added : Developing Logical Minds Concrete, Pictorial, Abstract

Situating Santana's beliefs in Ernest's framework shows that she prioritises 'mathematics centred' beliefs and also has 'authoritarian' beliefs. For the most part, she discarded the utilitarian beliefs although felt that the learning aim of 'skill acquisition and practice' was important.

With regard to the aims of mathematics education, although Santana prioritised 'back to basics numeracy' in line with her authoritarian beliefs, she also felt that 'creativity and self realization via mathematics' and 'critical awareness of society via mathematics' were also important:

'I think creativity is really important and a critical awareness of society. I put them together. These two are the 'back to basics' of numeracy and getting the mathematics knowledge is, like, the base. Then if we get this creativity and getting them to think critically about mathematics and everything, then that's where we would like them to get to'.

(Interview, 4.5.16)

Similarly, even though 'activity and exploration' and 'questioning and negotiation' were not prioritised within the theory of learning mathematics, Santana still considered them to be important:

'When you learn what an apple is, first you just have an apple and you can see what an apple is. Then you might learn the word apple, why it's called an apple and then learn how to spell it. I think it's like that in mathematics, so when you just give symbols and numbers and expect them to understand, that's not how people learn; it's why you need that concrete, pictorial manipulative sometimes. I guess that links to the activity and exploration.'

(Interview, 4.5.16)

Although Santana prioritised a mathematics-centred and authoritarian approach she taught an RWEI lesson in a real world context. It seems relevant that Santana has a degree in Politics and International relations, and this could be the reason she felt comfortable and confident to refer to political discussions in a mathematical context. There was further evidence of this when she discussed future lessons which would look at the correlation between countries' GDPs and their literacy levels and mortality rates.

(Post- lesson reflection, 7.7.16)

However, although Santana did situate RWEI in her lesson, the structure of the lesson reflected her authoritarian and utilitarian beliefs, as the mathematical element was not dissimilar to an exercise the pupils might do from a textbook. On the other hand, the discussion and questioning element of the real life data raised pupils' critical awareness of society via mathematics and resonated with Santana's socially aware

beliefs which, although not considered priorities by her in the interview, were still important to her.

8. Rachel

Rachel was a mathematics graduate and taught the subject in a larger than average size, mixed secondary school in East London. The majority of the pupils were from minority ethnic groups and there was an above average proportion of those who spoke English as an additional language. The proportion of pupils eligible for pupil premium was also above average. In their last Ofsted inspection, the school was graded as 'Good'.

(School's Ofsted Report, 2013)

How did Rachel position RWEI in her mathematics lesson?

Rachel taught the lesson to a lower set Year 9 class with about 30 pupils. Rachel's lesson objective was to criticise misleading graphs. The success criteria was that all pupils should be able to identify errors in bar charts, most pupils should be able to find reasons as to why a graph is misleading, and some pupils should be able to analyse the best statement or type of graph to represent a situation. *(Lesson Plan, 19.1.16; Lesson Observation, 19.1.16)*

The lesson addressed the following areas of the mathematics in the National Curriculum:

Working mathematically: Solve problems

- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 5)

Statistics

- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data

(Mathematics Programmes of Study: Key stage 3, National Curriculum in England, 2014, DfE, Page 9)

The lesson started with Rachel asking pupils to brainstorm the names of different graphs and charts. After a short discussion, pupils were shown a pictogram of sales for cat products over five years. The appearance of the image representations for one of the years had been adjusted using uneven spacing to make the sales look greater for that year. Rachel discussed with the class why this chart was misleading and why people and organisations might create misleading charts.

The pupils were then given sugar paper, which they folded into six sections. Rachel asked them to create a poster, answering questions about five misleading graphs which were passed around in a carousel. Rachel put some prompt questions on the board:

- What's the scale on the axes?
- What's the actual increase/decrease?
- How many people are represented?
- Are labels present and correct?

Pupils were also asked to create their own 'checklist' questions. Pupils were told that they would be selected at random to present findings on a particular graph.

The graphs were not in any particular context. Examples were: a bar chart of year-by-year sales not showing any particular product or company; two pie charts - one of boys of different ages and one of girls of different ages - with no reference to the background for the data.

(Lesson Plan, 19.1.16; Lesson Observation, 19.1.16)

Rachel felt that, by the end of the lesson, pupils had a better awareness of how visual data could be misleading:

'I wanted students to be able to spot when graphs or statistics in newspapers, and other media, are manipulated to support statements which aren't necessarily true. Although the examples I used had only one taken from real life, they were all intended to be the kind of "tricks" that are commonly employed. I think some of them have come away with a greater awareness of the importance of reading and scrutinising axes and scale in detail, and a few have picked up the fact that pie charts, in isolation, only tell you proportions,

not amounts'.

(Interview, 8.12.15)

Interestingly, Rachel had taught an RWEI lesson with little reference to the real world. She did discuss how the misleading aspects of the graphs, which the pupils had been analyzing, were the types of data manipulation that pupils needed to be aware of when presented with charts and graphs in real life; however, she presented no real life examples in the data pupils worked with. As such, the lesson could be said to be situated in Skovsmose's milieus of learning (Table 1), with reference to semi-reality. Although the pupils were interpreting charts, they were doing so with the intention of identifying misleading information. So the lesson did require pupils to engage in mathematics in an interpretive manner, thus positioning itself in 'landscapes of investigation' as opposed to traditional exercise based mathematics.

Rachel reflected that by using semi- real examples, pupils were made more aware of how real data could also be misleading, and my observation of the pupil discussion verified this as pupils were able to discuss ways in which the data could be used to misinform people.

(Lesson Observation, 19.1.16; Post-lesson reflection, 25.1.16)

One could argue, however, that using some examples from a medium like newspapers would have introduced a level of validity to the lesson for the pupils.

Although a theme of RWEI ran through the lesson, the examples used could just as well be used in a mathematics lesson with no reference to RWEI. Hence there could be no criticism that the lesson in any way compromised on the mathematical content.

Rachel mentioned how she had recently taught a Year 11 lesson, discussing how shops were pricing goods. She had used the example of the surface areas of pizzas with her 11P5 class, discussing whether it was better value to buy a 12" pizza or two 8" pizzas for the same price. She commented that:

'They seemed to feel that it was a somewhat contrived application, but were surprised by the outcome!'

(Post-lesson reflection, 25.1.16)

Why did Rachel position RWEI in her mathematics lesson?

Colour coded in hierarchical order (no colour means belief has been dismissed):

Table 13 : Summary of Rachel's beliefs

High  Low

	THEORY OF MATHEMATICS	AIMS OF MATHEMATICS EDUCATION	THEORY OF LEARNING MATHEMATICS	THEORY OF TEACHING MATHEMATICS
Authoritarian	A collection of rules and facts	Back-to-basics numeracy	Practice and rote	Transmission and drill, no frills
Utilitarian	Unquestioned body of useful knowledge	Useful mathematics with an industry centred focus	Skill, acquisition and practice	Motivate through work relevance
Mathematics - Centred	Structured body of pure knowledge	Transmit body of pure mathematical knowledge	Understanding and application is key to progress	Explain, motivate, pass on structure of knowledge
Progressive	A personalised activity	Creativity and self realization via mathematics	Activity and exploration are central	Facilitate personal exploration
Socially aware	A socially constructed practice	Critical awareness of society via mathematics	Questioning and negotiating meaning is essential	Discussion and questioning

Added : **Links/ Relationships** **Remembering previous knowledge**
Analysing / Applying

Positioned in Ernest's framework, Rachel's beliefs tended towards a mathematics-centred approach. In relation to an aim of mathematics education being the transmission of a body of pure mathematical knowledge, Rachel said:

*'I'm very much into that, but of the children I teach there's probably fewer than 10 that would appreciate that. I think it's really important that it's there, for their sake'.
(Interview, 8.12.15)*

When discussing themes such as 'critical awareness of society via mathematics' or 'creativity' Rachel referred to mathematics being interesting and creative for its own sake:

'I'm very much kind of "why would bother applying?"'. It's fun for its own sake, but actually there's quite a lot of nice things. Again, it's quite specialized. Again, I would probably downgrade that. It's a nice idea, but in a class of 30, probably not always applicable'.
(Interview, 8.12.15)

In relation to the theory of mathematics, Rachel tended towards the mathematics-centred approach,

"unquestioned body of useful knowledge", that's an interesting choice of words. I suppose because Math is very absolute, is very axiomatic, once it is true, it is always true. Supposing that sense is unquestioned. There's nothing in there about linking things together, because I think that's one of the most exciting things about Math. Yes, it is a collection of actual facts and rules, but actually they will relate to each other. Actually, yes, it is facts and rules, but actually it's also, how do you apply those and how do they all relate to each other. It's the relationships I think, the most exciting things'.
(Interview, 8.12.15)

When discussing 'facilitating personal exploration' within the theory of learning mathematics, Rachel returned to the difficulties of working with these ideas in large classes,

"Facilitating personal exploration'. I think that's quite interesting. I really like the idea of it but I find it very, very difficult to do in a class of 30. There needs to be real depth of understanding of the pedagogical side of what hints the children need, what are the right questions and the right phrasings'.
(Interview, 8.12.15)

Although Rachel had predominantly mathematics-centred beliefs and placed 'critical awareness of society via mathematics' low down in her hierarchy of beliefs, she did prioritise the socially aware beliefs relating to questioning:

'[D]iscussion means something very different in Math, as opposed to other subjects particularly. But no, I think questioning and discussion has its-- it definitely has its place. Discussion and questioning. I do that quite a lot. My typical example is-- my typical lesson is not the exploration, it's this: "Let's have a look at this problem. How might we approach it? Let me do one example. I do that quite a lot. Questioning is really useful because you can't open up a brain and see what's going on in there. It's kind of the next best thing we can do'.
(Interview, 8.12.15)

Rachel's mathematical beliefs were predominantly mathematics-centred with some elements of 'authoritarian'. Indeed, Rachel described mathematics as an 'absolute' subject. Her beliefs are reflected in the lesson, as her examples and exercises had little reference to the real world. Indeed, her lesson objective was to criticise misleading graphs; this was an objective which could have been taught without any reference to real world. However, she chose to introduce an RWEI into the lesson because, as mentioned in the previous section, she wanted pupils to spot when the media manipulate data to support statements which are not necessarily true.

Rachel thought that some of the more socially aware and progressive beliefs, such as 'facilitating personal exploration', 'creativity', or 'critical awareness', were interesting and nice ideas, but difficult to implement in large groups. This suggests that there could be issues of confidence when working with these ideas in the classroom.

6. Discussion of analysis

This chapter addresses the main research question by analysing ‘how’ and ‘why’ secondary school mathematics teachers might situate real-world equity issues in the classroom. As I discussed in Section 2.5 (Original contribution to knowledge), the eight case studies will be analysed in order to identify the different ways in which mathematics teaching can address RWEI and the reasons teachers in the study have chosen to take this pedagogical approach.

The analysis will be in two parts. Firstly, it will look at ‘how’ secondary school mathematics teachers might situate real-world equity issues in the classroom, and then at ‘why’ they might do so. Both sections will be considered in order to arrive at the overall findings.

6.1 How might teachers situate RWEI in the secondary mathematics classroom?

In order to identify ‘how’ teachers situate RWEI in the secondary mathematics classroom, this section will analyse the different ways in which the lessons were structured and the areas of the curriculum which were addressed by the lessons.

In Section 2 (Literature Review), I discussed that there is little in the way of research in the field of teachers’ practice of situating values based lessons in the mathematics classroom. I also discussed that critical mathematics education and social justice are framed within certain conditions (Skovsmose, 1996; Bishop, 2010; Nolan, 2009). While the conditions identified are a very important aspect of critical mathematics education, they might prove to be a barrier in certain circumstances. Therefore, analysing the structure of the lesson across the eight case studies identifies the different ways in which RWEI can be situated in the classroom, and explores if this can take place while compromising on the conditions associated with critical mathematics education.

In order to analyse the structure of the lessons across the study, I shall return to Skovsmose’s milieu of learning matrix as a framework. The eight case study lessons will be located in the learning matrix, and this analytical process is further detailed

later in this section.

Lessons which address critical mathematics education and social justice are often associated with statistics and data handling. For example, Gutierrez (2013) discusses the idea of 'political awareness' through mathematics as being able to analyse society and identify injustices in the world. In the mathematics classroom, she translates this as pupils making sense of data to analyse injustices in society. Nolan (2009, p210) too, while critical of simplistic models of teaching mathematics for social justice, mainly refers to pupils learning "traditional mathematics content through the "statistics and figures" of social justice, making them not only more mathematically literate but also critically aware and action-oriented citizens". If there is an implication that issues of critical mathematics and social justice are best addressed by a certain area of the curriculum, then this introduces a limitation to incorporating RWEI into mathematics. Therefore, it is important to note that RWEI can be addressed throughout different areas of the mathematics curriculum (Coles et al, 2013). I decided to use the mechanism of tabulating the curriculum areas addressed in the lessons to identify if some areas of the mathematics curriculum are more appropriate for situating RWEI than others. In order to do this, the curriculum areas covered in the eight lessons will be tabulated under the broader titles from the National Curriculum: number, algebra, ratio, proportion and rates of change, geometry and measure, probability and statistics.

6.1.1. Analysing the lesson structure using Skovsmose's milieus of learning.

Through an analysis of the case study data of how each teacher situated RWEI in their lesson (see Analysis of participant data: Section 5.1), this section will determine the different ways in which teachers situate these issues in their classroom.

In the literature review I discussed Skovsmose's milieus of learning. Within this framework, the 'landscapes of investigation' are classroom situations that support investigative work. Skovsmose (2001) defines this milieu as a teaching environment where teachers invite pupils into a process of exploration through questions such as 'what if.....?', and 'why is that?'. What is important is that pupils take charge of the lesson. This form of democratic participation reflects the type of conditions within which critical mathematics and social justice takes place in the classroom (Skovsmose,

1996; Bishop, 2010; Nolan, 2009). Based on the analyses in the previous chapter, the lessons have been placed within Skovsmose's framework to identify if these conditions are always necessary for critical mathematics to take place (Table 14). The limitations of the matrix have also been discussed, because within a lesson there are different possibilities of where a part of the lesson could be placed. Skovsmose (2001) gives the example of some mathematics exercises which can prompt problem-solving activities and move into genuine mathematical investigations. While recognising that the purpose of the milieu of learning matrix is not to provide any clear cut classifications (Skovsmose, 2001), and that positioning of the lessons within the matrix cannot be a definite or even a completely accurate representation, it is important to note that the matrix does provide a visual platform, showing how the lessons were structured in terms of the types of activities. For the purposes of this analysis, and in order to minimise the limitations of the matrix, the framework has been adapted to place lessons along each milieu to give an idea of how the lessons might have been different even within each milieu. For example, Minervia's lesson (M) was situated within the landscapes of investigation but, as discussed in the analysis of her lesson, it would have been better situated towards the traditions of exercise. Further, some lessons appear in more than one milieu, as there is a shift in the pedagogical approach during the lesson. For example, in Tao's lesson (T), pupils did some traditions of worksheet-based exercises, but later in the lesson, there was also reference to mathematics in a real-life context.

Table 14 : Lessons positioned within Skovsmose's milieus of learning

Key: (A) Aron*; (E) Edwin*; (F) Fabia; (J) Jason; (M) Minervia*; (R) Rachel*; (S) Santana; (T) Tao

*Mathematics degree

	Traditions of Exercises	Landscapes of Investigation
References to pure mathematics	T1	
References to semi reality	A1*	R* E*
Real life references	T2	A2* M* J S F

It would be expected that lessons involving critical mathematics and social justice would be positioned at the bottom right corner of Table 14, that is lessons with reference to real life taught within landscapes of investigation. Out of the eight lessons, three were taught like in this milieu; these were the lessons taught by Jason, Santana and Fabia. These lessons were taught in a more exploratory way with pupils taking charge of their learning and engaging in a process of exploration while working within real life contexts.

The remaining lessons from the study are distributed in other parts of the matrix, suggesting that RWEI lessons can take place even if they are not positioned in the right hand corner of the matrix. Most of these lessons also let pupils take charge of their learning and included elements of exploration, but these elements did not necessarily run throughout the lesson in the same way they did in the lessons by Jason, Santana and Fabia. This did not make them any less valid or effective as RWEI lessons. Indeed,

Edwin's (E) and Rachel's (R) lessons did not make any reference to the real world, though they still engaged the pupils in thinking about real world issues.

In Edwin's lesson, pupils had to find the optimum shape for a drinks can to contain a fixed volume of liquid. Although he mentioned the environmental impact of more than 7 billion cans being sold in the UK every year, he made no further reference to real life contexts, such as discussing the cans of soft drinks that are different in size from those of some energy drinks, or exploring which cans might be responsible for more wastage, or putting the problem in the context of a real company like 'Pepsi'. However, the worksheet Edwin had used, from the Core Mathematics website, had pictures of fizzy drinks cans (Figure 5) which, although not attributed to a real company, looked like Coke cans. As such, the context of the lesson was easily relatable to real life contexts by many of the pupils.



Figure 5 : Pictures from worksheet

(<https://www.stem.org.uk/resources/elibrary/resource/282472/max-cylinder>)

In Rachel's lesson, pupils were given a number of graphs and charts in which they had to identify why the information presented was misleading. As part of making pupils aware of data manipulation, Rachel discussed why people and organisations might create misleading charts; however, she made no reference to real world examples in the context of the graphs and charts the pupils worked on. At least one example of a real world misleading chart or graph would have introduced a level of validity to the lesson. However, the pupils were still able to discuss the idea of misleading data and, because of the discussion, understand how this could relate to real-world situations. So, even though the lessons had little or no reference to real life, they still engaged pupils in critically thinking about real world issues through mathematics.

Except for Tao's lesson, all the lessons were placed, within varying degrees, within the landscapes of investigation. Aron's lesson was initially positioned within the traditions

of exercise with reference to semi-reality, but then moved towards the landscapes of investigation. Both Aron's and Minervia's lessons were placed on the borderline of 'traditions of exercise' and 'landscapes of investigation'; Skovsmose (2001, p128) describes this vertical line as a very 'broad' line, representing a huge terrain of possibilities. In Aron's case, he had introduced some exercises involving payday loans. Based on different interest rates, time periods and fees, pupils had to calculate the repayment amount and decide which loan was the best and which the worst, and explain why. Although pupils made decisions based on the mathematics, there was limited exploration involved. The last question was placed in the context of a real loan company charging an interest rate of 292% APR for their loan. This prompted more discussion about both the mathematics and the ethics related to the high percentage rate. As there were elements of inviting pupils into a process of exploration through questions, the lesson was placed on the borderline of 'traditions of investigation'. Similarly, Minervia's lesson, whilst relating to airline flights in the real world and containing elements of exploration, included more questioning and discussion near the end of the lesson. The positioning of the lessons within the matrix has an element of subjectivity, but the purpose of the matrix is to enable discussion rather than to provide a clear-cut classification.

The positioning of the lessons is certainly more concentrated where lessons refer to real life contexts and are taught in an investigative way. However, there are also RWEI lessons that are taught in the other milieus, and therefore not framed within the conditions appropriate for critical mathematics education (Skovsmose, 1996; Bishop, 2010; Nolan, 2009). While the conditions, such as democratic participation, are desirable, many teachers may be working in environments where these approaches are not possible or are difficult to execute.

6.1.2. Analysing lessons by curriculum areas

Through an analysis of the areas of the curriculum addressed by each of the eight lessons (see Analysis of participant data: Section 5.1), this section will determine the different areas of the curriculum covered by the teachers in the case study. The topics covered in each lesson have been tabulated under the broader areas of the

Mathematics National Curriculum. Each entry is initialled in the same way as Table 14, in order to identify the teacher who taught the lesson. Where more than one teacher has covered an area of the curriculum, this will appear multiple times in the relevant column. For example, both Santana and Tao covered 'construct and interpret pie charts', so this features twice under 'Statistics'.

Table 15 shows that lessons were not confined to particular areas of the curriculum, and that all of the six broad areas of the curriculum were covered across the eight case studies. Although lessons that address social justice are often associated with the areas of data handling and statistics, there is clear evidence that other areas of the curriculum may be just as appropriate for situating RWEI.

The table suggests that probability and statistics have been taught, for the most part, by teachers who do not have mathematics degrees, whereas algebra, geometry and proportion have been taught by teachers who do have mathematics degrees. Although it is not possible to draw any conclusions from such a small sample of case studies, there is an implication that teachers with mathematics degrees are more confident to situate RWEI in wider areas of the curriculum. Future research, where there may be fewer limitations with regards to time and financial constraints, may be able to investigate this area further.

Table 15 : Curriculum content

Key: (A) Aron*; (E) Edwin*; (F) Fabia ; (J) Jason; (M) Minervia*; (R) Rachel*; (S) Santana; (T) Tao

* Mathematics degree

Number	Algebra	Ratio, proportion and rates of change	Geometry and Measures	Probability	Statistics
<p>1. Compare two quantities using percentages (F)</p> <p>1. Express one quantity as a percentage of another, compare two quantities using percentages (A)*</p> <p>3. Use a calculator and other technologies to calculate results accurately and then interpret them appropriately (E)*</p> <p>4. Interpret mathematical relationships graphically (E)*</p>	<p>1. Identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically (E)*</p>	<p>1. Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics (A)*</p> <p>2. Compound interest (A)*</p> <p>3. Using scale factors, scale drawings and maps (M)*</p> <p>4. Convert between related compound units (M)*</p>	<p>1. Interpret and use bearings (M)*</p> <p>2. Derive and apply formulae to calculate and solve problems involving prisms (including cylinders) (E)*</p>	<p>1. Predict the outcomes of future experiments (J)</p> <p>2. Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions (J)</p>	<p>1. Construct and interpret pie charts (S)</p> <p>2. Construct and interpret pie charts (T)</p> <p>3. Construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms (R)*</p>

6.2 Why might teachers situate RWEI in the secondary mathematics classroom?

The second section of this chapter analyses the participants' interviews, post-lesson reflections and observation notes to establish why teachers might situate RWEI in the secondary mathematics classroom. There are four questions I have identified to in order to explore this:

1. When situating RWEI in their lesson, what were the constraints or limitations identified in relation to situating RWEI in the mathematics lesson?
2. How did the teachers' beliefs influence why they situated RWEI in their lesson?
3. What other underlying reasons were there as to why teachers situated RWEI in their lesson?
4. Are there differences between the mathematical beliefs of early-career teachers from diverse academic backgrounds?

6.2.1. When situating RWEI in their lesson, what were the constraints or limitations identified by the teachers?

When considering why teachers decide to situate RWEI in their lessons, it is also important to discuss any constraints that were identified. There were two prominent reasons as to why the teachers felt they were constrained with regard to situating RWEI in their lessons. In the case of Aron, Jason and Fabia, the opportunities for teaching RWEI were limited by the demands of the curriculum:

'I will look to make lessons more real-world focused and I wish the curriculum was built around these ideas'.

Aron (Post-lesson reflection, 21.5.16)

Jason explained that he felt he could not situate RWEI regularly in his lessons because of the limitations of the curriculum:

'I would like to but I am under pressure to raise standards in my classes and that is measured by their performance in exams. The curriculum is designed to be taught by topics and not through individual problems., This makes it difficult to teach the subject like this. I think with the current system, it would be impossible to teach every lesson through a real-world equity issue, but I would like to raise them if it was appropriate. I

think the most suitable balance would be having one lesson a half term on a topic like this’.

Jason (Post- lesson reflection, 24.5.16)

Fabia identified that lessons like this did not feature much in her school because of an ‘exam focus’. However, she felt it was important to teach these lessons when the opportunity arose:

‘Activity and exploration’, I think they are important for learning but not used as much in schools, especially in secondary because they tend to take more time and tend to be less exam focused, so you don’t give pupils the chance to do more activities or explore.’

Fabia (Interview, 30.4.15)

Tao, Edwin and Rachel may have also felt that the demands of the curriculum were inhibiting, with regard to opportunities for situating RWEI in their lesson. However, their interviews revealed a lack of confidence when considering how to situate RWEI in their teaching. Edwin and Tao both implied that there was a lack of confidence when thinking of how to situate RWEI in their lessons.

‘I find it very difficult to think of real-world equity issues that I would actually address in my mathematics lesson; however, if I came across one that I deemed good and appropriate, I would be very happy to teach it’.

Edwin (Post-lesson reflection, 24.5.16)

‘It’s difficult to think of relevant examples. But if the opportunity presented itself, I would teach real-world equity issues as part of my lesson’.

Tao (Interview, 7.5.15)

In Rachel’s case, she did not mention difficulties in relation to thinking of examples of RWEI, but rather expressed a lack of experience or confidence when working with these ideas in large classes. When discussing critical awareness via mathematics, she commented:

It’s a nice idea, but in a class of 30, probably not always applicable.

Rachel (Interview, 8.12.15)

Further, with regards to facilitating personal exploration, she said:

“Facilitating personal exploration”. I think that’s quite interesting. I really like the idea of it, but I find it very, very difficult to do in a class of 30. There needs to be real depth of

understanding of the pedagogical side of what hints the children need, what are the right questions and the right phrasings.'

Rachel (Interview, 8.12.15)

There is further evidence of possible confidence issues in the fact that the three participants made little or no reference to real world contexts (see Table 14). However, in the cases where there was little or no reference to a real-world context, this did not make the lesson any less valid in terms of RWEI, as the examples used contexts which could be transferred to the real world. For example, Edwin wanted pupils to find the minimum materials required to make a 330ml drinks can. I discussed, in the last chapter, that Edwin could have introduced a real-world context by discussing different brands of drinks and which, if any, might be responsible for wastage. However, the worksheet did have pictures which were typical of many soft drinks cans (Figure 5) and this helped pupils relate to the real-life context.

6.2.2. How did the teachers' beliefs influence why they situated RWEI in their lesson?

In the study, the data for teachers' beliefs had been gathered through interviews and card-sort arrangements, where an adapted version of Ernest's model of Mathematics-Related Belief Systems (1991) was used as card-sort prompts for the interview. Ernest recognises that it is common for teachers to combine positions in the model. Therefore, when analysing interview and card sort data, it is not surprising that the teachers in the study had identified with beliefs across the five belief systems (see Analysis of participant data: Section 5.1). However, in all cases the summary of participant's beliefs tended to prioritise a particular belief system and, in some cases, there were some belief systems which, even if they were not prioritised, were certainly prevalent. Further, beliefs were not necessarily consistent, as we know that particular beliefs may only apply to certain situations (Swan, 2006). For example, Jason mentioned that his opinion is different for different classes. He used his lower set Year 11 class as an example of how his beliefs could be different for a class with different aspirations, where there is an element of 'hoop jumping' to get the required qualifications (Jason, Interview 27.4.15). However, these beliefs were limited to particular situations and classes and did not influence the predominant mathematical beliefs of the participants.

Table 16 shows the predominant belief system of each participant. Where there are other prevalent beliefs, they have been listed in order of hierarchy.

Table 16 : Participants predominant belief system

Name	Predominant belief 1	Predominant beliefs 2	Predominant beliefs 3
Aron*	Utilitarian	Mathematics centred	Socially aware
Jason	Mathematics centred		
Fabia	Socially aware	Progressive	
Minervia*	Progressive	Socially aware	Mathematics centred
Edwin*	Authoritarian	Utilitarian	Mathematics centred
Tao	Authoritarian	Utilitarian	
Santana	Mathematics centred	Authoritarian	
Rachel*	Mathematics centred		

* Mathematics degree

As critical mathematics and social justice are associated with questioning, discussion and personal exploration, we could be led to believe that all teachers who situate RWEI in their lesson would have ‘progressive’ and ‘socially aware’ beliefs prioritised within their belief system. However, only one participant, Fabia, had prioritised the socially aware approach in her belief system. Although Aron and Minervia had identified a socially aware approach higher up in their belief system, Aron had prioritised a ‘utilitarian’ approach and Minervia a ‘progressive approach’.

Six of the eight teachers in the study prioritised beliefs other than ‘socially aware’ or ‘progressive’, suggesting that these teachers can still have some type of commitment to situating RWEI in their lesson. In the case of Santana, who prioritised ‘mathematics centred’ and ‘authoritarian’ beliefs, the lesson involved pupils participating in decision-making mathematics throughout the lesson, deciding how money from taxes should be distributed by the government and drawing pie charts to represent their ideas, so positioning the lesson firmly within an investigational landscape with reference to real life. She also mentioned how she planned to compare GDP, wealth and poverty figures

across different countries, in a future lesson. Similarly, for Aron, who prioritised a 'utilitarian' belief, the lessons started as traditional exercises and then moved towards an investigation-based study, where pupils were making decisions based on the mathematics they were engaged with. Therefore, teachers who have 'authoritarian' or 'mathematics-centred' beliefs can still have commitments to situating RWEI in their mathematics lessons.

As well as teaching a lesson which situated RWEI for this study, most of the participants discussed how they had taught such lessons in the past or were planning to teach some in the future. Rachel mentioned how she had taught a recent real world lesson to her Year 11 class. In the case of the other participants, Jason and Fabia gave examples of RWEI lessons they would be teaching in the future. Santana, Aron and Minervia were able to discuss the RWEI lessons they had recently taught as well as future lessons they were planning to teach. Edwin and Tao did not discuss any lessons, but did mention they would teach RWEI if the opportunity arose; Tao had recently taught a lesson on climate change to his Year 11 class. Hence, all eight participants had not merely simply taught a one-off RWEI lesson for this study, but had some commitment to addressing issues of social justice in their teaching.

Therefore, there was no theme arising with regard to the predominant belief systems of the participants and why they situated RWEI in their lesson. However, situating RWEI in their lessons was not for the sake of the study, as other RWEI lessons had been planned or had been taught by the participants. There also seemed to be underlying reasons as to why the teachers were situating RWEI in their lessons. This was related to something they felt, as teachers, was important for their pupils. Although not completely unrelated to beliefs, this is something that was evident in their interviews, and could be seen as something they saw as the underlying responsibility they have as teachers preparing pupils for the real world. I have discussed this in the next section 'underlying reasons'.

6.2.3. What other underlying reasons were there as to why teachers situated RWEI in their lesson?

Most of the participants did not prioritise 'socially aware' or 'progressive' beliefs, but, as mentioned in the previous section, they were all committed to situating RWEI in their lesson, albeit to different degrees. As well as their beliefs there were underlying reasons as to why some participants situated RWEI in their lesson.

Rachel's mathematical beliefs were predominantly 'mathematics-centred' with some elements of 'authoritarian'. As discussed earlier in this section, Rachel also mentioned a lack of experience or confidence when working with these ideas in large classes. However, she still felt that it was important for her pupils to be aware of how graphs or statistics in the media could be manipulated to misinform people.

'I wanted students to be able to spot when graphs or statistics in newspapers, and other media, are manipulated to support statements which aren't necessarily true'.

Rachel (Interview, 8.12.15)

Aron prioritised 'utilitarian' beliefs and identified the structure of the curriculum as a constraint to situating RWEI in the lesson. With regard to critical mathematics, he remarked: *'I don't know if that is what I would do in my mathematics education'*. However, he identified that it was a priority that pupils were mathematically literate:

'I want kids to know which mortgage is best for them, I want kids to know how many litres of paint they need to paint their house and figure out that kind of stuff, and that's not necessarily the focus of education as it currently stands..... I think that it is a priority is that kids are mathematically literate. Pupils aren't taught about taxes, pupils aren't taught about retirement or interest rates. There's no class for that and the incentives aren't there for the mathematics departments to teach that, but there should be, because we are sending pupils out into the world, not teaching them how to figure out 'if I got a 1200% APR what am I going to pay next month?', and that's ridiculous. And a lot of pupils in the demographics of this school will be going for a payday loan or pawning something or taking a job and not realising how little they will be paid'.

Aron (Interview, 7.5.15)

Fabia had prioritised the 'socially-aware' and 'progressive' approaches as part of her belief system. Beyond this, she had concerns about her pupils and their knowledge of real-world issues such as elections and the voting system:

'Pupils' prior knowledge of politics in the UK was not strong, and so I had to adapt to make sure that pupils understood all the key terms. In particular, they found it difficult to grasp what a constituency and a seat were'.

Fabia (Interview, 18.12.15)

Jason prioritised the 'mathematics-centred' beliefs and was critical of constructivist approaches in teaching mathematics. However, he had been teaching in the school for three years and seemed to be in agreement with the school ethos with regard to improving pupils' aspirations through their academic development.

What I thought was particularly beneficial was how the relevance would increase their aspirations. This is something that I will be able to refer back to in future lessons.

Jason (Interview, 27.4.15)

The four examples I have used imply that for some teachers the reason for situating RWEI in their lesson is an underlying concern for their pupils, which is not necessarily reflected in their mathematical belief system. This concern is not only in relation to areas of critical mathematics education. For example, Jason taught RWEI lessons as they would increase the pupils' aspirations. However, he changed his pedagogic approach to 'teaching for the test' for his Year 11 group:

A lot of my Year 11s right now have aspirations that they want to go onto college; some of them have football coach type aspirations some have picked a school where they can go to do plumbing and they need to be able to get their C in mathematics in order to do that. And my responsibility is not to make them understand the mathematics in that situation; my responsibility is to make sure they have the opportunity to be able to go to that college – so it is a hoop jump.

Jason (Interview, 27.4.15)

In both cases Jason's underlying motivation is his concern for the pupils. As a practitioner-researcher, Gutstein (2003) expresses similar underlying beliefs when he describes his larger goals as a teacher. As a mathematics teacher, Gutstein (2003) explains that, in addition to his mathematics-specific objectives, he identified larger goals such as helping develop the pupils' social and political consciousness, sense of agency and social and cultural identities. Hence, beyond the mathematical beliefs of teachers, there is also a concern that teachers have for other aspects of pupils' development.

6.2.4. Are there differences between the mathematical beliefs of teachers from diverse academic backgrounds?

The summary of the participants' predominant mathematical beliefs (Table 16) does not reveal any emerging themes. This suggests that teachers' mathematical beliefs are not related to their academic backgrounds. A larger sample with a fuller range of participants from diverse academic backgrounds may have revealed some themes with regards to differences in mathematical beliefs of teachers from diverse academic backgrounds.

6.2.5 Are teachers with non-mathematics degrees are more likely to engage with RWEI than teachers with degrees in mathematics?

In relation to the participants' academic backgrounds, Chapter 5 (Analysis) discusses if teachers with non-mathematics degrees are more likely to engage with RWEI than teachers with degrees in mathematics. In particular, this could be the case as the participant might feel more confident about using knowledge from their degree subject in a mathematical context. Table 17 shows the degree discipline of the participants with non-mathematics degrees and the RWEI they situated in their mathematics lesson.

Table 17 : RWEI situated by participants with non-mathematics degrees.

	Degree discipline	RWEI addressed
Jason	History	Government and NHS
Fabia	Classics	Voting system
Tao	Bio-chemistry and Natural Sciences	Time and Motion Studies
Santana	Politics and International Relations	Government spending

Table 17 shows there is no obvious link between the participants' degree disciplines and the RWEI they situated in their lesson. Only Santana's degree in Politics and International Relations had a direct relation to the RWEI taught. However, there are a number of factors to consider here. As identified in Section 6.6.2, these were not a one-off lesson for the participants, so it is unlikely that they will always teach RWEI lessons

relating to their degree subject. Further, this sample only covers some subjects. Curriculum content in subjects such as geography or economics might provide more opportunities for teachers to situate RWEI in their lesson.

7. Findings and implications

The aim of the study was to develop an understanding of how and why Secondary School Mathematics teachers might situate real world equity issues in the classroom. Based on the analysis of the data, the following points in this section, address the findings of the study

1. RWEI can be effectively situated in mathematics lessons even if there is no reference to real life contexts. In the introduction, I defined real world equity issues (in the context of mathematics education) as real world issues which might be critically examined in the mathematics lesson in order to encourage pupils to be democratic citizens who are critically literate through mathematics. I also discussed the advantages of using real world examples in mathematics. However, in the analysis (Section 6.1.1) there were two examples of lessons which did not refer to any real world contexts. I discussed that although the lessons would have benefited from the inclusion of real world examples, they were still successful in initiating the pupils to think critically about their world. As such, 'Real-world Equity Issues' needs re-defining. I would still define RWEI as 'real-world issues, which might be critically examined in the mathematics lesson and so encourage pupils to be democratic citizens who are critically literate through mathematics'. However, I would add that in order to critically examine real-world issues, the lesson does not necessarily have to be in a real-world context. Skovsmose (2001) gives the example of a semi-real situation of a grid representing eleven horses in a race. Pupils are asked to bet on a horse and two dice are thrown to move the 'horses' forward on the grid until one reaches the finishing line. Although this has no reference to a real-life context, it can still prompt an RWEI discussion about betting. For example: 'Who goes betting? 'Who are the winners?'; 'Is the national lottery also a form of betting?'

2. Although conditions of democratic participation are seen to be necessary for critical mathematics education to take place in the classroom, not all RWEI lessons need to be framed within these conditions.

3. RWEI lessons are not confined to particular areas of the curriculum. Although lessons that address areas of critical mathematics and social justice are often

associated with data handling and statistics, there is clear evidence that other areas of the curriculum are just as appropriate for situating RWEI.

4. Teachers can feel constrained with regard to situating RWEI in their mathematics lesson. One group of participants in the study identified that opportunities for teaching RWEI were limited by the demands of the curriculum. Interviews with other participants in the study revealed that some had a lack of confidence when considering how to situate RWEI in their teaching.

5. Some teachers situate RWEI in their lessons because they have an underlying concern for the pupils. This underlying concern is not necessarily reflected in the teacher's mathematical belief system.

6. Analysis of the participants' predominant mathematical beliefs suggests that teachers' mathematical beliefs are not related to their academic backgrounds.

7.1 Implications for mathematics teaching

Situating RWEI in the mathematics lesson implies that the lesson should make references to real world contexts. Further, when working within the broader concepts of critical mathematics and social justice, the classroom should be an environment of democratic participation where pupils can engage in discussion and make choices (Skovsmose, 1996; Bishop, 2010). That is not to say that mathematics should not have periods of 'consolidation' and work on exercises in a purely mathematical context. Referring to the milieus of learning matrix, Skovsmose (2001) supports the movement of mathematics education between different milieus. Nevertheless, the conditions of democratic participation and reference to real life contexts seem to be necessary for RWEI to be situated in the classroom. I agree that these are the ideal conditions for issues such as critical mathematics, social justice and RWEI to take place. Indeed, there has been criticism when issues of social justice have taken place in the mathematics classroom without these conditions in place (Nolan, 2009). However, in Section 6.1, I discussed how, in certain circumstances, these conditions might be a barrier to critical mathematics and social justice. The educational landscape is constantly changing and RWEI may have to be situated in contexts where compromises have to be made.

Many new academies and schools hold beliefs which are strongly in line with the 'absolutist' philosophy of mathematics education.

Head Teacher Katherine Birbalsingh explains the style of teaching at Michaela school which involves 'imparting' knowledge:

'We have the teacher standing at the front and imparting knowledge. We believe the teacher knows more than the children. Most teachers in Britain do not believe that. They believe that the children and teachers all know pretty much the same stuff, which is why the children just need to be guided by the teacher as opposed to being taught by the teacher'.

(The Guardian newspaper, 2016)

If issues such as critical mathematics, social justice and RWEI are to exist within this new order, they may have to do so with an element of compromise, otherwise they may remain as unrealisable ideals in mathematics education which we rarely see practiced in the classroom. The eight case studies in this research include examples of RWEI situated in a range of ways in the mathematics classroom by teachers with differing mathematical beliefs and from a range of academic backgrounds.

This study demonstrates how secondary school mathematics teachers can situate RWEI in their classroom. It also presents reasons as to why these teachers situate RWEI in their lessons from eight different perspectives. Through the eight case studies, there is clear evidence of several different ways in which RWEI can be situated in a secondary school mathematics lesson. In the 'Introduction' and 'Literature Review', I have discussed how education relating to morals and ethics, for the most part, does not feature in the mathematics classroom. Under pressure to perform in exams, schools sideline moral, social and cultural issues, so making it difficult for teachers to situate such practice in their classroom. The eight case studies in this research have demonstrated different ways in which it is possible to situate RWEI in the secondary mathematics classroom and why some teachers decide to do this. These case studies provide mathematics teachers examples and also inspiration in the area of teaching mathematics from a critical mathematics education perspective.

In Chapter 5 (the Literature Review) I discussed how mathematics teachers may be reluctant to take risks and encourage learning processes such as RWEI. Therefore, important to this study is the fact that the participants have an interest in RWEI, but are not experts in the area. Indeed, the study established that some of the participants had identified their difficulties in situating RWEI in lessons because of the demands of the curriculum or a lack of confidence in this particular area. In some cases, teachers might find it difficult to come up with specific examples of real life contexts, but still be able to teach an RWEI lesson.

Also important is the fact that the participants had different mathematical beliefs. These ranged from Minervia prioritising 'socially aware' beliefs to Rachel, describing mathematics as an 'absolute' subject. Hence, teachers need to realise that situating RWEI in mathematics lessons is not a pedagogical approach accessible exclusively to teachers who have 'fallibilist' or predominantly 'socially aware' beliefs.

In conclusion, the implications for mathematics teaching is that teachers understand that situating concepts of critical mathematics and social justice in the secondary mathematics classroom is a pedagogical approach which can be practised by teachers from wide ranging mathematical beliefs. RWEI applies to all the areas of the National Curriculum and can be taught without reference to real world contexts. Although conditions of democratic participation are desirable, there are environments where this may not initially be possible. That is not to say that situating RWEI in the classroom is reduced to referring to a set of examples taught, or even delivered, to pupils, hence treating critical pedagogy as nothing more than a teaching technique rather than a means to an end (Ernest, 2016).

7.2 Limitations of the research

This was a case study research where conclusions were drawn through the exploration of eight cases. The case studies were conducted through observations and semi-structured interviews. In Section 3.3.2 (Credibility and dependability of interviews), I have discussed some of the possible limitations of interviews. I have also discussed how I have addressed these limitations in order to maintain a level of credibility and reliability. Further, the two frameworks used for the purposes of analysis in the study,

namely Skovsmose's milieu of learning matrix (Skovsmose, 2001) and Ernest's model representing range of teachers' mathematics related belief systems (Ernest, 1991b), have limitations. Skovsmose's milieu of learning matrix was used to analyse the eight case-study lessons by situating them within relevant parts of the matrix. For the overall discussion of the analysis across all eight case studies. the lessons were then situated in an adapted version of the milieu of learning matrix, where lessons could be positioned at various points of the matrix to allow for comparison of the structure of all eight lessons. The limitations of the matrix have been discussed in Section 6.1.1, and it is recognised that the purpose of the matrix is not to provide definite classifications. The purpose of the adapted version of the matrix is to allow for a visual comparison of the structure of the eight case-study lessons. Similarly, an adapted version of Ernest's model representing a range of teachers' mathematics-related belief systems (Ernest, 1991b) was used as prompts for the interview (Section 3.3) and also to visually summarise the participants' beliefs (Section 5.1). Ernest (1991b) identifies that the system has limitations as it makes several assumptions and could be seen as simplistic. However, as a theoretically well-grounded model. it provides an appropriate framework for teachers to consider their belief system. Further, in Section 2.2 I have discussed the possible complexities of addressing beliefs in educational research; therefore. a simplified model of mathematical belief systems might be more appropriate for this study. as it would be easier for participants to relate to.

A further limitation was that only eight teachers were interviewed and only one of each teacher's lessons was observed for this case study. Although the intention of this study is not to make generalisations, a larger sample size would have provided further insights for the conclusion. For example, in Section 7, when identifying if there was any connection between the participants' degree subject and the RWEI topic they taught, no theme was identified. However, a larger sample for the case study would have covered a wider range of subjects. and so there might have been themes arising. Observing the same sample over a number of lessons, as opposed to just the one taught by each teacher, might also have identified themes in this area.

Ultimately, taking into account factors such as cost and time means that there will be limitations, in varying degrees, in most studies. However, I have identified the limitations of this study and taken steps to minimise these limitations as far as possible.

7.3 Contribution to the field

I propose that the findings of this study are ethical, credible and dependable. Further, appropriate tools are used for the methodology and analysis of data. Indeed, for the purposes of interviews and analysis, the study uses frameworks based on the research of leading academics in the areas of critical mathematics and teachers' beliefs.

Dealing with mathematics education and values is seen to be problematic as there is a lack of research evidence relating to social justice and values teaching. Although there have been a limited number of significant studies in the field of critical mathematics practice in the classroom, these mainly report on practice in the area of critical mathematics and social issues but do not relate practice with practitioners' beliefs. This study was motivated by a lack of research in the area of teachers' beliefs and related practice in the context of critical mathematics education and so makes an original contribution to this field by focussing on how and why secondary school mathematics teachers might situate real-world equity issues in their lessons.

Although other studies have referred to Ernest's model of mathematics-related belief systems, this study makes an original contribution to the field as the methodology and analysis have used an adapted version of the model and combined it with Skovsmose's Milieus of Learning matrix in such a way as to reveal teachers' beliefs, thought processes and classroom practice in order to conjecture links.

In Section 1.2 (Mathematics and the curriculum), I discussed how successive governments have addressed areas such as citizenship and morals in the national curriculum. Currently, schools are legally obliged to provide a spiritual, moral social and cultural education (SMSC) and all schools which follow the National Curriculum must teach the Citizenship programmes of study. Some of the content the government has stipulated that pupils should be taught about include:

- the different electoral systems used in and beyond the United Kingdom and actions citizens can take in democratic and electoral processes to influence decisions locally, nationally and beyond
- the rights, responsibilities and role of the media and a free press in informing

and influencing public opinion

- the different ways in which a citizen can contribute to the improvement of their community, to include the opportunity to participate actively in community volunteering, as well as other forms of responsible activity
- income and expenditure, credit and debt, insurance, savings and pensions, financial products and services, and how public money is raised and spent

(National Curriculum in England : Citizenship Programmes of Study, Department for Education, 2013, p2-3)

Some of these areas were RWEI and were situated into the mathematics classroom by participants in the study. Fabia had taught a lesson about different electoral systems, Santana had engaged pupils in discussion about how public money should be spent, Aron had discussed credit and debt in a lesson about compound interest and Rachel had taught about misleading graphs. All of these lessons were taught in mathematical contexts. and the study has included the areas of the mathematics national curriculum addressed in these lessons. The teachers in the study did not necessarily know that this content was part of the Citizenship Programmes of Study but felt they were important issues which could be addressed in a mathematics lesson.

However, the RSA report that despite schools' legal commitment towards providing spiritual, moral, social and cultural education, the issue of developing pupils' broader human qualities are sidelined as schools are under the pressure of a 'teach to the test' exam results culture (RSA Report, 2014).

This study makes a significant contribution to the field by demonstrating that RWEI can be situated into mathematics classrooms, and so work towards fulfilling the legal obligation schools have to provide a robust citizenship, spiritual, moral, social and cultural education.

The eight case studies consisted of teachers from different schools and different academic backgrounds and with different mathematical beliefs. For example, the study included teachers who believed mathematics was an 'absolute subject' as well as teachers who believed mathematics to be a subject which can raise social awareness, but both types of teachers had underlying beliefs, which went beyond their

mathematical beliefs, concerning other aspects of pupils' development which related to RWEI. As such, the study demonstrates that situating RWEI in the mathematics classrooms is a possibility for all mathematics teachers.

The study, therefore, contributes to an understanding of teachers' pedagogical approaches in the wider field of real-world equity issues, a concept which lies in the domain of critical mathematics education. Real-world equity issues and the wider areas of critical mathematics and social justice are not ideas which are a threat to academic mathematics, but use mathematics as a tool to develop pupils' understanding of their world from a critical perspective.

7.4 Implications for future research

The study highlights how mathematics teachers can situate real-world equity issues in their lessons. As I have mentioned, the case studies should provide teachers with ideas and inspiration to situate real-world equity issues in their lessons. However, time and financial constraints have resulted in some of the limitations identified in Section 7.2 (Limitations of the research). These include the limitations of a small sample size and limited observations. These limitations could be addressed by involving practising teachers in future research in a longer study. This could take the form of participatory action research project, initially with a small group of teacher researchers interested in situating real-world equity issues in their classroom. Whereas the case studies in this research provided important snapshots of teachers implementing real-world equity issues in the classroom, a longer project involving teachers could work towards implementing change over time.

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Appendices

Appendix 1 : Ethical approval

**APPLICATION FOR ETHICAL REVIEW OF RESEARCH PROJECTS
INVOLVING HUMAN SUBJECTS**

(It is highly recommended that you consult <https://my.lsbu.ac.uk/page/research-degrees-ethics> before completing this form)

This form should be completed by the following:

- ◆ Students undertaking research for a higher degree
- ◆ Staff employed by London South Bank University who are undertaking research, whether externally, internally or self-funded.
- ◆ External researchers wishing to base all or part of their project at London South Bank University

(Undergraduate students and those on a taught postgraduate programme are advised to see their Course Director)

State the title of your study and the date you wish to commence data collection. NB you should allow six weeks from the date of submitting this application, because we may ask you to revise and resubmit some of your documents. You are reminded that it is unethical, and may be unlawful, to commence data collection without having all necessary ethical approvals in place.

1. Title of Application

The place of real world equity issues when teaching mathematics in secondary schools.

2. Contact Details of Lead Applicant

Title: Mr Surname / Family Name: Ghosh	Forename(s): Suman
Address for Correspondence: 43 Buckingham Road South Woodford London E18 2NH	Telephone Number: 07941 560 596 Fax Number: E-mail address: suman24@me.com

3. Contact Details of Supervisors or Associate Applicants

Title: Professor Surname / Family Name: Lerman	Forename: Steve
Address for Correspondence: London South Bank University 103 Borough Road, London SE1 0AA	Telephone Number: 020 7815 7440 E-Mail address: Fax Number:

	(Please <input checked="" type="checkbox"/> as appropriate). Supervisor X Associate Applicant <input type="checkbox"/>
Title: Dr Surname / Family Name: Courtney	Forename(s): Jane
Address for Correspondence: London South Bank University 103 Borough Road, London SE1 0AA	Telephone Number: 020 7815 5434 Fax Number: E-mail address: (Please <input checked="" type="checkbox"/> as appropriate). Supervisor X Associate Applicant <input type="checkbox"/>
Title: Surname / Family Name:	Forename(s):
Address for Correspondence:	Telephone Number: Fax Number: E-mail address: (Please <input checked="" type="checkbox"/> as appropriate). Supervisor X Associate Applicant <input type="checkbox"/>

4. Ethical guidelines
In addition to following the LSBU code of Practice for research Involving Human Participants, which set(s) of professional or association guidelines have you read and do you intend to follow? I have read, and intend to follow, the British Educational Research Association (BERA) guidelines.

5. Safety

a) Please indicate any possible risks to the investigators, participants, other personnel or the environment:

(Please tick as appropriate).

- | | | |
|---|--|---|
| <input type="checkbox"/> use of environmentally toxic chemicals | <input checked="" type="checkbox"/> potential psychological intrusion from questionnaires, interview schedules, observation techniques | <input type="checkbox"/> defamation |
| <input type="checkbox"/> use of radioactive substances | | <input type="checkbox"/> misunderstanding of social / cultural boundaries |
| <input type="checkbox"/> ingestion of foods, fluids or drugs | <input type="checkbox"/> bodily contact | <input type="checkbox"/> nudity; loss of dignity |
| <input type="checkbox"/> refraining from eating, drinking or usual medication | <input type="checkbox"/> sampling of human tissue or body fluids (including by venepuncture) | <input checked="" type="checkbox"/> compromising professional boundaries with participants, students, or colleagues |
| <input type="checkbox"/> contravention of legislation on any of: gender, race, human rights, data protection, obscenity | <input type="checkbox"/> sensory deprivation | |

b) If you have ticked any of the boxes above, please describe the actions which will be taken to minimise the risk.

Considering my relationship as the subject tutor to some participants from the previous year, there are ethical considerations I need to be aware of which are particular to interviewing. I am aware that participants may want to share control of the interview and ask me questions. Further, they may be concerned about 'right' or 'wrong' answers. My relationship with the participants is such that I have given them feedback on their lessons through a process of self reflection and discussion. Within these sessions the participant will often ask for my perception of a situation or my opinion about a particular episode. Therefore, I need to be clear about my role in the study and that it is beliefs and opinions of the participants that are important. I have experience of interviewing tutees in previous research I have undertaken and found teachers to be forthcoming about their views. In the interviews I approached similar issues from a viewpoint that it is the debates surrounding these issues that are important in mathematics education and there is no 'wrong' or 'right' view.

It is important to note that although some participants were my students in the previous year there is no longer a student / tutor professional relationship with them as they are no longer students at the University. Therefore the risk of compromising professional boundaries is substantially minimised. Further there may also be participants who were on the course but were not my tutees in which case the student / tutor relationship will have little influence.

All Participants are informed that they are able to withdraw from the research at any stage if they feel uncomfortable without suffering any consequences.

c) If this project requires the use of any special procedures or techniques, please describe any training or competency assessment to be undertaken by investigator (s).

In order to be able to analyse data efficiently I am undertaking some training in NVivo software.

d) Does the applicant or any member of the research team require a Disclosure and Baring Service Check (DBS, previously CRB) in order to conduct this research?

(Please ☒ as appropriate).

☒ Yes. State why here: I will be going into schools to do interviews and observations.

☐ No. State why here:

If a DBS disclosure is required please indicate whether

(Please ☒ as appropriate).

☒ The DBS check has been completed

☐ The DBS check application is submitted, awaiting outcome

☐ No DBS application has yet been submitted

Note: if a DBS disclosure is required, the Secretary of the Research Ethics Committee will need to see evidence of a satisfactory disclosure before final ethical approval can be given. In order to save time, evidence of satisfactory disclosure need not be submitted at the application stage.

6. Anonymity / Confidentiality

Please indicate measures that will be taken to protect and maintain the anonymity and / or confidentiality of participants.

All participant data will be confidential and anonymous. In the case of interviewing, the requirement for confidentiality and anonymity is more acute as I see the participants during the interviews and know their names and identities. Interview records and extracted descriptors will be separated from the personal data of the participant as all records will be anonymised when transcribed.

The digital recording device and hard copies of the observation notes will be kept in a locked storage unit. Digital recordings will be transferred to my computer and copies of observation will be typed and stored on the same computer. The computer is password protected. Data will be backed up on secure 256 Bit encrypted USB drives and recordings on the original recorder will be erased. All data will be destroyed one year after the viva.

7. Informed Consent

a) How will potential participants be invited to take part in the study? If by letter please include a copy of the letter, if by poster please include a copy of the poster and make clear where the poster will be displayed.

I will be writing to students on The Teach First Secondary Mathematics PGCE course at the Institute of Education. The student data I have access to has the names of the schools the teachers practice in so I can send letters to their schools. Further, I continue to visit many of the schools I did last year so I can meet my students from last year and can have a conversation to see if they would be interested in participating in the study. The Teach First Secondary Mathematics course consists of students with mathematics and non mathematics degrees and I have access to this data as a tutor on the course. Once I have made initial contact with the participants I will ask them if can use this data for the purposes of this study.

Potential participants will be invited to take part in the study by letter. A copy of the letter has been included in Appendix 1.

b) When will participant receive a copy of the participant information sheet? Please include a copy of the participant information sheet. If you are not providing a participant information sheet please explain why.

I will send the participant information sheet to participants at least two weeks before I plan to set an interview date with them. As I plan to start the interviews in mid June I will send the participant information sheet to participants in mid May. A copy of the letter has been included in Appendix 2.

c) How long will participants have between receiving information about the study and giving consent?

Participants can give consent at any time between receiving the information and a week before organising their interviews. Therefore they have up to two weeks between receiving information about the study and giving consent.

d) Who will obtain informed consent from participants?

I will send letters and consent forms directly to the participant.

e) Please indicate what form of consent will be used in this investigation.

(Please ☒ as appropriate).

- ☒ Written (note a copy of the consent form must be attached)
☐ Verbal
☐ Implied

If consent is verbal or implied, please explain why you are not obtaining written consent.

f) Are you offering any incentives or rewards for participating? (Note, normal travelling expenses are not regarded as an incentive or a reward.)

(Please ☒ as appropriate).

- ☐ Yes
☒ No

If yes, what are they?

g) Are there any issues related to the ability of participants to give informed consent themselves or are you relying on gatekeepers to consent on their behalf?

Participants will be giving informed consent themselves. There are no issues with this.

8. Research Proposal

Please attach a copy of your research proposal to this form using the headings below. *Your proposal should normally be limited to 12 sides of double-spaced A4. This limit is exclusive of references and supporting documents, which are also required.*

- Background / rationale – this should include a review of key literature and outline the theoretical or conceptual framework for the study
- Aims / objectives
- Study design – this should include the methodological approach and the start and completion dates
- Methods – should include how site and/or participants are selected and accessed; how many participants are to be used and how that number was decided including where appropriate the details of sample size calculations; details of how and where any interviews or observation will be conducted and how any questionnaires will be distributed and returned; details of the information to be collected including copies of questionnaires, interview guides and observation guides as appropriate; details of how the data will be analysed; details of how and where the data will be stored and for how long and plans for disposal or archiving; the process for obtaining informed consent; details of ethical issues for this study
- References
- Details of persons / agencies / organisations whose permission will be required in order to conduct this study
- Details of persons / agencies / organisations who may financially benefit from it outside their normal terms and conditions of employment
- Appendices containing copies of documents to be used in the study (include all that are appropriate for your study, whether or not they are listed here): letters seeking permission to conduct the study; recruitment poster or other recruitment material; letter of invitation to the participant; participant information sheet; consent form; questionnaires; interview guide; observation guide.

Ethical Guidelines

- Association of Business Schools Ethical Guidelines (<http://www.the-abs.org.uk/files/2010%20Ethics%20Guide%20AGM%20version.pdf>)
- Association of Social Anthropologists of the UK and Commonwealth (<http://www.theasa.org/ethics.htm>)
- British Computer Society (<http://www.bcs.org/server.php?show=nav.6030>)
- British Educational Research Association *Revised Ethical Guidelines for Educational Research (2004)* (<http://www.bera.ac.uk/files/guidelines/ethica1.pdf>)
- British Psychology Society: *Code of Ethics and Conduct (August 2009)* (http://www.bps.org.uk/the-society/code-of-conduct/code-of-conduct_home.cfm)
- British Society of Criminology (<http://www.britsocrim.org/codeofethics.htm>)
- British Sociological Association: *Statement of Ethical Practice for the British Sociological Association* (<http://www.britsoc.co.uk/equality/>)
- College of Occupational Therapists: *Code of Ethics and Professional Conduct (2005)* (<http://www.cot.co.uk/MainWebSite/Resources/Document/Code-of-Ethics.pdf>)
- Institute of Engineering and Technology (IET) (<http://www.theiet.org/about/governance/rules-conduct/index.cfm>)
- Institute of Career Guidance: *The Code of Ethics for Members of the Institute of Career Guidance* (http://www.icg-uk.org/code_of_ethics.html)
- Institute of Business Ethics (<http://www.ibe.org.uk/index.asp?upid=57&msid=11>)
- General Social Care Council *Codes of Practice for Social Care Workers* (<http://www.gsccl.org.uk/NR/rdonlyres/3CFA9CAE-0A38-48B9-8BD2-B513A0A5024F/0/GSCCCodeofPractice.pdf>)
- National Health Service National Patient Safety Agency: *National Research Ethics Service* (<http://www.nres.npsa.nhs.uk/>)
- Royal College of Nursing: *Research Ethics: RCN Guidance for Nurses* (http://www.rcn.org.uk/__data/assets/pdf_file/0010/78742/003138.pdf)
- Social Policy Association (http://www.social-policy.org.uk/downloads/SPA_code_ethics_jan09.pdf)
- Social Research Association: *Ethical Guidelines* (<http://www.the-sra.org.uk/documents/pdfs/ethics03.pdf>)

World Medical Association *Declaration of Helsinki: Ethical Principles for Medical Research Involving Human Subjects (latest revision: October 2008)*
<http://www.wma.net/en/30publications/10policies/b3/17c.pdf>

9. Approval by Executive Dean or Dean's authorised signatory (required for applicants who are members of LSBU staff)

I confirm that the applicant may seek ethical approval for this proposal.

Signed

Name.....

Date

10. Signature of Lead Applicant

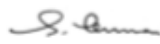
I confirm the information supplied is correct and understand that failure to provide accurate information can invalidate ethical approval. I will inform the Research Ethics Committee **in advance** of any **proposed** changes to the research design **or** methods.

Signed 

Date : 14 April 2014.....

Signature of Student's Supervisor (where appropriate).

I confirm the information supplied is correct and understand that failure to provide accurate information can invalidate ethical approval.

 Signed

Date27 April 2014.....

Appendix 2 : Letter to Participants

Suman Ghosh
UCL Institute of Education
University College London
20 Bedford Way
London
WC1H 0AL

Tel: 07941 560 596

Email: s.ghosh@ioe.ac.uk

[Date]

Dear [.....],

Re: 'How can Secondary School Mathematics teachers situate real world equity issues in the classroom?'

I am formally writing to invite you to participate in a study entitled:

'How can Secondary School Mathematics teachers situate real world equity issues in the classroom?'

You have been chosen to be invited to participate in this study as you are a Mathematics teacher in a secondary school. Further, it may be that your degree is not mathematics related and this is an important factor for the sample. Initially, approximately 10 people will be included in the study.

I have included an information sheet about the study and your potential role. I have also included a consent form for you to send me if you agree to take part in the study. Once completed please email the consent form to the above address within the next two weeks.

I would very much appreciate your participation in this study.

Yours sincerely,

Suman Ghosh

Participant Information Sheet

UREC No 1423

'How can Secondary School Mathematics teachers situate real world equity issues in the classroom?'

You are being invited to take part in a research study. Before you decide it is important for you to understand why the research is being done and what it will involve. Please take time to read the following information carefully. Talk to others about the study if you wish. Ask me if there is anything that is not clear or if you would like more information. Take time to decide whether or not you wish to take part.

The study aims to investigate the place of real world equity issues (RWEI) when teaching mathematics in secondary schools. RWEI in the context of mathematics education are real world issues, which can be critically examined in the mathematics lesson and so allow pupils to be critically literate through mathematics. The study also aims to examine if mathematics teachers, from diverse academic backgrounds, have different pedagogical approaches relating to the place of RWEI in teaching mathematics in secondary schools. This will involve an audio interview with you and may be followed up by some joint lesson planning and classroom observations.

You have been chosen to be invited to participate in this study as you are a Mathematics teacher in a secondary school. Further, it may be that your degree is not mathematics related and this is an important factor for the sample. Initially, approximately 10 people will be included in the study.

It is up to you to decide whether or not to take part. If you do, you will be given this information sheet to keep and be asked to sign a consent form. You are still free to withdraw anytime up to the submission of the dissertation and without giving a reason. There are no consequences if you decide to withdraw.

If you are willing to participate, I will visit you at your school for an interview lasting approximately 30 minutes at a mutually agreeable date and time. During the interview, I will explore with you, your beliefs in relation to teaching mathematics and your opinions in relation to RWEI and their place in teaching secondary mathematics. For ease of later analysis, I will make audio recordings of the conversation with your permission as well as take notes. If you do not wish to be recorded but are still willing to participate, I will take notes only. Following the interviews I will ask some participants if they are interested in planning some mathematics lessons with a focus on RWEI and if it will be possible to observe the participant teach a lesson. Further I will also ask you to give me an anonymous sample of your assessment of pupils' work from the lesson. I will be writing to your Headteacher to request permission to observe you in the classroom.

It is not anticipated that you will be at any disadvantage or suffer any risk from this study.

It is unlikely that you will gain any personal benefit from participating in this research. However, the information you share with the researcher will be important in exploring how

RWEI can be integrated into secondary school mathematics. Further, the information will be useful for secondary mathematics initial teacher training courses who are interested in addressing RWEI as part of their programme.

You are free to withdraw from the study and not have your information included, at any time up to the time of completion of the dissertation. However, after that time, it would be impossible for the researcher to comply. There are no consequences if you decide to withdraw.

All information received from you will be handled in a confidential manner and stored in a locked filing cabinet and on a password protected computer in an environment locked when not occupied. Only the researcher and supervisor will have direct access to the information. Any reference to you will be coded. This information will be held for four years after the study.

This study is being completed as part of a Doctorate in Education (EdD) at London South Bank University. It has been reviewed and ethically approved by the London South Bank University Research Ethics Committee.

If you have a concern about any aspect of this study, please do contact me (Suman Ghosh at 07941560596 or s.ghosh@ioe.ac.uk). If you wish any further information regarding this study or have any complaints about the way you have been dealt with during the study or other concerns you can contact: Steve Lerman at 0207 815 7440, who is the Academic Supervisor for this study. You may also contact the University Research Ethics Committee (ethics@lsbu.ac.uk) as someone who is not related to the study.

Finally, if you remain unhappy and wish to complain formally, you can contact the Chair of the University Research Ethics Committee (<http://www.lsbu.ac.uk/research/governance/ethics>)

UREC No 1423

'How can Secondary School Mathematics teachers situate real world equity issues in the classroom?'

WRITTEN CONSENT FORM:

University Research Ethics Committee No 1423

Title of Study: 'How can Secondary School Mathematics teachers situate real world equity issues in the classroom?'

Name of Participant:

- I have read the attached information sheet on the research in which I have been asked and agree to participate and have been given a copy to keep. I have had the opportunity to discuss the details and ask questions about this information ☐
- I agree that I will be interviewed and that the interview will:
(Please tick one box)
 - be recorded on an audio device ☐
 - be recorded in written note form ☐
- The Researcher has explained the nature and purpose of the research and I believe that I understand what is being proposed ☐
- I understand that my personal involvement and my particular data from this study will remain strictly confidential. Only researchers involved in the study will have access ☐
- I understand that the data collected will be used for the purpose of the stated research and related follow up publications. All data will be anonymous. ☐
- I have been informed about what the data collected will be used for, to whom it may be disclosed, and how long it will be retained ☐
- I have received satisfactory answers to all of my questions ☐
- I hereby fully and freely consent to participate in the study which has been fully explained to me ☐
- I understand that I am free to withdraw from the study at any time, without giving a reason. There are no consequences to my withdrawing from the study. ☐

Participant's Name:(Block Capitals)

Participant's Name: Signature

Date:

As the Researcher responsible for this study I confirm that I have explained to the participant named above the nature and purpose of the research to be undertaken.

Researcher's Name:

Researcher's Signature:

Should you wish to contact someone not related to the study I have provided contact details of the University Research Ethics Committee (ethics@lsbu.ac.uk)

(Please sign both copies, keep one copy and send one back to me at Suman Ghosh, Institute of Education, 20 Bedford Way, London WC1H 0AL)

UREC No 1423

'How can Secondary School Mathematics teachers situate real world equity issues in the classroom?'

Appendix 3 : Letter to Headteachers

Suman Ghosh
UCL Institute of Education
University College London
20 Bedford Way
London
WC1H 0AL

Tel: 07941 560 596

Email: s.ghosh@ioe.ac.uk

[Date]

Dear [],

I am currently doing my Doctorate in Education at London South Bank University. The study aims to investigate the place of real world equity issues (RWEI) when teaching mathematics in secondary schools. RWEI in the context of mathematics education are real world issues, which can be critically examined in the mathematics lesson and so allow pupils to be critically literate through mathematics. Part of this study will involve some and classroom observations of teachers. It will also involve my analysis of teachers' assessments of pupils' work from the observed lesson. For this purpose the teachers will copy a sample of pupils' work and anonymise it before it is passed on to me.

I have asked [Participant name] if s/he would agree to a classroom observation and s/he has agreed to this. I am writing to request your permission to observe [Participant name] teaching a lesson. If you are happy for me to do this you need not reply. However, if you have any questions or would prefer me not to do this then please do email me on s.ghosh@ioe.ac.uk within the next two weeks.

I have a valid DBS check and, as a lecturer in Mathematics Education at University College London (Institute of Education), I visit a number of schools to carry out classroom observations of student teachers.

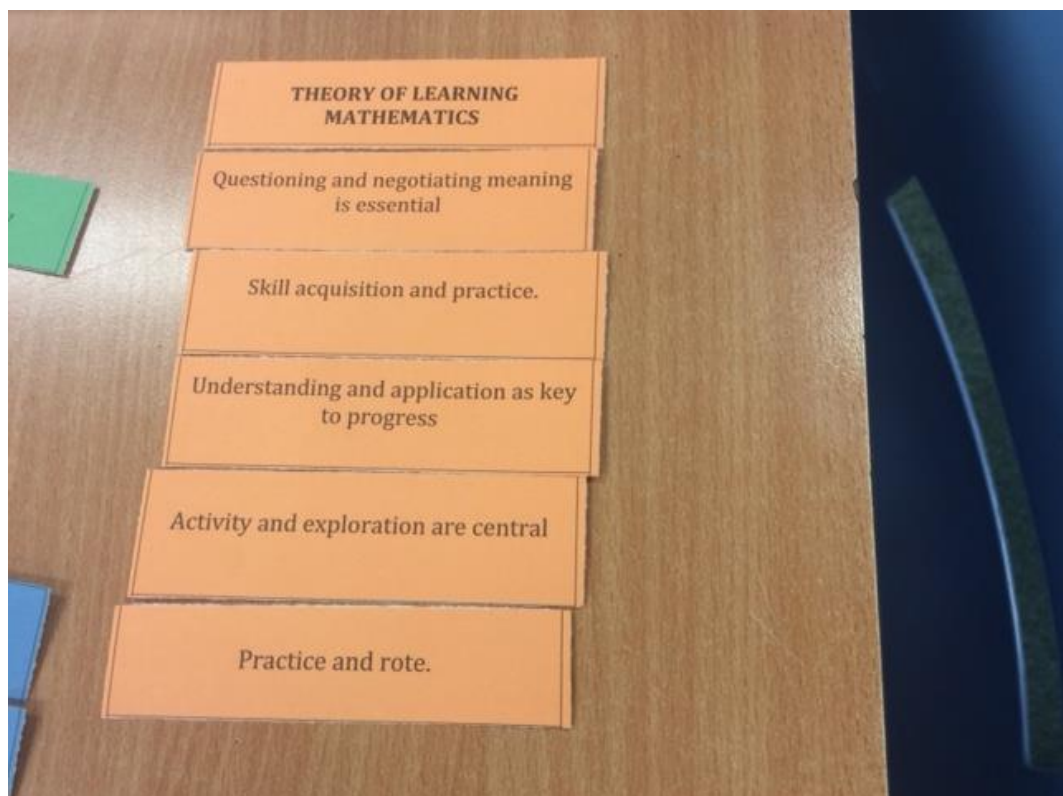
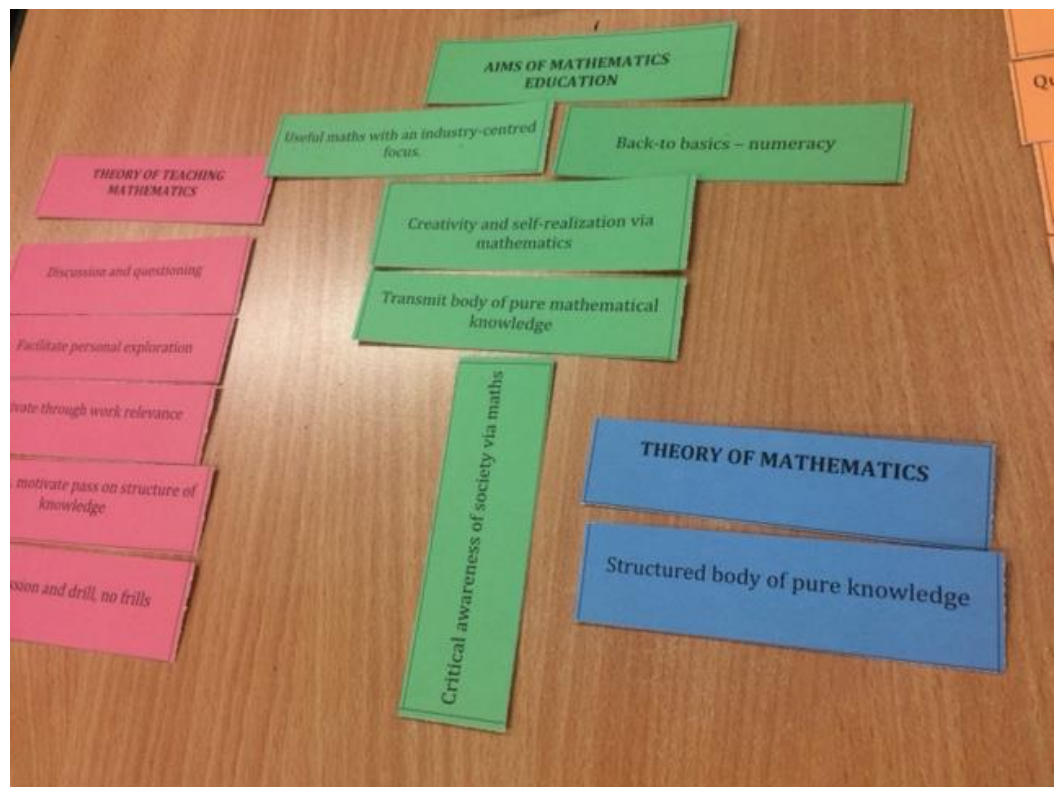
Please do contact me if you have any questions about this.

Yours sincerely,

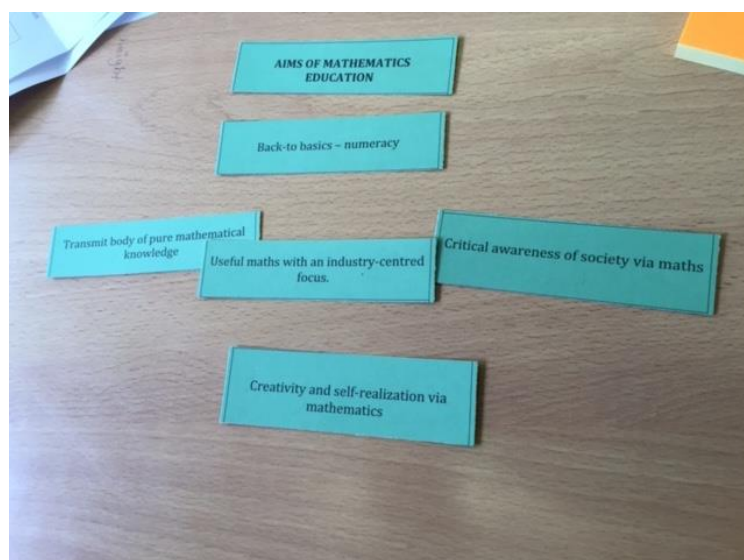
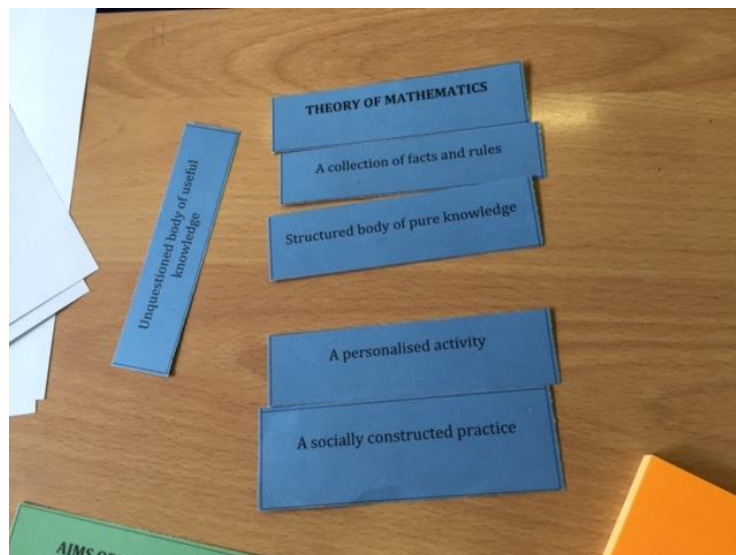
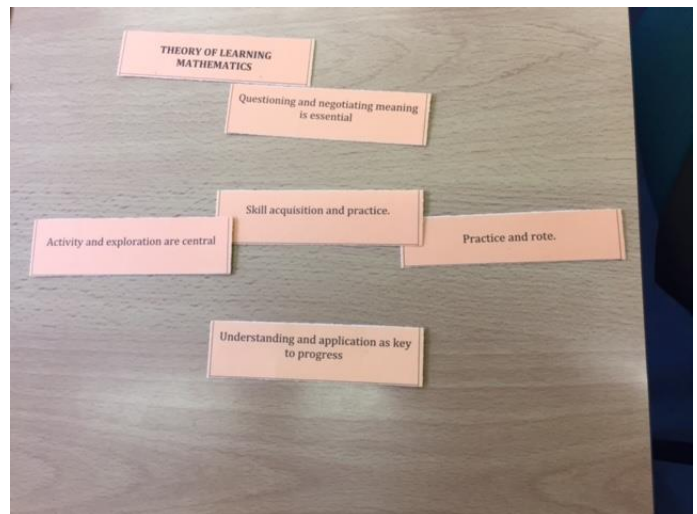
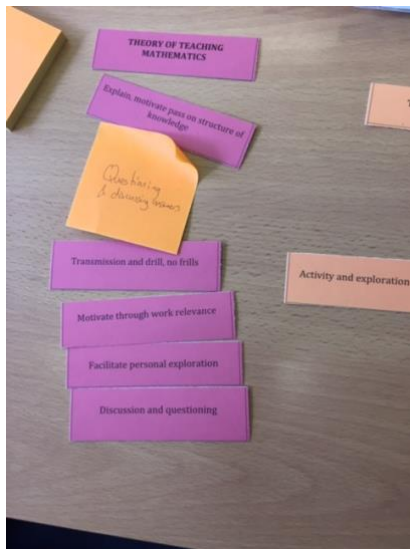
Suman Ghosh

Appendix 4 : Participants' arrangements of card sorts

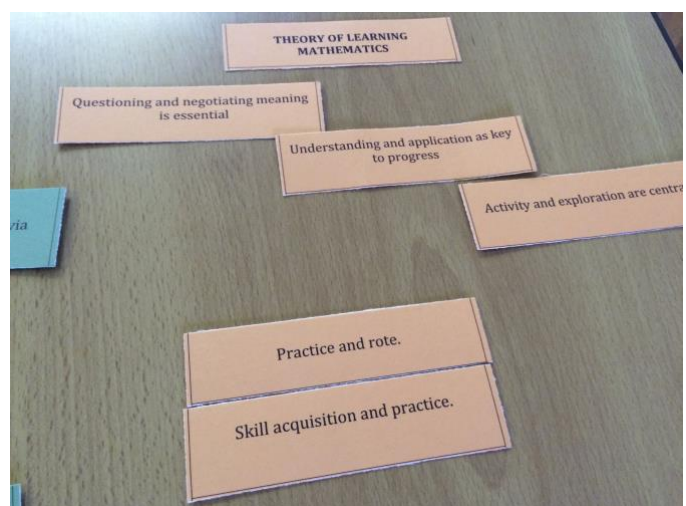
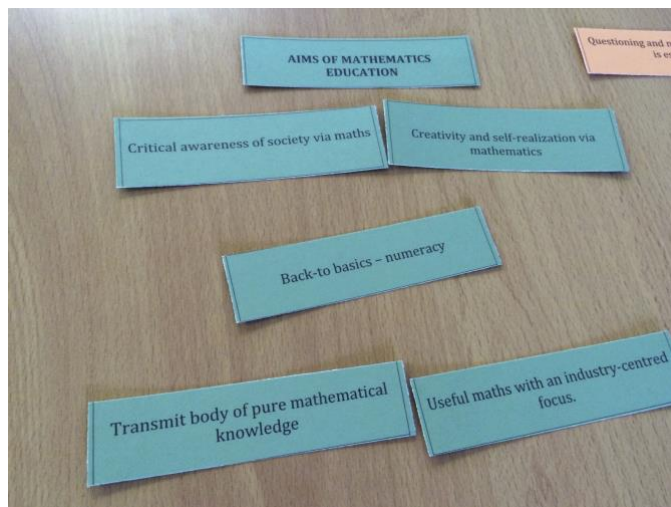
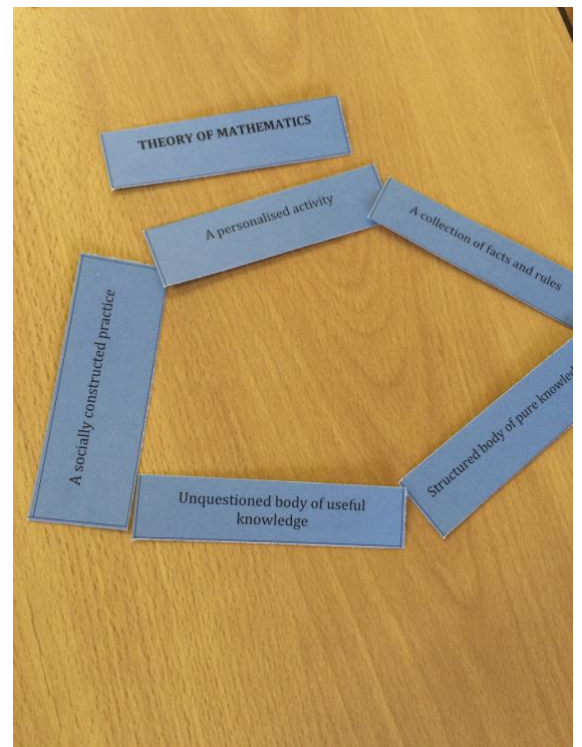
Aron's Card sort (7.5.15)



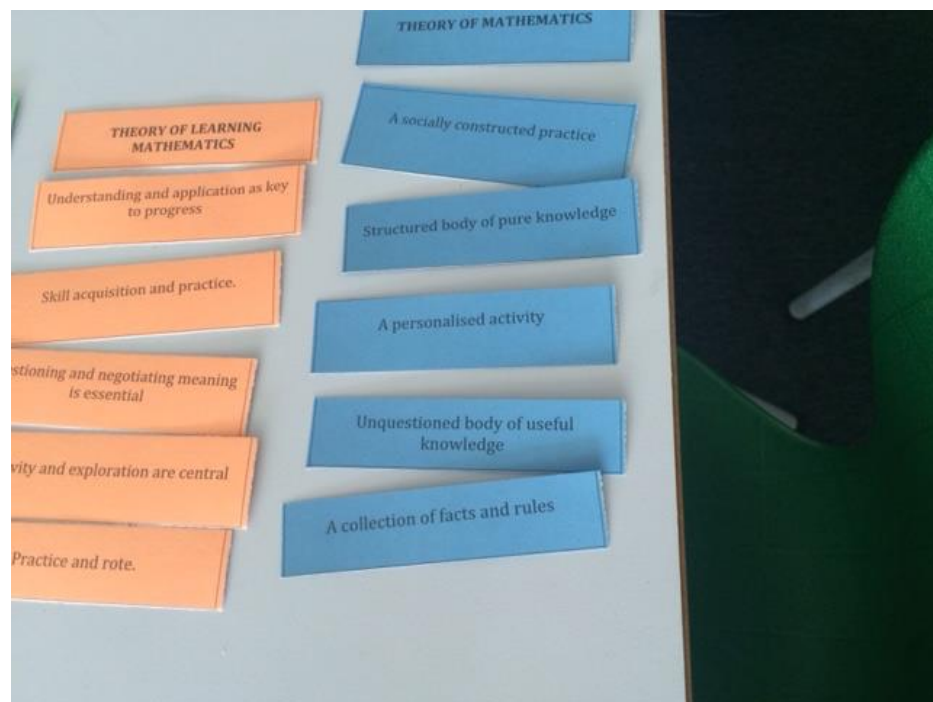
Edwin's card sort (10.2.16)



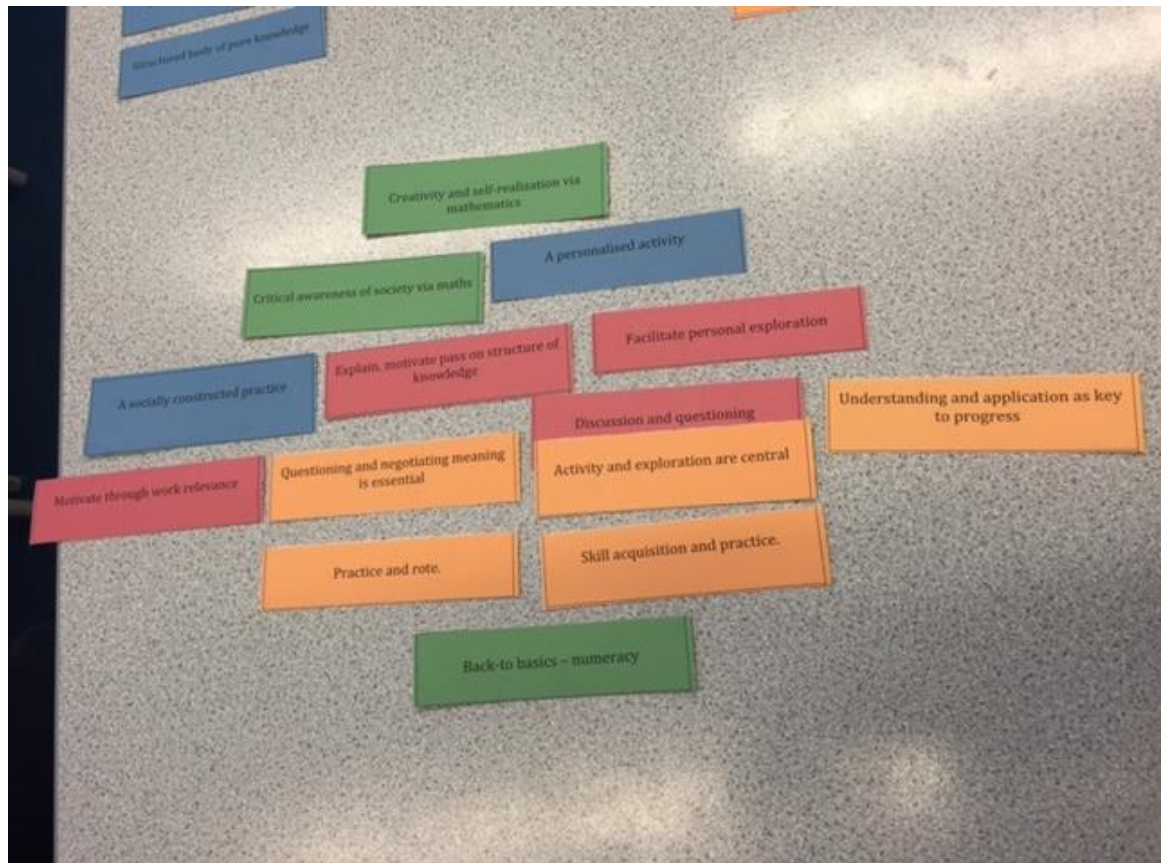
Fabia's card sort (30.4.15)



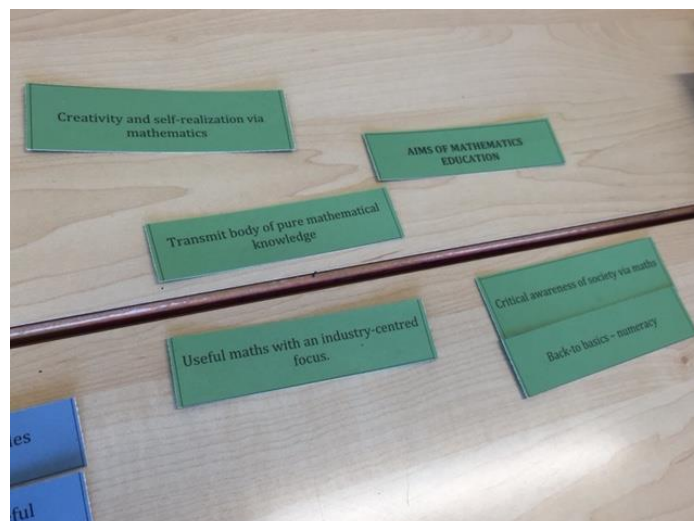
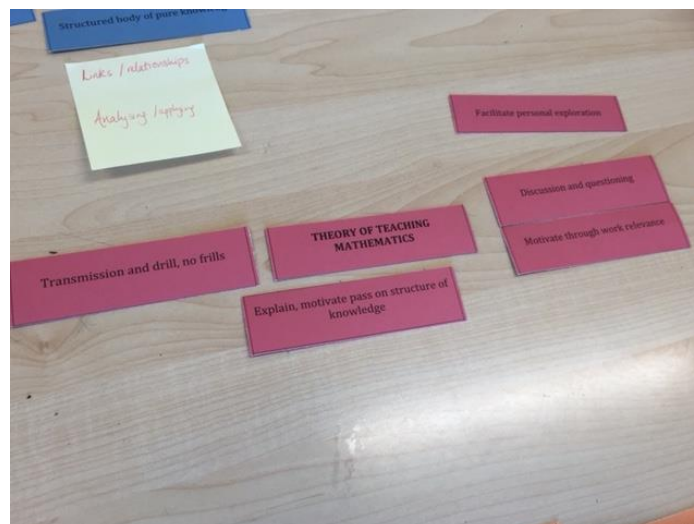
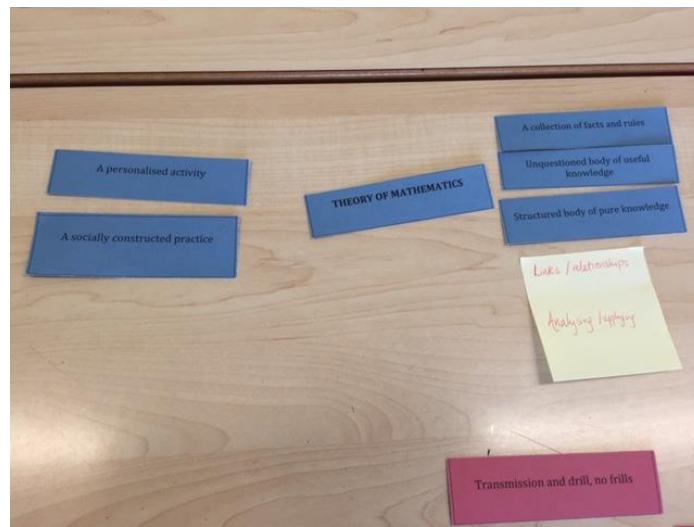
Jason's card sort (27.4.15)



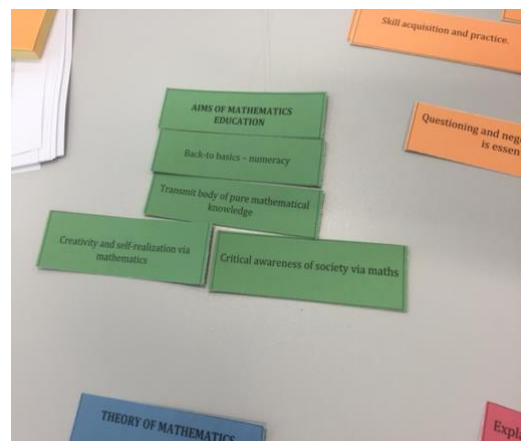
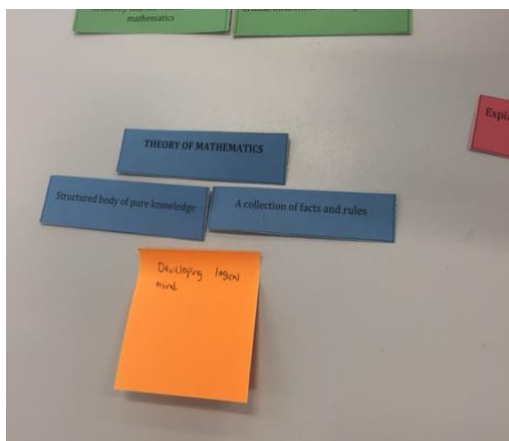
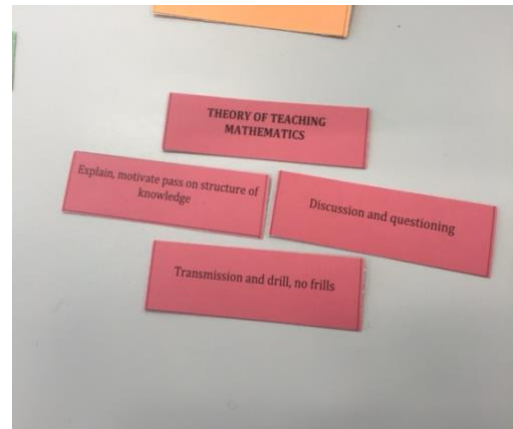
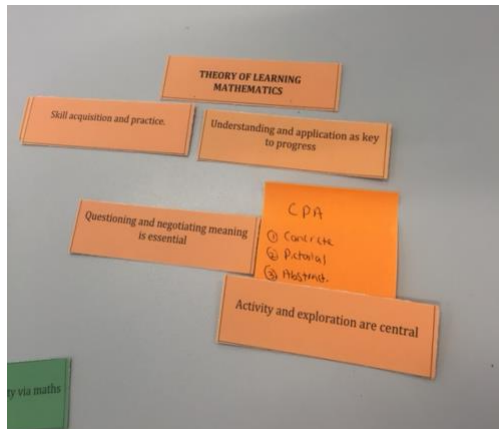
Minervia's card sort (14.4.16)



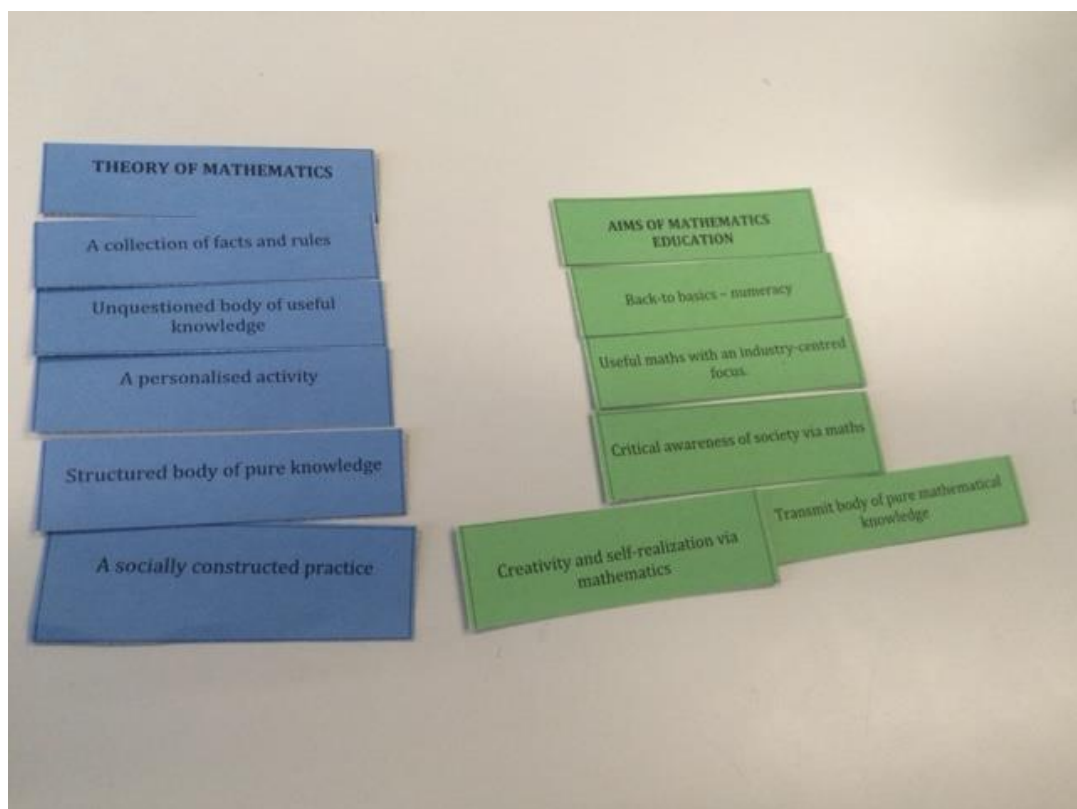
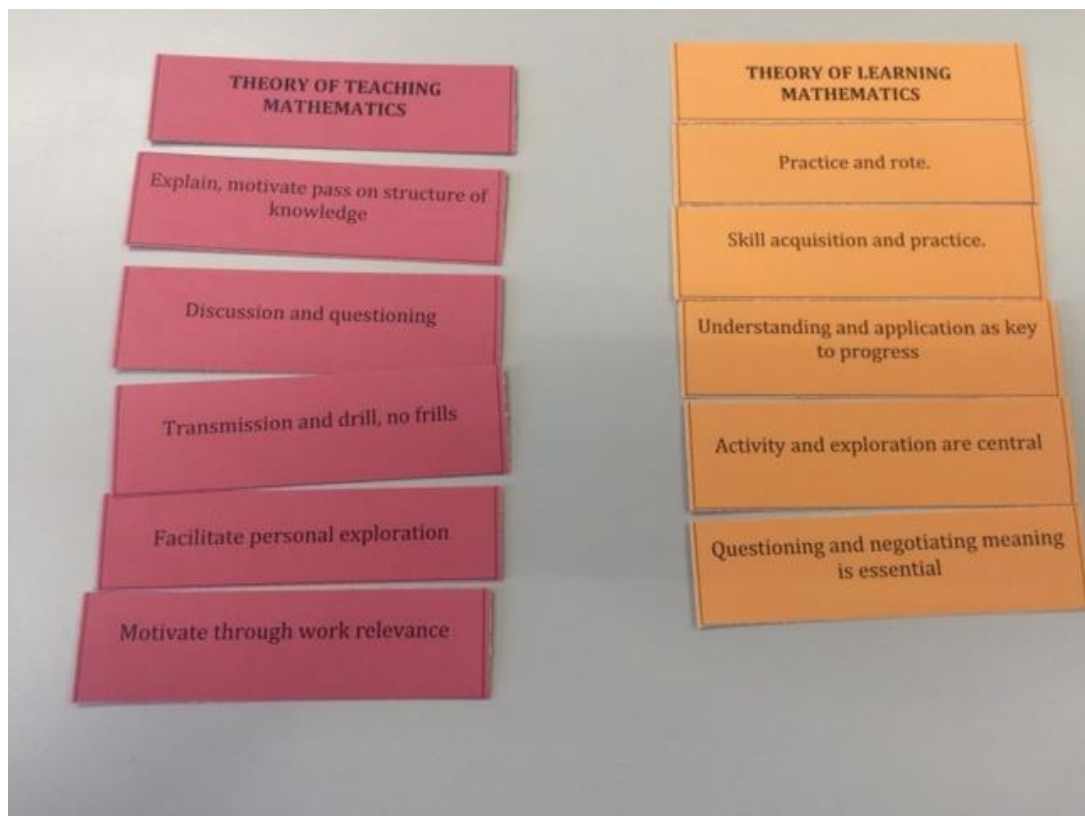
Rachel's card sort (8.12.15)



Santana's card sort (4.5.16)



Tao's card sort (7.4.15)



Appendix 5 : Transcription of participants' interviews

Aron's Interview (7.5.15)

Transmission and drill and no frills is important but not what I think is the most important

The issues with all of these is that, or the issue with this task, is that it depends on the pupil and I think and with higher ability pupils I much prefer to teach them by facilitating personal exploration because if you have a lower ability pupil who struggles with maths, struggles to internalize and adopt (4.31) some of the main ideas of maths and doesn't really have a strong concept of number then facilitating by personal exploration is rather fruitless and transmission and drill no frill, whilst its not ideal, the pupil is not in an ideal position to understand mathematics.

All these things are valid. My ideal way that I would teach mathematics is through essentially discussion, personal exploration - I think that personal exploration follows from discussion and questioning. I think you cant just give someone a pen and paper and say go figure out the maths unless they are motivated or prompted by discussion (5.47) I think motivating personal exploration is important for it to be quite motivated through

Having a task or competition because you are trying to achieve a goal of some kind drives , maybe not to be better at the maths, but to work harder and get more done and be more efficient

(6.29) *Structure of knowledge* – I think that's really important, that gets more and more important as kids grow up, so I think that the structure of maths is far more important at GCSE and much more important at A-Level than it is at Key Stage 3 because I think this can be taught very quickly, it doesn't have to be embedded from day 1

(7.20) *Aims of mathematics Education* I have come to the belief recently that if you start to look at the statistics of the situation the number of pupils you have in any one class that go on to be mathematical academics is zero and whilst its important to transmit a body of pure mathematical knowledge it is not the goal of a KS3 or KS4 class. And I also believe that the way the kids are tested is not necessarily fair because they are tested in a nearly pure mathematical setting which is, sort of, a bit silly for a vast majority of them

How do you think they should be tested?

I don't think they should be tested, I think that testing kids gives terrible incentives and leaves people horribly institutionalized I think its much harder to judge people based on projects but I think that is a lofty aspirational goal that society should have, additionally I think University exams should be scrapped I think University exams are terrible, I think people who leave University are suddenly left with a very massive hole where they don't quite know what to do with themselves because suddenly there are no more official exams, I've got my piece of paper but now will someone pay me to do something that's real please.

So, with those ideas in mind I think that useful maths with an industry centred focus is definitely my number one priority. I want kids to know which mortgage is best for them, I want kids to know how many litres of paint they need to paint their house and figure out that kind of stuff and that's not necessarily the focus of education as it currently stands.

(9.30) I think second to that is creativity and self realization via mathematics. I think thatpriority is that kids are mathematically literate and that's quite a basic priority but its number one, number two is that once they are literate they are able to essentially express themselves through mathematics, that could be computer programming which I think would be so valuable for kids.

The back to basics numeracy is the backbone of a useful industry centred focus, I think they definitely sit side by side for what I am trying to achieve, I would like kids to have critical maths but if they can't have basic numeracy and mathematical literacy then critical maths is useless.

Transmit a body of pure mathematical knowledge, they are not unrelated all these things, and a critical awareness of society via maths – I don't know if that is what I would do in my mathematics education I think that other subjects should speak numeracy, should speak number and that is where that would from, I don't think there is time unless you gave maths teachers far more time in class to be able to do this and also get kids to be where they need to be.

(11.45) Theory of learning mathematics

What is the number one for learning maths - I think its questioning and negotiating meaning. I think if you put yourself in the thought where mathematics is a language you can practice and do rote till the cows come home but you are not teaching them anything this is pattern recognition and telling the teacher I know what I am doing but its not actually very useful in terms of really learning mathematics, its useful in terms of becoming an accountant.

Conversation is so important. I think part of mathematics is having a really really strong

Activity and exploration are central – yes they are but I think these two sit parallel. These aren't for me - that's not true. What this is for me is the motivator, the activity or the exploration kind of like 'oh I can do those myself' is about independence in mathematics, which I think is really important but I don't think that number 2, I think in terms of learning mathematics you need to have the skills and you must practice them. I think these two are such a back to basics style, have a conversation do the maths have a conversation do a work sheet but that because I think to do the harder maths your basic skills need to be so solid. You know if you are stating to do transformations at GCSE but your coordinates aren't very good then you are stuffed and the reason you are stuffed is because you are trying to do two things you find though at the same time but if your coordinates are second nature because you have done enough skill acquisition and practice then it allows you to do the harder maths . maths is very linear that's very important. I'd like to think that activity and

exploration are important and I just don't think that they are a day to day way in which you can teach maths and that kids don't necessarily appreciate that.

I put in understanding application as key because I don't think they are separate from the questioning and negotiating meaning but the application side of it is really important.

Theory of Mathematics

For me the toughest one

What is maths? Maths is not a collection of facts and rules, maths is definitely not an unquestionable body of useful knowledge because it's not unquestioned, maths is not a personalized activity, maths is not a socially constructed practice, maths is and only is a structured body of pure knowledge. All these other things, they are misconceptions about mathematics when people have been taught mathematics by practice and rote. Socially constructed practice – you have to hate maths to think that. Collection of facts and rules, taught by rote and a personalized activity, I don't know, you have done too much philosophy.

I think what I would like to say is that mathematics is just the language of the world numerically and the world does have structure and the world does have rules but it's not as if those rules are not there to be unquestioned and much of mathematics as is taught in school is simply historical convention which is the best solution so far, but up until 1000 years ago people did not count 1-100, they used different bases and different numerals, it's not exactly fixed. The area of a triangle has not changed but the way you get there is different.

Anything else?

Theory of teaching maths I think this idea of discussion and questioning is just so vital and that questioning is at the top of my theory of teaching mathematics and also at the top of my theory of learning mathematics and I don't think that's by coincidence I think that's because my thinking of mathematics is that it's has its own language it's so important that everyone has clear understanding of the words that are being used and they really mean and so mathematics is a language and it's important to have a shared understanding and definition

(17 57 left)

mathematical literacy

Could you give me an example of how Critical awareness of maths could be addressed in other subjects?

Statistics, statistics so people are constantly throwing around meaningless statistics and saying 'we've done a survey and suddenly it destroys you to eat red meat after 7.30pm on a Wednesday' but they asked six people. And understanding statistics in other subjects because statistics are used so flippantly and people are so blind to the

fallacies of statistical thinking and I think that teachers in other subjects are not numerically literate enough to make those connections themselves so they can't help their pupil with that. Statistics could be embedded into the humanities

Pupils aren't taught about taxes, pupils aren't taught about retirement, interest rates there's no class for that and the incentives aren't there for the maths departments to teach that but there should be because we are sending pupils out into the world not teaching them how to figure out 'if I got a 1200% APR what am I going to pay next month?' and that's ridiculous. And a lot of pupils in the demographics if thos school will be going for a pay day loan or whether its pawning something or taking job and not relasing how little they will be paid

I think the way the school is structured is too comparmentalised.....kids really suffer from that because they walk into Year 7 and they get 'ok as a maths department our entire goal is to get you to pass your GCSE at Year 11 – alright everyone the race has begun, you got ot get your C and it's a race from day one which is always there and kids aren't prepared for things that are outside the curriculum because there's no external motivation. Whether or not its taught by a maths teacher or it could be someone trained in maths. Any school I come across do it as a one off novel as opposed to necessary . Time isn't made, there is an illusion of a lack time and I think there's other ways we can teach kids to make vey fas progress but its not GCSE focused as, obviously, the Jo Boaler books talk about.

There's other ways we can teach kids where they can make very fast progress

Edwin's Interview (10.2.16)

Interviewer: It may well be that you don't agree with any of those, or don't agree with some. The idea is to an extent that if you don't agree with something you discard the card, if you do agree with it you keep it, or you might agree with all them but in the -- most people decide, "Well, I agree with it," but in that order. You might just want to put in order of preference or like I said, you might just want to introduce your own ideas. But the idea is this is about your thoughts about maths, and I say very little in this interview because of that. But what I'd like you to do is just explain your choices.

Edwin: Why I'm thinking about one instead of another?

Interviewer: Yes, your thinking process as you're doing it. Then the other thing I will do is just take a picture of your arrangement at the end. Start with any of them.

Edwin: I'm going to start with theory of mathematics. I think that's probably easiest to clarify that because then I will probably affect my answers to this one, the theory of teaching mathematics. So theory of mathematics personalized activity, social and constrictive practices. Structured body of pure knowledge. This one sounds least interesting to my students but collection of facts and rules, I'm going to put that right up there with theory of mathematics, with the axioms, and I think that is very important. I think that structured body of pure knowledge is a - I would say probably another similar way of saying the same thing, maybe a more poetic way, essentially it's a collection of facts of rules.

The pure knowledge, I like that because I do agree that maths is knowledge in it's simplest form. Personalized activity, socially constructed practice, Unquestioned body useful knowledge, I think. This one question, body of useful knowledge could be true, could not true depending on how look at it. The collection of facts, that's what we're trying to do when we prove theorems and mathematics, we should prove it's absolutely true so that there is no question of doubt about it.

However, I think definitely math should be questioned, I think whenever I learned the maths asking why. This one is - I'm going to put that slightly outside because that can be interpreted differently. Personalized activity and socially constructed practice, you do maths by yourself and then you do maths in groups.

Interviewer: And of course they don't necessarily contradict each other.

Edwin: No, I'm just trying to think to what extent I think it should be done by yourself or should be done in groups. I can't remember what was the name of the mathematician, but I read a book on him at university. You do hear a lot about mathematicians just working by himself and that guy who prefers [crosstalk].

Interviewer: Andrew Wiles.

Edwin: Yes, hid himself in his attic -

Interviewer: For years.

Edwin: - for years. In that sense of second personal activity. I think it's a personalized activity and then brought into groups. You try it by yourself, you do it by yourself, and

then you bring into groups. When I had courseworks to do in University, we would - I always try to do it by myself and then share what I had with my friends.

Interviewer: Because of course you've got a master's degree?

Edwin: Yes, with the masters degree and I think to really understand I needed to go through the motions by myself. I think probably if I was to do a group work activity I would always try and do it by myself. Then if I got to a point where I couldn't do it anymore, I would then ask around to see what other people got. I would leave personalized activity on top and this is it - and my personal view as well on how I would do it.

Interviewer: That's fine. It's all about that. It's about your personal take on it as it were.

Edwin: Aims of mathematics education you've learned through teaching. I'm going to do aims first. Probably the teaching would probably be more interesting or there's more argument to that. I think probably I should start with that if I [crosstalk] be teaching math with the Aims.

Interviewer: That's actually fine.

Edwin: Creativity in self realization around mathematics transmits body of pure mathematical knowledge. Back to basics numeracy, critical awareness of society about maths, use for maths industry-centered focus.

I think I'm going to put - and it's very different to when I started teaching, but I would put back to basics numeracy right at the top because I think before I get on to any of these, they need to have this level of numeracy. You can't really do anything without this, so yes, put that right at the top.

Creativity in self realization about mathematics. As much as I wouldn't - I don't think the aims of maths education in this country is creativity. As much as I believe, you can have that. I don't think that's necessarily an aim. I'd put that at the bottom. There can be aspects where there's creativity, but probably at a higher level. Aims in mathematics education depends on obviously coming from a school's focus being a math teacher.

Interviewer: It's interesting you say the back to basic numeracy is not something you would sort of -

Edwin: No.

Interviewer: Now your philosophy is changed to an extent.

Edwin: Yes, because I was in the top set there about for most of my life. I didn't really experience that people of year 9, year 10 not being able to do basic skills and you're thinking about, "Well, they're going out into the world and they're not - some of the stuff they can't do is really basic." If I was to stop my year at the middle set class now, their math education now, they would still be able to function well in society. But the ones without the numeracy, they're the ones who really go about.

The transmit body of pure mathematical knowledge, useful math with an industry centered focus and critical awareness of society by maths. I am going to put use for math in industry-centered focus low just because I really hate the idea of -- well I think, a lot of times people talking about industry-centered or real-life maths. The maths involved isn't very interesting. They're not really learning anything. You've got two options. You either doing a normal question and you just changed A and B to the name of a company or something.

Or..... maybe -- a lot sounds quite obvious stuff and I don't think it necessarily needs to be the industry focus. I would really tie all of these three together - transmit body and pure mind useful. I would tie the industry-focused one as well with the transmit body for maths knowledge and the critical awareness of society environmentalist. Because I think, I would try to fit in as much as I can all of three of these in my lessons. I couldn't really say that for the creativity, but when I see an opportunity I would try and fit these in. I'm just going to -

Interviewer: And if I've interpreted you correctly, it's a genuine opportunity rather than -

Edwin: A genuine opportunity for - [crosstalk]

Interviewer: - for the sake of it.

Edwin: Yes, definitely. I'm going to stay low of this because I think this one can get abused a bit. I think that's the key point, it's got to be genuine. I think probably now as I'm progressing with this teaching I'm a bit more confident with - what is learning and what's going to get -- I might actually, if I find something really really interesting, I might get it in the lesson. It might not even necessarily be to do with the topic, but if it's something really interesting maybe just a small paper or video, I might put that in because I think that can make them interested in maths as well.

I'll move this one here. Then I'm going to go teaching first because I think teaching -- you could say learning should come before but I'm someone about learning before.

I'm going to go teaching first. The theory of teaching maths motivate through work relevance and that links to what we were talking about before. When I was talking about that I've got my own personal bias, I didn't enjoy that at school. I know there will be some people who will find that more motivating if it's related to work.

Facilitate personal exploration, discussion and questioning. Discussion I'm going to put down below, because - we talked about in theory of maths, we talked about when you can have mathematical conversations and definitely in my classes, the students can work together to get them discuss their answers. I think discussion probably works a lot better - it's a little easier in other subjects and I can't advise because we've got observations and we set out the rows even doing a discussion. That's meant to be a feature of the lesson, and sometimes not relevant. Questioning actually though, because it's got questioning, I really think that's very important.

Interviewer: Yes. You could just do questions as you said. You're not that - if you want to set that's absolutely fine.

Edwin: Yes. I'll do. Questioning, I think is very important. I'm going to put questioning and discussing answers is good. I think just trying to get kids to have a discussion about maths. They're not really something mature enough or interested enough in the

subject or have the subject knowledge to do it. That's why I'm putting discussion down here. Again, I think this links to what we said before, I'd be happy to deal with the transmit pure mathematical knowledge or industry-focused or the critical awareness in essence, as long as it was relevant, as long as it was purposeful for you doing that.

Motivate through work relevance, facilitate personal inspiration that's, I'll put that there, transmission drill, no frills, top one definitely. Explain, motivate, and pass on structured of knowledge, that's definitely what I'm trying to do more of my teaching is now is do maybe one example and then pass it over to the students quickest there, so a lot of time they might struggle up a bit more with it but I think they're gaining more from actually having that independence and trying to doing themselves.

Transmission drills and no frills, I think that definitely has its place. Questioning, discussing, explain motivate, past knowledge, questioning, discussion and answers, transmission drill, knife rules, motivator, I think that is important. Here it's different because you're motivated, so it's definitely a focus to motivating and is motivated, that's important. Then facilitate personal inspiration, and then I've left discussion at the bottom.

Interviewer: Do remember you don't have - if you don't agree with something you can just get rid of the cards as well.

Edwin: Yes, I will leave it on because I'm not going just - that might be a thing, definitely some opportunities for it are probably rarer than these. Facilitate personal inspiration, yes, because we have that, we've had some discussions with students and you can have that. The students are talking about how they feeling and what their views are on it. Yesterday my year 11, I had to stop a student to spit their gum and they started talking about how it wasn't anti-Marxist
[laughter]

Interviewer: Really? Which year is this?

Edwin: Year 11.

[laughter]

I wanted a more interesting explanation for why they were chewing gum, and then I guess I wasn't necessarily linked teaching of mathematics, but was interesting to have that timed discussion [laughs], yes, very surreal moment.

Interviewer: Yes, I'm sure, yes.

Edwin: [laughs] Theory of learning maths. Activity and exploration central, questioning and negotiating, meaning essential, practice and rote. That's probably been the easiest one to agree with all, all five of them. Activity and exploration central, questioning and negotiating.

[silence]

I don't necessarily see all of these as mutually exclusive. For example the practice and rote one, I think is an unfair representation. In China they're very key on making sure everyone practice and rote. I think through that as well like depending on what the questions are, there can be exploration and you're going to have to negotiate meaning when you're practicing.

Skill acquisition and practice, so see the difference between skill acquisition and practice. Its skill acc maybe would be in examples in the textbook if they are changing, and then that they're not just asking you to do the same thing. We've got these new textbooks actually for the new GCSE and so they all - the questions are quite awesome, that's the back page. I was using them today and the questions were very good, they're very varied and they're so it was like-

Interviewer: I think I've seen that one actually.

Edwin: We did a look at - I preferred a different, I preferred the minors one I think, because I preferred the minors one because it had a section for students to practice the skill just concretely, to get the skill solid, and then they could go into the problem-solving activities. Some students couldn't be able to get straight to the problem-solving activities but, some kids that was really good so, they got to just practice the right skill and then move on.

Interviewer: Do you remember, I don't know if we did it in your year but we introduced these words sheets called *Practice, Technique, and then Challenge*, and the idea was the exact say use of - you do some sort of straightforward practice and then you go deep in those techniques, and then you have a challenge question and it's a good way of differentiating.

Edwin: Definitely and I'm going to put questioning probably up high, because - and negotiating means - because I think that's actually really when there's a lot of learning going on and checking.

[silence]

I think these three are the actual doing activities and I put them all as equal because, I don't think they are mutually exclusive. Then understanding and application is key to progress.

[silence]

I'm just trying to think about different sets. I think it's probably very much true for a bottom set possibly if you got higher ability class they don't necessarily need to practice the application to progress. I leave this one.

Interviewer: All right, thanks very much. Was there anything else you want to add? Nothing, but we don't necessarily have to stop it because the cards have been arranged. Was there anything else you wanted to say? Like I said you don't have to but -

Edwin: Possibly how my interpretation of this is changed is I've developed as a teacher, I've become more confident as a teacher. Coming in I wanted to try and I do fun lesson all the time and trying to do activities, and make the lessons really exciting, and then I probably moved to more the - teaching and get into practice, trying to put together some skills, and now probably is simply doing more of this. Maybe just do one small example on the board and then try and get people to get on with the work as quickly as possible. I developed as a teacher, my understanding of teaching and what teaching should be, has changed. And I probably recognize I don't know, if you came and asked me this like in a year's time. I might have different

answers. Because I definitely would have had different answers at the beginning and probably at the beginning of my first year.

Fabia's Interview (30.4.15)

Questioning and negotiating meaning in terms of the way that I teach for students understanding is a major part of my teaching it's incredibly important for pupils to be questioned and pupils be able to question themselves. I think if you question a pupil they learn it more deeply because they are being questioned about their actual understanding.

The danger with maths is that they can repeat a sum or module you have shown them but they have not learnt how to understand it. So for me that's really important.

I mean they are all important. Activity and exploration, I think they are important for learning but not used as much in schools, especially in secondary because they tend to take more time and tend to be less exam focused so you don't give pupils the chance to do more activities or explore. So yesterday, because it was the last day before half term we did a really great activity and actually it's the most engaged I have seen them all half term. Everyone was working, everyone was doing stuff and yet they weren't doing exam questions and they were Year 11!

Practice and rote, I do think that rote learning does have a place in most subjects. Certainly, I do think it helps in speed and fluency in Maths but in terms of learning mathematics I would put it there – maybe third. These two are quite similar for me, skill acquisition is gained by practice and I do think it is important to practice and to have things learnt by rote because it does make their ability to tackle harder problems easier but I don't feel it's central as how I want them to learn.

Understanding application is key to progress, yes I do think that is important for me. This is more important than practice and rote because I feel that helps them focus their learning and what they have learnt is what they are modelling as opposed to practice and rote which can mean that they have learnt to copy a teacher's description or example, that does not necessarily mean that they have learnt the maths but learnt a process. So I don't value these as much, but they do have a place, but I would value these three more – more central to pupils learning of maths.

Aims of mathematics education

If we are talking about the broad overall aims of maths education should be more (more motivating for the pupil) about the discovery of maths and how that helps them live in the real world and that does not mean that they have to do maths that is directly associated with an industry but that they can apply it to being a scientist or being a statistician but they can use their mind and the skill that they gain from maths to assess, to live well, in the real world. So these two get that sort of aim than 'Useful maths with an industry centered focus'. I just feel that will turn so many children off maths, like if you aim maths education towards you should be able to become something that uses maths in your job, I feel it will put so many pupils off maths. Like if you aim maths education towards 'you should be able to become ...something that uses maths in your job.

It takes away some of the exploratory aspect of maths and gaining of a mathematical mind rather than just knowledge for the sake of industry. And for the same sort of

reason I don't think that the aim is to just have a body of mathematical knowledge, its to be able to use maths and think about maths not just know a set of mathematical facts. I think that numeracy, I feel it should be a separate thing to maths education. I think maths as in what we do in the classroom at KS4 and numeracy are not really the same thing. If you get a GCSE you don't have to be that numerate, whilst it is very important to be numerate in real life and I think lots of us are not. I think numeracy needs to be taught but I don't know whether it is or should necessarily be an aim of Maths education, whether it should be numeracy as a separate qualification. If you can do a free standing qualification in numeracy then I think that should be for people that don't get on well with KS4 maths that those can be very very useful

I feel like when people go to the shop have a Year 11 class not a single girl would be able to tell you that if they bought three items in a shop roughly what change they should be expecting, none of them would be able to. So they could easily be ripped off - that's an incredibly important skill but I don't think it's the aim of secondary maths education but I do think it's the aim of primary - where it is called numeracy.

Theory of teaching mathematics. So I think that discussion and questioning for me is a central part of my teaching and how I think, how I believe about teaching anything but especially maths because actually every other subject that they do is so centred around discussion and one of the things I find about maths is that pupils don't expect to have to talk about it or to reason or to explain their answer. The same with writing things down, If I ask pupils to justify their answer they say well we don't write in maths. But I do believe that its really central in their learning and in the way I teach it get the best out of the pupils, I find it really really important. Questioning is completely central to all of my teaching and how I assess and how I differentiate. I think my way of trying to teach pupils maths is to motivate them by the skill they are learning and the processes they are gaining an understanding of which I think is what this is about. So I feel that one of the aims of maths is the ability to use a mathematical mind, to be creative and to have self realisation and that is helped by the processes that you are learning. As a complete opposite to this I don't think that pupils are going to be that motivated by thinking they are going to use the maths they are learning but by using the skill they are gaining from persevering and from picking a problem apart and using an easier example to consider how to do a harder example or generalising, I think that all of that is actually incredibly useful in the workplace rather than the maths content itself. Facilitate personal exploration is connected with what I was saying, transmission and drill like the practice and rote wit teaching I do think it has a place but I don't value it as highly as these ones. In an exam focused exam system, it is necessary.

And then, how I feel about mathematics. I will put these in a circle because it all of these things. I feel it is a collection of facts and rules, it is socially constructed should be taught in a way that they realize that there is pure knowledge in there and it can be personalized but I think it (maths) is a specific way of working and using your brain and the process, mathematics is about processing and how you get to an answer and how you tackle a problem and for me that is a really central part of mathematics.

Fabia then mentioned she wanted to look at the HS2 and the impact on the community and the school. Where to put a new airport

Jason's Interview (27.4.15)

Aims of Mathematics education

I think my first thing I will say is 'Useful maths with an industry centered focus', I don't think that is so important. The way I think about maths is that the reason why it's unique and important as a subject is because pupils can understand and go through the process of learning maths and getting their head around it that does something that makes them more intelligent makes their IQ higher and makes them able to make links in other areas and we live in a world that's changing as well so we could teach them some industry centred maths if we wanted them to, however it is not necessarily going to be applicable in four five years time plus not doubt the industry themselves can do it.

So looking at the other, I'd say transmitting, thought I'm not sure I'd use the word transmit, like the pure mathematical knowledge and that I would still include your other areas of applied maths anyway, is quite important.

Creativity and self realisation, I think if you can work something out on your own not necessarily undirected, but often it's more effective guided can really improve and get the pupils to direct their own studying, then that will in turn make them more intelligent therefore more capable of studying, not only maths, but other subjects.

Numeracy is essential as a starting place for everything, so if a pupil comes in weak then they can just get back to basic numeracy then that is really important but it shouldn't be at the top of the list because a lot of people can master that quite early on and you've got to look at what you build on from that.

I think critical awareness is more important, we live in a world where the newspapers lead us into different ways of thinking and we need to be really careful when they do that so pupils can be really aware of statistics and percentages and how that kind of data is gathered and used that will make them more capable of making their own decisions when they are older.

Theory of learning mathematics

My opinion is different for different pupils. I said maths is about making them more intelligent but as a teacher you do have that role of...for instance a lot of my Year 11s right now have aspirations that they want to go onto college, some of them have football coach type aspirations some have picked a school where they can go to do plumbing and they need to be able to get their C in maths in order to do that. And my responsibility is not to make them understand the maths in that situation, my responsibility is to make sure they have the opportunity to be able to go to that college – so it is a hoop jump.

Activity and exploration essential I would put towards the bottom. As I said in the last one, creativity and self-realization is important but I think the a lot of the mistake with maths education is to think that that needs to happen in a constructivist and

indirect way and I don't think that's an effective way to teach what is essentially a large body of knowledge.

Practice and rote is interesting I believe that practice is incredibly important and I think if they work hard, if they are taught well and they work hard they can do well at maths. I am going to put that below activity because of the word rote. But I would put 'skill acquisition and practice' higher because that sounds like they are actually getting it and they are not just memorizing and that is more effective. You do something rote they will get it and they will repeat it but you'll be lucky if they can do it six months later.

Understanding and application is key to progress, I think if that is talking about applying maths to the real world then going on what I said I don't think that is so important but in terms of understanding something and being able to apply it to a different context in terms of the body of pure mathematical knowledge. I do see as important so I'm going to put that at the top.

Questioning and negotiated meaning as essential I think that this is something that I probably more important for those higher ability, higher attaining students. I'd expect all my students to question but even my Year 12s who have been doing AS maths we've still got a lot to get through and we have a lot of basics to recover and often the meaning. Has got to be dropped and I ask them to accept something that's just beyond their reach in order that we can consolidate something that's more basic. For instance with integration between a curve and a line we'll spend a lot of time just talking about which one will be taken away from the other and the shapes that make them up. Because we spent the extra time on that we then drop the idea that understanding the actual process of integration and the mathematics that is actually happening. We'll leave that and if they want to do maths next year it can be picked up then. So put that in there above activity and exploration.

Theory of Maths

I think this is where, not studied maths at degree level, my opinions are less strong and less clear to me. However, it's obvious that a 'collection of facts and rules' is not true, I would go as far as to say that is not what maths is.

A personalised activity, I don't really like the sound of that because it's not too far of a jump from that to go to does not matter if you don't get the right answer it's the working that counts and I don't believe that is the case I think people who think like that need to be challenged. There might be more than one right answer, there's many ways to the right answer but we do need to challenge pupils when they don't know how to do multiplication and don't know how to do lots of things – I suppose it could be personalized in the sense that you might have your own way.

Unquestioned body of useful knowledge. Well that's just silly because I know people who are doing PhDs and if it's unquestioned, then it's not going anywhere. The maths I teach is similar to the maths I did and, I am sure, the maths my dad did. However, definitely at the higher end it clearly must be changing and computing etc, those kind

of applied things that technology has had a big impact on has changed a lot of the way we think.

Socially constructed practice. So people have thought up what it is whereas a lot of mathematicians would believe that there is a naturalness to maths. Whenever things get difficult I think like a historian (13.56 – 14.34) I am going to put socially constructed practice at the top.

Theory of teaching mathematics.

I think that's an interesting one to think about, the work relevance. Pupils can be really motivated particularly with things like interest rates and compound interest they find it absolutely fascinating. Particularly when I am teaching GCSE statistics, particularly like the kids love the idea of the FTSE 100 and they particularly like the idea of – if I did that I could earn this much money so they like the idea of banking.

However, I think if that is one of the most important things then you set you up to fail as soon as the first pupil asks you the question of 'im never going to use this' – for lots of things there's no answer to that. Inequalities, for example. If you say 'this will help you' the whole time and they get a diet of that then they are clever enough to spot when it doesn't. A lot of maths is relevant but as a teacher you are not always necessarily aware of that.

I think they learn maths that is going to set them up, just the actual aspect of learning it is going to help them. So whenever they ask that's why I'm always saying, that's how it helps you in later life.

Transmission drill and no frills, it's interesting maybe I am rejecting this out of hand but some of my lessons are very much transmission drill and no frill, I'm not going to lie about that but it's not necessarily the way I would want them to be but I don't think that's what maths should be. A lot of my lessons are more like this, explain motivate and pass on structure of knowledge. We look at something and we discuss it, we do some well meaningful AfL to understand who knows what and then we go into some practice of it, maybe something that might resemble drills but that's only because we have had that opportunity to get an understanding of it. So in that sense questioning and also that wider questioning, questions on the board challenging questions.

Facilitate personal exploration. Constructivism is a mistake by the maths educational body. You want pupils to be active and their brains to be making connections and thinking because that's what's going to help them learn and as a profession we made a mistake by thinking that the best way to do that was to give them a task and let them get on with it and in fact what is more important and more effective is to give them some of the information and ask them the right questions and definitely don't involve discovering the rule for yourself. You have the rule and now you can apply it to the problem. Maybe you have learnt how to collect like terms You have shape and you have algebraic terms around the outside - that personal exploration is very good.

When I'm thinning about my Year 7s often I allow them to have that kind of time, but when the crunch comes in Y9 and Y10 Y11 and you are held accountable for their grades then you just have to say we are going to learn this and we are going to learn it properly.

Minervia's Interview (14.4.16)

Interviewer: The way the interview is done is that you think out loud about the decisions you're making, and then I'll know. At the end of it, I'll take a picture of the arrangements you made.

Interviewee: Can I mix them up or does it have to be this one, this one, and this one?

Interviewer: Yes, you can mix them up absolutely. It doesn't matter which one you start with. So, it's very open in terms of how you want to do it. Pick whichever one you want first, but the idea is to have a look at all of them and then –

Interviewee: I'm just going to outlay them?

Interviewer: Sure.

Interviewee: I've been to this training the other day and they were talking about a Diamond 9. Have you ever heard of that?

Interviewer: Yes.

Interviewee: Maybe you will have a look if it's appropriate, but there might be too many in there. So, you're talking about my sort of view about mathematics and --?

Interviewer: Your view of your maths and how it relates to -- so, this is your view of theory of mastery, what the aimed message should be, your view of the learning, theory of learning maths and the view of teaching maths.

Interviewee: You want me to pick those as a mathematician or as a math teacher?

Interviewer: As a maths teacher, but if you feel there's a difference, then that will be quite important to as part of the interview. If you think for example as a maths teacher, I'd pick that one, but as a mathematician I'd pick that one, then that would be a very important part of the interview.

Interviewee: What do you mean by a socially constructed practice?

Interviewer: Socially constructed practice means that maths is not something that's an isolated subject and it relates to the social world around.

Interviewee: I'll definitely pick this one because I'm reading and questioning body of useful knowledge and a collection of facts and rules. I totally disagree with the end questions because, again, I did a four-year degree so I did some research in my last year, and we get explained that research is all about pushing the boundaries. I would definitely take this one out, but maybe as a math teacher because the kids need the basics, sometimes it would be better than to talk about this.

Kids would not be in a place to question how do you add fractions; or multiply fractions; or things like that; but definitely this one. I would take this one as well. I'm thinking about different changes in general and how would you address

mathematical facts and knowledge to different kids in different ways. I saw this one earlier. I teach a Year 9 set seven and definitely this is something they need. This is something I would focus a lot, but not with all my sets.

Interviewer: Yes, because this is the back to basics numeracy.

Interviewee: Yes. Making sure I equip them with the basics in numeracy for them to succeed in life and be able to do the basic things in the supermarket or whenever they're going sell or things like that. I'm just going to put with this on the side for now. I like this one, but I don't know. It seems a bit static, structured body of –

Interviewer: Okay, structured body of pure knowledge.

Interviewee: It sounds a bit like, that in a question of body of useful knowledge and you can't really touch it and model it. Transmit body of pure mathematical about useful math with industry-centered focus. I will just put this one there for now.

I don't know. I'm a very vigorous person. I really like this, but again, I think this doesn't really imply that you would still teach creativity; explain, motivate, pass on structure of knowledge, because you want to be passing on this structure, and you want to be passing on the rigorousness, but that doesn't really imply that you're being creative and maybe we can just put it there. It just completes each other. What does it mean transmission and drills? No frills, as in no creativity?

Interviewer: Yes. I suppose to an extent, some people would see that as if you think back to things like when we did Skemp, and the idea of instructional learning in a very transmission-based and this drills and frills –

Interviewee: I'll take it for now. This really goes with creativity in a person, so I'm going to put it there. Now, discussion and questioning—

Interviewer: So, it's the facilitate learning exploration I think.

Interviewee: Yes, personal exploration. Yes. There was this TED Talk actually from a math teacher, an American math teacher. He was saying how the books; the mathematics books were really not well written, in a sense that every problem would be deconstructed into many steps, so that the pupils will be able to solve the problem; but if faced with the problem in real life, they would never be able to come up with the steps. He was saying all about how we should teach the students instead of looking at a math problem, and finding the steps for themselves. I thought that was really good.

Interviewer: Do you remember when that was?

Interviewee: I can't remember, but I can send you the link.

Interviewer: That'll be great.

Interviewee: At least I can put it on my Facebook.

Interviewer: Yes.

Interviewee: It was really interesting. He was saying how he showed one of his classes, a big sort of pool thing, and he showed himself pouring some water into it, and he just showed that video and then at some point a kid said, "Oh man, how much time is that going to take?"

Interviewer: It wasn't Dan Myers, was it?

Interviewee: Maybe. He's an American young teacher.

Interviewer: Yes. It could be Dan Myers Sounds like the sort of thing he'd do.

Interviewee: Also, you know that the event was really interesting. Why did I say that again? Facilitate personal exploration so that the kids would be encouraged to actually work out from there. That's quite good. We did lessons on graphs and I applied it to finance quite a bit, but then that was difficult for some of them because some of them were not interested in finance. I had to think of other ways to motivate them. We'll just put them on the side for discussion and questioning.

Interviewer: That's to motivate through work relevance?

Interviewee: Then this one I put here for now, but I really think that this could go there anyway; the discussion and questioning. Understanding an application as key to progress, but not as practice in road. What does it mean, road?

Interviewer: It's like road learning. Learning just by constant practice, but not doing anything else.

Interviewee: I'll just put this one with the back-to-basics. You might see that these two are quite good together. Activity and exploration are central, definite. It should be central, but really it's not at the moment.

Interviewer: Yes. That's quite important to mention, yes.

Interviewee: Skills, acquisition, and practice, and I think this just could be there on the same level. Questioning and negotiating meaning is essential. Yes, this makes me think of the activities you made us do in the truce, always true sometimes, but it was never true. I think these are actually really quite good for students, because they make them think about -- it's discussion and questioning as well.

Now, let's try and make this into a Diamond 9. I would definitely put this one as the first one; creativity and self-realization for mathematics. I like to motivate. It's not just giving on the knowledge, it's all about giving the passion as well for the subject, and not just the plain knowledge. It's more going to be like a diamond 15 perhaps[laughs].

Discussion and questioning, these two go together, understanding an application that keeps progress; together. Two to an exploration are central. Can I put these two together because for discussion and questioning?

Interviewer: Of course, yes.

Interviewee: Seems to be like that. There you go. I'm missing a row, but I think we could -- it's fine?

Interviewer: No, that's fine. These you've discarded them?

Interviewee: Yes, these I'm not sure I want to put in. It's because this one for example is -- I like what this is saying, but I think explain, motivate, pass on the structure of knowledge is much better than just transmit body of pure mathematical knowledge; because this is quite static, whereas this is implying that you pass on your passion for the subject as well. This is the industry centered focuses with the work. They motivate through work or whatever, so we already have that.

Interviewer: Great. That was basically it. Thank you very much. What I'm going to do is take a picture of this.

Interviewee: Can I take a picture of it as well?

Interviewer: Of course, you can, yes.

Rachel's Interview (8.12.15)

Rachel: I might do this two together.

Interviewer: I've started recording, so just like I said, the best thing to do is read them out but have a holistic look at them and then start thinking about what you think really.

Rachel: Its discussing these views with a friend who had recently gone on a course to look at what the theory of Mathematics is.

Interviewer: Right. Okay. Yes.

Rachel: Actually, we came to the conclusion that, we both did Math degrees and we're quite keen on the pure knowledge part. Actually, it's funny with Math being such a cornerstone in society. Everyone needs a good Math GSCE. It's actually, well, they don't need pure Math GSCE. They need numeracy, basic understanding of how numbers work and how we use numbers, rather than that say-- I love math because I love the pure stuff, but actually I don't necessarily think that's the best thing that we can teach especially on this level. Where to start?

Interviewer: Start where you feel comfortable, and you might want to just take each one at a time. That makes it easier, rather than seeing the whole picture. Obviously you can change your mind whenever you want.

Rachel: Okay, so if we start with the big stuff, the aims.

Interviewer: Yes, absolutely.

Rachel: Like I was saying, I really like the pure stuff, but I'm aware that that only appeals to a certain type of brain.

Interviewer: Right.

Rachel: I'm very much into that, but of the children I teach there's probably fewer than 10 that would appreciate that. I think it's really important that it's there, for their sake. actually because Math is so very different once you get further up. I think with a lot of other subjects, you do get, kind of a feel for what it looks like A level and for University. You don't really get that with Math. Yes, I think it is important to have some pure stuff.

Yes, like i was saying before the critical awareness in society and the numeracy stuff. I think that's what most students would expect out of Math. I think what most parents would want for their children, and what Math GCSE is meant to stand for really. Use of math so that ind cen focus. Is that looking at things like, Trigonometry being useful for engineering, things like that?

Interviewer: Yes, absolutely.

Rachel: I quite like that there are some applications for things. I'm very much kind of why would bother applying. It's fun for its own sake, but actually there's quite a lot of nice things. Again, it's quite specialized. Again, I would probably downgrade that. It's a nice idea, but in a class of 30, probably not always applicable. Creativity and self realization, I think it's because I'm not very good at the creative side myself. I find that quite hard to teach. We're trying to do a lot more investigations at the moment, and I just find it very hard to kind of, give a structure without dictating what needs to be done. I think actually, there's a lot of Mathematics creativity is just kind of, playing around numbers, just problem solving, just having a bit of fun.

Interviewer: Math can be creative for its own sake, you're saying.

Rachel: Yes. I was showing you my further Math script, and tuckers self refreential formula. I don't know if you've come across that. It's basically a graph that plots the same formula. It's so exciting, and actually it's that kind of thing that I really love and they really love, but most people would just think it's a bit hard. I think that's quite nice, but I just-- again, it only appeals to certain students and given that we're operating under schemes of work, quite often, I find it hard to actually find where can I put these in. thing and where can we have this. Put that there. If kind of, I rambled enough about the aims? [laughs]

Interviewer: That's fine, and if you feel you've missed something, do go back to it.

Rachel: Just talk about Math, Math is great. Maybe come to this and then come to teaching of it, so let's look at theory. "unquestioned body of useful know," that's an interesting choice of words. I suppose because Math is very absolute, is very axiomatic, once it is true, it is always true. Supposing that sense is unquestioned. There's nothing in there about, linking things together, because I think that's one of the most exciting things about Math. Yes, it is a collection of actual facts and rules, but

actually they will relate to each other. Like my final project at Uni was linking. Hyperbolic Geometry and linear algebra and number theory analysis. That's one of really nice things I liked about it.

Actually, yes, it is facts and rules, but actually it's also, how do you apply those and how do they all relate to each other. It's the relationships I think, the most exciting things.

Interviewer: That's not really here, but in the course, but it's something you think is part of the theory of Math? That's fine.

Rachel: Definitely. A lot of the facts that we use, say like, for a long time, I didn't really understand why we taught circle theorems, because they're just an archaic set of facts that don't really relate to anything, but actually, the way I see it now, is that the point is that we are teaching this-- I kind of took it, and we're teaching the analysis behind it, so it's looking at this new problem and thinking, well, actually, I know all these different facts. Which of these can I pull out, and which of these can I apply to it. It's that kind of analysis and the things behind it and what you do with the facts and rules that make it Math. "Maths is a personalized activity", not really sure what that means.

Interviewer: Well, is talking about Math being something that a lot of people would tend to think it's very-- sort of, isolated, personalized activity. In schools obviously, and classroom, because obviously sort of lot of group work and discussion going on. Someone say-- you know, relating back to the idea of an unquestioned body of useful knowledge. What is there to discuss in Math, because of the-- so it's personalized activities rather-

Rachel: Its kind of individualized, as opposed to a collective.

Interviewer: Yes.

Rachel: Okay, I would disagree with that. Because I found-- so obviously I haven't done a PhD or any kind of long-term research, but I definitely found that I work a lot better with other people. Normally its kind of, study groups of two or three other people.

Interviewer: You're sort of doing research every day when you're in your class, aren't you?

Rachel: Yes, that's true. Actually I talked to members of the Math department almost constantly. I don't really have a disagree part or the-- that's going to be my new disagree part. The big example of that would be Andrew Weils being very personal, but that actually being completely different. Many papers that you read have three or four names on them, because people do work together.

These two are quite close, they didn't tell me the facts and rules and useful knowledge and, I covered that. Socially constructed practice. I suppose a lot of the ways that we see Math in everyday is social constructed by, I think it is still there. Even if we didn't use the names we had for them, we'd still want to get use a lot of the concepts that we have, and I disagree part. "A structured body of pure knowledge". I agree with that,

that's not all there is to it, there's the links and there's the applications and the reasoning between all. I feel like I'm just rambling, is this useful?

Interviewer: That is exactly what this is about, so it is. We're having me-- you're having very little input. Basically talked about your beliefs within this. You don't believe this for example, it's actually a sort of practice, you do strongly believe these. I don't know if I mentioned to you that part of this study is looking at people with Math degrees and people without Math degrees and it's quite interesting how Mathematical beliefs may or may not be different in that sense. This is actually very useful. Just you discussing this.

Rachel: Good. I'm glad. I might stick a post-it on that section and just say-- kind of links in relationships between things. Not just applying but analyzing and applying. Kind of my caveat for those three.

Interviewer: Okay.

Rachel: Let's do teaching next, theory of teaching. If I could get away with no frills, then my life would be significantly easier. But a lot my children just can't cope with that. I think this is the kind of thing you can get away with more mature students. It's kind of almost lecture style but you need the maturity and you need an attention span. You need an ability to actually understand what's going on as you go. That works for very few students of this age, I think.

Facilitating personal exploration. I think that's quite interesting. I really like the idea of it but I find it very, very difficult to do in a class of 30. Actually, I had an obstacle a couple of weeks ago on Linear nth term. Actually, my feedback was, "Yes you did kind of discovery together" but actually it would have been better for them to have done it in small groups. I think my worry is more on the practical side of kind of actually—I almost don't trust my children. There needs to be real depth of understanding of the pedagogical side of what hints the children need, what are the right questions and the right phrasings.

I personally don't feel like I'm quite there yet. If that is done well, then that is very very useful. Cause I think it's just two things students remember it better. It's that thing "I did myself." The other thing is, it's that kind of feeling of success. That is just paramount. If you can have a feeling of success in the classroom, then that means everything else in the classroom fits together, that would be better.

Discussion and Questioning. I do that quite a lot. My typical example is-- my typical lesson is not the exploration, it's this: "Let's have a look at this problem. How might we approach it? Let me do one example. How do you think we're going to use this example can help the next example?" then off you go. I've try to do a lot more, "Why is this happening? Why have you said your answer is this? What's wrong with this other answer?" Yes. Questioning is really useful because you can't open up a brain and see what's going on in there. It's kind of the next best thing we can do. Questioning, again, it is quite hard because you have to find the right question to get the answer you're looking for. Sometimes it's kind of "Guess what this really means," questions. Sometimes it is a kind of actual, "what is in your head" questions.

These two about motivation through work relevance. Is that kind of looking at things in the real world that's saying this is where you're going to use this?

Interviewer: Yes. Often childrens at school, wherever they use this and relating it to, like you said, with the trigonometry and the engineering. How does Math relate to your lives and how the work you do?

Rachel: Yes. Actually this is one of my targets at the moment. I'm conscious that I don't do it very much because I see so much in the kind of pure sense. I'm content with that. I forget that people aren't, but no, I do think it is quite important for students to realize. Actually, this is one of things that is directly relevant to you. Things like, I found that our students are so bad at estimating, cause I think that's one of the really, really important and kind of basic numeracy expectations. Things like, if you're spending X amount on flights and this amount on hotel, what's your rough budget going to be, can you afford it? Or is going around the shops, how much money have I got in my purse? I think that is really, really important. Explain motivate pass on structure of knowledge. Can you just clarify what that one means?

Interviewer: Yes. I suppose, it's slightly related to just passing on a structure of knowledge really without, I suppose the opposite of that, looking at the work relevance, so it's just seeing-

Rachel: Its a kind of, quite abstract.

Interviewer: Yes, seeing Math as a structured knowledge, and Math in itself is justification for the subject.

Rachel: I think it is important, but I'd be wary of which classes I would do that with. Probably, higher sets I would do that with a little bit more, because they're kind of capable of seeing the bigger picture, they're capable of holding it all in their heads. Actually, I really like the links between lots of different things, but for some students I think that would just confuse them. Things like, so is that looking nth terms and determine how that would relates graphs?

Interviewer: Right.

Rachel: That kind of thing. I know there's a lot little challenge. Teach them together, teach them back to back. Look at the links between them, and I would love to do that with kind of higher set, but my mindset for it at the moment. I think I just-- I kind of teachings discreetly, because otherwise there's too many processes going on that they just can't quite cope with. I think this relies on every single individual item being very secure, and being able to kind of look, and see the bigger picture, somewhere in the middle.

Is there anything else I want to add? I suppose I could. Learning Math. I think practicing gets a bit of a bad wrap. I think there's some things that, you do just need to have a little bit of time to get your head around. I was doing kinematics with my interns this morning, and they were very, very lost, and actually, all we need to do is just, let's do another example together, again, now, you talk me through this one, and now can you go, and it is just that kind of processing time. It's repressive in terms of, if we did that every lesson, then they'd get bored, and they just-- motivation would go out the window, engagement will go plummeting, but I do think it has it's place.

Interviewer: There's a place for it.

Rachel: Definitely. Skill acquisition practice. It's quite similar to that isn't it?

Interviewer: Yes.

Rachel: I was thinking of that, also just kind of highlighting what other skills and what other facts as a different.

Interviewer: Yes.

Rachel: Different set of things you have learned. I think those are quite useful and actually, getting students to identify the skills. Because I found-- when I do, when I get to do self reflections quite often, I have done well because I got them all correct, rather than what skills have we being practicing today? I've been doing questions, I've been doing algebra, no. [laughs] I try to do that in my kind of differentiated learning objectives at the start. I try to kind of say, this is the main learning objective, before we do that we need to be able to do X, Y, Z.

Things like, expanding brackets you need to multiply algebraic terms, and then kind of differentiate looking at, kind of what the possible negative numbers, because for a lot of the students that's-- answer is no. [laughs] Activity and exploration are central. I think they're important. I wouldn't necessarily have said central. I guess like I said before about facilitating personal exploration. It is it is a really nice way of doing it, but, I find it pedagogically quite hard. There are some things to learn yourself to it and there are some things that just don't. I think that's one of those that varies quite a lot. Yes. They're useful.

The activity bit, the kind of, them actually doing stuff. Its really useful, because actually and with Math, we don't really learn by reading. You don't learn Math by reading an example, you do learn Math by doing it yourself. Yes. Definitely, questioning and negotiating meaning. Is that between students or between student and teacher or?

Interviewer: I think it could be both but the idea that there is some sort of questioning and negotiating, meaning as parts objects where the—

Rachel: Okay. I personally haven't really thought about but I do quite like-- cause sometimes, some of the best points we make in a lesson from things that I just kind of, never even thought about. Students have come up with but what if this happens. Yes. I agree with that actually.

Interviewer: I suppose, in a way it relates to that as well, in terms of discussion, questioning, so in terms of theory of Math. Is it a subject where you can have questioning and some sort of negotiating meaning of something.

Rachel: I think definitely, discussion means something very different in Math, as opposed to other subjects particularly. But no, I think questioning and discussion has its-- it definitely has its place. I tend to do a lot more of the kind of you ask any question, I answer it well then you ask me a question and get someone else to answer. Which is something else that's being pointed at me and it's quite a nice idea actually.

Then understanding and application is key to progress, definitely. Its that kind of, you know, you've got to do it, you got to practice it yourself.

I think what I have struggled with my lower sets is the retention. It's kind of, if you learn Math, it's kind of can you do Math? Can you do it? This lesson. If you can do this lesson, that's great but can you still do it next week and depressingly frequently the answer is no. I'm still kind of trying to work out which of these would help with that. Theory of learning mathematics, I guess. Can I add on remembering things?

Interviewer: What's important is that you said, which of these might help do it, because there would be two schools of thought on that. On scale, I think they're both right. I don't like the idea of Math being sort of, polar ends. But some would say practicing route is a way of remembering, someone say well, actually children being active and their learning and doing something like that, is its best way of remembering. Its quite interesting that you put remembering because people have different ideas of how children might remember as well.

Rachel: Yes. Actually this is kind of a prerequisite for them, how do you teach, cause you can't teach things as a bigger picture. You can't teach the whole structure if they don't remember each individual little bits.

Interviewer: Was there anything you wanted to look back on and sort of comment on or holistically, if you want to sort of say anything about the whole thing or?

Rachel: Yes. That's fine. I think I do struggle as a Math teacher with that balance between, I want them to understand things, and I want them to pass the exam. Because unfortunately, they're not always the same thing.

Interviewer: I think that's quite a common struggle with most Math teachers.

Rachel: I like the idea of the new GCSEs being much more problem solving. I feel like I haven't come to terms with it yet. As a mathematician, I like, as a teacher I'm terrified. I think most of I wanted to say, I said to you.

Interviewer: All right. Thanks very much for that. Like I said, the whole point of this was that, it was you doing more or less all the talking, because then I can ask any leading questions, and then, it becomes quite sort of unbiased interview, apart from the fact that I've given you lots of prompts which sort of introduces a level bias straightaway. When I was looking at this, I was just thinking this is probably the best way of someone openly being able to talk about their beliefs, which is why I did it like this. Great, thank you very much, so I'm going to take a picture of these now. Then very quickly just discuss-

Rachel: The next steps. [crosstalk]

Interviewer: Yes, the next steps which should be looking at a lesson, which I suppose if I was looking at these cards it would be looking at things like this idea of, critical awareness of society by Math. How Math can be an important tool to make children sort of, I suppose it's just simple mathematical literacy, so they can go in the real world, and they can see data or they can see certain things.

Rachel: I feel like it is most obvious with data.

Interviewer: Knowledge wise, they can use this. Their mathematical knowledge can help them to sort of interpret these things and make decisions. Right, so I'm going to take a picture, if that's alright.

Rachel: Fine.

Santana's Interview (4.5.16)

Interviewer: Okay, so that started recording now. It's hard to talk about beliefs so I've done it as a sort card exercise. This is based on a framework by Professor Paul Ernest which has been adapted.

These cards are about theory of learning maths, these are about the theory of teaching maths, these are about the theory of maths itself and what you think about them. Now because these are Paul Ernest's, mainly, things, it may be that you don't agree with any of these, in which case that's absolutely fine. There's some post-it's here. You can put your own ones in. It may be that you only agree with some, in which case you discard the others. But people do it in different ways. Some people do it sort of like, order in which they believe in it. They believe in all of them but in -- it's up to you. The only thing I'd ask is, as you're doing it, if you can talk me through it. Then that becomes the interview because then that gets recorded and at the end of it, whatever you've organised, I'll just take a quick picture of so I've got record of that. It's a more open-ended interview than me just sort of saying things like, "What you believe in maths", because that's very difficult. It's for example, if you're going through this and you say, "Well, I don't think, for example, social construction practice is what maths is", and things like that and just explain that, then I've got a recording of your views.

Is that okay? You can use these in any way you want. You can either take a pile and go through them first, or you can look at all of them in one go. I'll set them out for you and then you're free to discuss them and put them in some sort of order or get rid of all of them or whatever you want really. This is all very opened. There you go, there's that if you want to try that as well.

If you've got any questions just ask me and I'll explain any one-

Santana: Okay, I'm going to start with the **aims of mathematics** education. I think that obviously crucial to it is having core numeracy skills. I think that that's like the most important thing we probably try to get as a base. If we think of everyone, you want everyone to be schooled with some basic numeracy skills. That's where we want everyone to get to. I would say that ones the most important.

Then it's transmitting a body of pure mathematical knowledge in a way. They need to get the knowledge so they can get their qualifications as well. I wouldn't say industry center focus is important to me. Obviously I haven't come from a maths background or anything. I don't think that's the aim of mathematics education, to some extent, that might come later if students then decide they want to go into that. I think at a school level, until university, even at A level, I don't think that's the most important thing so I'm going to discard that one.

I think creativity is really important and a critical awareness of society. I put them together. These two are the 'back to basics' of numeracy and getting the maths knowledge is like the base. Then if we get this creativity and getting them to think critically about maths and everything then that's where we would like them to get to.

Then, I'm going to do Theory of mathematics because I think it links nicely to that. I would say structured body of pure knowledge because obviously there are methods, rules and formulas we follow but I wouldn't really agree with a lot of these. I wouldn't say it's necessarily unquestioned body of useful knowledge, because you can still

think about why they work. Even if we know certain things are fact. We know zero times zero is zero, but we could still question it and think about it.

Interviewer: Right, okay.

Santana: One thing I always think of maths -- maths is a different way of thinking, a logical way of thinking, which you might not get that developed in other aspects. I think maths is important, because it develops our logical thinking because it's a logical approach to learning. I guess itback, but yes, so developing our logical mind. I'll say the freedom of maths is yes, around the structured body of pure knowledge. I guess you can say a collection of facts and rules, and then you're also developing your logical mind, which you might not get to in other areas of education or in life.

Okay, I'm going to do Learning Maths then I think, how you think that will link into how you teach it. I think understanding an application is key to progress. It's important for acquisition and practice, because you need that first and then I think question and meaning is so important. I think unless -- you find that with brighter kids, when they question why things are how they are, they develop the understanding of the maths better. I guess the way the theory that I would look at, like is more like CPA and obviously we've been doing that like the concrete pictorial abstract approach to maths.

Interviewer: All right, okay.

Santana: I think that's really important and that's made a lot of sense to me, , it links into how people learn stuff in life. When you learn what an apple is, first you just have an apple and you can see what an apple is. Then you might learn the word apple, why it's called an apple and then learn how to spell it. I think it's like that in maths, so when you just give symbols and numbers and expect them to understand, that's not how people learn it's why you need that concrete, pictorial manipulative sometimes. More and more I'm starting to appreciate that and how important it is. I write CPA, but I guess that links to the activity and exploration.

Interviewer: Yes, okay. All right. Can you just write that out in full?

Santana: Concrete. There's three stages.

Interviewer: You've said it on there so it's fine.

Santana: Pictorial, abstract and that links to question and meaning, so we're not just representing maths as an abstract thing, which is why I probably discarded these things where it's like-

Interviewer: Yes, that makes sense doesn't it?

Santana: Yes, unquestioned body of knowledge, so they fit together.

Interviewer: These link together?

Santana: Yes. All of them link together.

Interviewer: Yes.

Santana: I guess that links to skill acquisition and practice, it's the same thing.

Interviewer: Yes. Anyway this reflects that isn't it? Because what you're saying is you need these to then go on to-

Santana: Yes. Then I think teaching maths, it's about explaining, motivating and passing on the knowledge.

Interviewer: All right.

Santana: Then transmission and drill I guess, you do just need them to -- I guess when you get to key stage four that's more important, where as key stage three I would focus more numeracy skills and the CPA, and getting them to understand more. Maybe by the time they're near 10 and 11, you just want them to be able to do the maths. This changes with their age group as well, so maybe I'll put it lower but I think with everyone it's about motivating and passing on knowledge and discussion and questioning. I don't make it that personal guess, so I don't think work relevance is -- I don't ever really do that in my practice, or don't really facilitate personal exploration that much. There'll be bits in my questioning that might do that but not in itself. I'm going to get rid of it too. I think that's like my final thing.

Interviewer: Yes, and because obviously you've got here things about critical awareness of maths and social -- society and things like that. Could you think of the sort of lessons you might teach which would make children be more critically aware?

Santana: Last week I taught year eight compound and simple interest but it was such an interesting lesson because they were so engaged by the ideas. We just started by look at what happens? Would you put money in a bank, and what interest is. What compound and simple interest is, and obviously some of them didn't even realise that you can get money from putting money in a bank, so you can get interest. They were genuinely really interested in that. Then one of them brought up and said like when babies were born around 2003 they were given money to put in a bank. So some of them realised that their parents have that for them but they never really knew what it was.

Interviewer: All right. That's quite interesting.

Santana: They brought that up themselves. Then we looked at like increase and decrease and we looked at car values. But he was getting into first year, he was like, "Oh, so you lose a lot of money from a car every year". Then first he made a comment like, "Oh, I want a Ferrari or something". I was like, "But then aren't you going to lose more money?" and he said, "Oh, my God yes, because the percentages is -- because the price is higher." That's how we lose so much money in a few years and I think they really enjoyed that. They really got thinking about it and it just grouped them. A lot of

it like that car one it wasn't even I had a plan for it but it was just them thinking and asking the questions they wanted to.

You need the room for that as well. That they know they can just ask me something like that and we can talk about it. Whereas if you have a very rigid classroom structure they might have not felt comfortable doing that. And at first they were like - at the beginning where I said a 3% that's all a bank gives you, that's not a lot. Then we did a question where the money that was put in was really high. Then they realized, "Oh, now I get thousands of pounds over a few years". They made that. I never even made that link them or initiated it. They made that link themselves. Some of them were like, "Oh, this time we actually get a lot of money".

Them starting to think about that was really good. A lot of it was just reading the question and then thinking about it for themselves but then being able to like say what they thought out loud. When you did that topic it's easy to be like, "Here's the formula to use there. Just apply the formula to the compound interest". But actually looking at the question and talking about the question, I think they learned a lot from that. Some of them never had an idea of banks and interest and that you could actually get money from saving in a bank. It was really interesting. That was something I did last year and I really enjoyed teaching that as well and gained a lot from it.

Interviewer: Did they did they go onto things like taking out loans or anything or?

Santana: No. We didn't look at that that much.

Interviewer: But even that's quite interesting that they didn't realise they can get some money by leaving it in the bank, yes.

Santana: Yes, and then realising that even if it's 2%, it could actually be a lot of money over more years or something.

Interviewer: Yes, yes. Okay. I'm going to take a picture of this, because it reflects your own mathematical beliefs. You've explained them. The other question I've got is, do you have a lesson coming up or if you don't then just let me know by email where you think you'll be appropriate, not just for the sake of this, but where you think you would have looked at real world issues-

Santana: We're going to do it with that same classroom. They've done like percentage work and stuff, then I might introduce pie charts, drawing pie charts then because they haven't. We might look at the budget and get them to look at -- make a pie chart and look at percentages of where the money is allocated.

Interviewer: All right. Okay. That's good, yes.

Santana: Then I get them to discuss -- because they probably don't know like, "Do you think that's right that the most money is given to this? Or the most money is given to this? What would you want to give it to?" I might do a lesson around that.

Interviewer: That'll be really interesting yes.

Santana: Yes, because with a class like that if I taught them how to draw a pie chart they'll learn it quite quickly because they're really able. I can bring that into it. And get them to actually think about so when they draw the pie chart, the questions they have will be like, since they're just asking where does the government give the most money to what area? I'll just ask them a question say, "Okay, what does your pie chart show about where the money is allocated? Do you think it should be more or less to one or the other?" Get them to think about it a bit more. I have like 100 minute lessons with them as well, so I'd probably do that in 100 minute lesson where they can really explore it.

Tao's Interview (7.4.15)

Interviewer: Right. The recording started and the cards go through a number of titles like aims in Math Education and the idea of the cards is that you have a look at what the aims of Math Education are here, and they're quite broad. You can either think, actually none of these go along with my ideas what aims of Math Education and Theory of Math Education or the Theory of Learning Mathematics is, or some of them do and I'll disregard some of them or they all do and I'll put them in some order, is entirely up to you.

Interviewee: Okay.

Interviewer: They're adapted from a book by Professor Paul Ernest, who's an expert in the Philosophy of Math Education. It's adapted from that and that's where they come from. Like I said, the idea is have a look at each one and do anything you want with them. You might feel they're all relevant, none of them are relevant and they're all relevant in a progressive manner. It's how you want look at them. I usually find the best thing to do is take each one and if you could just—

Interviewee: Sign whether I agree or- [laughs] [crosstalk]

Interviewer: Yes.

Interviewee: -or disagree.

Interviewer: Also maybe for the interview, because this is the interview basically. Just tell me why you might think this is a more thoughtful process. Go behind those decisions and then just maybe somewhere down there- or organize main field.

Interviewee: Alright. Where shall I start? I'm going to start with the aims of Math Education. I think that's probably the easiest one. I'd say that. In terms of the amount of time that I spend doing Maths teaching, I'd say most of the time that's what I'm doing. That's basically my field. In terms of what I'd really like to be doing, I'd say is that creativity and self-realization via mathematics and I think that comes in more the older the students get, and if you are giving them more some investigative tasks.

Interviewer: Yes.

Interviewee: That's what I'd like to be doing. This critical awareness of society via maths, I think when I use that it tends to be as context for probably transmitting mathematical knowledge and this creativity and then in terms of useful maths with an industry center focus, I'm not sure that I spent much time [laughs] actually doing that, because I never learn Maths that way and I've never seen it taught that way.

Interviewer: Okay. That makes sense.

Interviewee: Even though I'm not a mathematician by degree.

Interviewer: What was your degree, by the way, I forgot to ask.

Interviewee: Biochemistry.

Interviewer: Biochemistry. Right. Okay. [crosstalk]

Interviewee: I did extra Sciences. First year I did Physics, Chemistry, Maths, Biology in a whim. I really enjoy Biology. Second year, I did Maths with T-Biology modules and then I ended up doing straight Biochemistry for third year.

Interviewer: When you applied to teach first, did you initially applied to do mathematics?

Interviewee: Yes. Absolutely. I can't see myself teaching anything else.

Interviewer: Alright. Okay. There you go.

Interviewee: That's what [crosstalk]

Interviewer: And I'm going to leave that because what we are going to- Just take a picture of-

Interviewee: Okay fine. I'll leave Maths up to them. [laughs]

Interviewer: -how you'd organize these- after you've done it, and part of the reason also for this is that, by not interviewing you in the traditional way, I'm not prompting you. These are, sort of open prompts.

Interviewee: Yes, That makes sense. I'm going to leave these ones for now, because I think it's easier to go.

Interviewer: Fine. Yes.

Interviewee: I mean I'll have probations on these ones first

Interviewer: Yes.

Interviewee: Theory of Mathematics next. Collection of facts and rules. I'm not going to start, I'm just going to put that right at the top because I think that's how students tend to see it. When they're first coming across Maths. I think in terms of, and that unquestioned body of useful knowledge and again I think that's how students will quite often see it.

Interviewer: How would you see it?

Interviewee: That's where I'm going to. [crosstalk]

Interviewer: That's where- That's good actually.

Interviewee: It's how they see it, I would say. In terms of how I see it, now it's harder. Okay. Right. I'll use all three of these ones, socially constructed, structure body of pure knowledge, and personalized activity. That personalized activity, that's how I learned it when I was at school. Spending time thinking about it for myself, maybe talking to somebody else about it. Structure body of pure knowledge, I think that's closer to how I see Maths as a whole. It's kind of this way, this entity of information is nice.

Interviewer: Yes.

Interviewee: That's why I think it's nice, because it's structured. It's a structured thing, so I like that. Then in terms of how I ended up learning Maths, that's socially constructed thing. I used to have- work always during on a Thursday when I was a year, and we used to spend all Wednesday night- me and a friend of mine that I lived with, doing Maths together and that's the way that I learned Maths properly. I would say that's probably how I see Maths best. So, it's socially constructed.

Interviewer: That's brilliant.

Interviewee: Okay. That's where I think students tend to be and that's how I see it. Let's do learning Maths next, because I think that probably leads to that best, and so Theory of Learning Maths. I think- okay, I'm just going to put it in order, again similarly thinking how- what it should be like, versus what is often seen as. I'm going to go that way. Strange because I wouldn't have thought of it this way, normally. I think, thinking about it in terms of again how I've learned, and how I last used and I think actually again, that negotiation is really important.

Interviewer: Right.

Interviewee: Again, practically some students tend to see it that way and it's obviously, very important that they can do that. Skills acquisition practices similar idea, obviously it's a little bit more the idea of learning skills. That's how I see it. When you're teaching individual- when you're having individual lessons, you have this skill that you'd like them to know how to do, and then at the end of the lesson, you want to see whether they can do it. I think that's generally how my lessons tend to be social. That's how they learn it. Understanding an application, this kind of building on that.

Interviewer: Yes.

Interviewee: You send in an application, and then activity and exploration as central. There is not so much scope for this, in terms of the actual schemes of work that we give it, and at terms of the curriculum. But, obviously, this is absolutely important because if they're to understand and apply, they have to have done the work for themselves, and to have that exploration. Then in terms of how it sticks once they've learned it, once they've acquired those skills, this is the important thing and it's to say that, that links how I learn Maths in the first place. That idea of having to discuss what's going on and I think some students really like that. They like to say, "So, what about this", and it's not necessarily into ability or to like Maths set, whether they like that or not.

Interviewer: Yes.

Interviewee: It's just some students really like that, while some students like the idea of just going in and learning themselves, one by one. That's theory of learning and then I guess hopefully, should inform this. I'm going to go, how should I order this one. It's a little harder. Okay. I'm going to go talk to- about where I'm going to go. Why I think they tend to spend time doing and also why I tend to spend time- how I spend time teaching data, how I really like to spend time teaching, but not necessarily how much I spend time doing. So probably that's why I actually do most, that's why I've got hold up for doing in- observations, in fact, quite a lot-

Interviewer: Yes.

Interviewee: -because I would stand in the front and I would be like, look at this amazing idea, and then I'll show them how to do it. Then hopefully, that leaves- It's funny someone out there probably need to ask myself- I need a team. Then I don't know so much about this one, obviously there is some of that then it's just transmission, I think this motivation explanations closer to why I actually, do most of the times. In terms of facilitating personal exploration, sometimes I do that, but I don't do as much as I would like to. Motivate through work relevance as in, is that providing context to stimulate them to do work. Yes, that makes sense. I think probably, yes, "This time I'm doing it right".

Interviewer: There's more- sorts of connections with the industry, for example.

Interviewee: I see, as in relevance to work as in careers in industry. Okay, fine. So, I don't spend a huge amount of time doing that, but when I do, it tends to be as context for, something else so for example, everything I've done, the course where that was based on climate change, so that was relevant to use stats in that particular field.

Interviewer: Yes.

Interviewee: So, I think that tends to be how I teach, less of this probably, do you know actually I'm tempted to put it this way around, because I think actually in terms of the amount of time that students spend, I think actually, I spend more time talking

and students talking to me than they do just repeating question over and over again. I've done this upside-down compared to rest [laughs] -but, never mind.

Interviewer: No, it's fine, because it's done yourself.

Interviewee: I don't know why I put them in that order [laughs].

Interviewer: No, that's really good and it's useful and part of the reason for doing this is because I think a lot of people think that teachers are very polar in their- Maths teachers particularly, in their way they feel teaching should be done, whereas as I'm going through these interviews most teachers, in fact all teachers sort of think, well actually all of these are important in teaching. It's not just the sort of instruction teaching, which- and there is a place for that in the same way there is a place for more relation teaching form, which what this is sort of looking at. I'm aware of time because you said-

Interviewee: No, don't worry, good. Yes, 20, 25, or so.

Interviewer: You said you've done a climate change project. Could you just tell me a little bit about that?

Interviewee: Yes, of course. So, the year tens have to do stats, they're doing GCC statistics, they have to do SAT scores because the only bit of course, where they got left, so it's the only bit of time where we actually have protected time where students can go and do right by themselves, because the rest of the schemes of work tend to be 'topic, topic, topic', rather than you have this time to go and do one of yourself, and the students found it really hard because they're not really had any practice in doing Maths for themselves without prompting. They got into it, I think it took a couple weeks for them to get into it, because at first they were like, "I just don't have any structure, I don't know what to do."

Interviewer: Yes.

Interviewee: So anyway, they got a hang of it for a while, and so there we had a choice of three briefs, one of them was puzzles and games, one of them was travel I think. So that was more to do, I think that kind of theme there was to do with how students travel to school and that kind of thing, and then the other one was climate. So we picked that one, partly because it's easiest data to get, it's easy to get huge data. We used the one from the Maths office and partly also, I guess, because it linked well with their Geography and their Science curriculum, and then partly because that's what they wanted, [laughs] I found it quite interesting.

Interviewer: Right. Okay.

Interviewee: So yes, that was good because it got students thinking about differences, the way that we focused it was- So first of all, we looked at changing over time, so 1970's to now, what's changed with temperature, that kind of thing and so they come up with a hypothesis, I think that average temperature, average maximum

temperature is increased since 1970. or there's a correlation between rainfall and sunshine, something like that.

Interviewer: Yes.

Interviewee: And then, it also got them thinking about climate different regions, so a lot of them, when they first came with the hypotheses would say, "It will be hotter in Scotland", they'd come with things that wouldn't necessarily think, that you'd think with a bit of general knowledge they might not have come up with. So it's good to be able to first of all, to get them to do the work and then to show that hypothesis wasn't correct. So it's good, I really enjoyed doing it to be honest, it's a shame that was not so much time to do that and then for me to assess it in the curriculum.

Interviewer: Yes, no, that's quite interesting. And do you think there was an element of- was it a project where they came out with a lot of knowledge about climate change? Or did they come out with almost a critical perspective of climate change, so, because we hear various arguments about climate change and whether they would appreciate both those arguments.

Interviewee: Yes, I think a lot of them was surprised. I think not a lot them showed that there was a very small increase, that some of the strongest ones came up with gradients on moving average on time graphs and things, which is really good to see. I think a lot of them was surprised, they did see, for example that there was a positive correlation with temperature overtime, but I think they were surprised it wasn't as big as they would think, so if you ask them they'd say, "It's probably increased by five degrees"-

-over 30 years, which is obviously completely impossible. I think they probably did come away with a better understanding of the scale of climate change. It is obviously a significant change, but it's not necessarily a perfect, straight line positive correlation.

Interviewer: Yes.

Interviewee: I think they probably did. This seems to focus on that part of it. I think definitely the ones that had had initially a [unintelligible] climate in the UK, I think they definitely came away with a better idea of how climate varies within a region. I think that's definitely true. In terms of asking them about it though, I didn't spend much doing it.

Interviewer: No.

Interviewee: The very best ones, the very best evaluations said things like, "I initially thought that there would be a stronger correlation between temperature and rainfall", say in Scotland, whereas in the UK. "But actually I found that, because there are a wider array of factors affecting temperature in", Scotland say, "Actually, that correlation was weaker". They came up with a pretty plausible suggestion for why that happens, as this is wrong.

Interviewer: That's good, that's useful.

Interviewee: Some of them did write that down. There were a number of students who did some good work, who did some good Maths and some good stats but who didn't necessarily evaluate that aspect of it, the climate change aspect of it. They just used the numbers and then turned the numbers into a table and that was it.

Interviewer: Purely mathematical I suppose, yes.

Interviewee: Yes, exactly. Which of course is valuable for them, because in their exam they have to be able to do that. But, I think they didn't quite necessarily get the idea that they had to be thinking about climate change as almost like a scientific brief. It's a range of things, I think.

Appendix 6 : Observation guide

Observation notes from Aron's lesson (18.5.15)

1/5

Name :

[Redacted]

Date:

18.5.15

Class:

7b Mat.

School:

[Redacted]

- What is the mathematical aim of the lesson?

Understand + calculate compound interest.
Effects of interest
Interest in the real world.

- What aspect of RWEI is being addressed in the lesson?

High interest loan companies
Payday loans.

- To what extent is RWEI situated in the lesson?

Some references throughout lesson but
most of lesson exercise based w/
some ref. to interest rates + loans +
savings. Ref to wages near end.

- How relevant is the RWEI to the mathematical content of the lesson?

Highly relevant - not add on.

highest to lowest -

$$\left[\begin{array}{ll} 21\% \text{ of } 400 & 35\% \text{ of } 30 \\ 2\% \text{ of } 280 & 75\% \text{ of } 28 \end{array} \right] \quad (A)$$

9.55 - Emphasis

Percentages as decimals.

$$46\% = 0.46$$

$$7\% = 0.07$$

$$130\% = 1.3$$

$$0.2\% = 0.002$$

↑
Fractions

(C)

Mr. ... £7300 into son's bank acc. at start of year. Acc't pays 3.2% interest per annum. How much will be in the acc't in 3 yrs time after interest if no more money is added?

$$\text{Amount} = \text{principal} \times (1 + \text{interest})^n$$

Time period

↑
original Amount

1% is a decimal

BIDMAS -

$$\begin{aligned} \text{Amt} &= 73000 \times (1 + 0.032)^3 \\ &= \text{£ } 8023.46 \end{aligned}$$

So 3.2% Interest is a lot of more.

(D) Jack put £100 into a Savings acct w/ an annual compound interest of 6%

[How much will he have in the acct at the end of 4 yrs if he does not w/draw any money?]

(E) £1000 invested for 5 years at an int. rate of 9%.

May 2005

6
7
8
9
10

£1538.62

(f) And total amount owed on a loan of
 £1000 lent over 3 years of
 25%
 $1000 \times (1 + 0.25)^3 = 1,953.13$

Comments about HSBC from pupils

A loan of £2500 is repaid for
 2 years at 7% per annum.
 Amount owed? 2633.27

What would be a 'good' loan.

It addresses the maths

Diff't loan companies - which loan is
 best (sheet)

Use of video

Reference to Wanga (see sheet)

Adrian

5/5 Loans Investigation

Company	Principle	Interest APR	Time Period	Repayment Amount
Bob's Loans	£1,000.00	20%	4 Years	
Money E-Z	£1,000.00	15%	5 Years	
Quick Cash	£1,000.00	30%	4 Years	
Dream Check	£1,000.00	40%	3 Years	
Monkey Money	£1,000.00	85%	2 Years	

- 1) For each of the loans above calculate the repayment amount
- 2) Which is the best loan? Why?
- 3) Which is the worst loan? Why?

You find out that some companies charge fees for their loans

Company	Fees	Cost of Fees	Total Cost of Loan
Bob's Loans	Set up cost of £30		
Money E-Z	Charge every year of £20		
Quick Cash	Charge every month of £1		
Dream Check	No fees		
Monkey Money	Charge every week of 10p		

- 4) Calculate the cost of each loans fees
- 5) Calculate the total cost of each loan
- 6) Which is the best loan? Why?
- 7) Which is the worst loan? Why?

Real Case Study:

At Wonga.com you can borrow £400 for 1 month at 292% APR. How much do you have to pay back at the end of the month?

Appendix 7 : Participant's evaluations

Aron (12.5.15)

I really enjoyed teaching that lesson. By motivating the pupils with the real world problem of borrowing/saving money at a particular interest rate I was able to engage them in solving problems which lay at the edge of many of their abilities. Planning the lesson and making sure my focus was on both conveying the real world truths as well as the maths was a far more pleasant experience for me as I am often troubled by how little school can prepare pupils to effectively tackle real world problems. This lesson gave the pupils a tool they can use to start thinking about the benefits of saving versus the problems of credit.

Though I can't comment too much on whether the behaviour was better because of more thought provoking subject matter or because OFSTED was inspecting my school at the time of this lesson it is my opinion that the pupils were more focused on this content than they would have been otherwise. Additionally their comments to me after the lesson was how much they enjoyed it, especially in talking about the maths in a qualitative way, i.e. what mathematical conditions make a loan 'bad' versus 'good'.

Where opportunities arise in the future I will look to make lessons more real world focused and I wish the curriculum was built around these ideas.

Edwyn (4.6.16)

I wanted pupils to work out the optimum radius and height for a 330 ml can. To be able to confidently use formula for the surface area and volume of a cylinder and be able to use formulae in excel. I also wanted them to think about the cost of choosing a particular design, financial and environmental.

I would like do more lessons like this to but I am under pressure to raise standards in my classes and that is measured by their performance in exams. The curriculum is designed to be taught by topics and not through individual problems, this makes it difficult to teach the subject like this. In addition, it is hard to differentiate problems such as the one I taught meaning not everyone would be able to make suitable progress/finish the work in the time provided. I think with the current system it would be impossible to teach every lesson through a real world equity issue but I would like to raise them if it was appropriate.

For these reasons I find it very difficult to think of world equity issues that I would actually address in my maths lesson, however if I came across one that I deemed good and appropriate I would be very happy to teach it.

Fabia (22.5.15)

I think the lesson went well. I really enjoyed teaching this topic, and it was great to see how engaged pupils were. I didn't really have any expectations of how pupils would respond, or engage with the material, and I was pleasantly surprised by how pupils worked in the lesson.

Pupils' prior knowledge of politics in the UK was not strong, and so I had to adapt to make sure that pupils understood all the key terms. In particular they found it difficult to grasp what a constituency and a seat were. It was really interesting to see pupils who usually really struggle in Maths getting settled and approaching activities more quickly and confidently. This may have been because pupils didn't see the work as 'Maths'.

I would definitely teach in this way again. Both my class and I really enjoyed the lesson, and it gave me a chance to see different strengths and areas for improvement in my pupils, whilst still giving them opportunities to do maths that stretched them, like ordering percentages and decimals and organising and collating data.

My intention was to allow pupils the chance to engage with the election data, and show them the data in different ways, like percentage of vote share against number of seats and then show them how the way in which we count the votes plays a part in this. I feel that pupils did learn a lot in the lesson, and are more critical in the way they can think about voting as a result of this lesson.

Jason (2.6.15)

The lesson covered the interrelationship between maths, government, medicine and science. Looked at different auto-recessive conditions. The end of the powerpoint covered them evaluating this learning to work out whether the government should test everyone for whether they are carrier, and whether parents who are carriers should have children.

Overall I was surprised how much they got from it. At the start they really bought into the idea and were excited by what they had to do. The depth of the task did mean that some of them lost focus towards the end, but the same can will always be expected in a 100 minute lesson! What I thought was particularly beneficial was how the relevance would increase their aspirations. This is something that I will be able to refer back to in future lessons – which may not contain real world equity issues. I think the most suitable balance would be having one lesson a half term or topic like this, with other more content/knowledge heavy lessons. Where learning is perhaps faster and progress is consolidated.

Once a half term will aim to use more of these examples, such as architectural design and frequency tables and graphs relating to politics.

Minervia (10.6.16)

The objectives were for pupils to come up with a flight timetable as well as a financial plan for it by using their skills and knowledge on bearings, maps and scales, percentages, and converting between units of money. From a mathematical point of view, the aim was for pupils to independently choose which skill and method to use.

I will regularly look at applications of the topics that we cover. For example, we were learning about compound percentage change last week and we looked at different banks and their saving plans to decide which bank we would rather invest our money at. I am also planning to do a lesson on encrypting and decoding messages using algebra.

Rachel (25.1.16)

You're very welcome to have visited - thank you for putting up with the insanity that was trying to deal with all the injections on that day!

My objectives for the lesson were for students to criticise misleading graphs. To identify key features of pie charts and bar charts, answer prompt questions to determine the validity of statements which appeared to be supported by graphs, and to come up with their own questions which could be used to interrogate graphs. I had intended for students to investigate in small groups and then give them a chance to develop their presentation and explanation skills, but this didn't work as the students were not able to match up the hint questions from the board with the graphs presented to them on paper. I would change this in future by having each hint question printed alongside the relevant graph and erroneous statement.

I wanted students to be able to spot when graphs or statistics in newspapers and other media are manipulated to support statements which aren't necessarily true. Although the examples I used had only one taken from real life, they were all intended to be the kind of tricks that are commonly employed. I think some of them have come away with a greater awareness of the importance of reading and scrutinising axes and scale in detail, and a few have picked up the fact that pie charts, in isolation, only tell you proportions, not amounts.

I have used the example of areas of a pizza with my 11P5 class: comparing whether it is better value to buy a 12" pizza or two 8" pizzas for the same price. They seemed to feel that it was a somewhat contrived application, but were surprised by the outcome! There could also be real world issues addressed when introducing trigonometry, such as using angles and horizontal distances to calculate the heights of tall structures or trees.

Santana (7.7.16)

I wanted the students to understand why we use Pie Charts and how to construct them. There was a focus on government public spending, with questions to get the students to think about their own opinions on where they think public spending should be allocated. Mostly, I feel the maths objectives were covered and the following lesson when the students were asked to complete a pie chart for the starter and they were able to do this successfully, as well as in the recent assessment. It was interesting, the students found it difficult to articulate and think about their views on the equity issue, but with prompting and time they started to get into discussions. It was useful giving them time to discuss with partners where they perhaps felt more comfortable.

I enjoyed the lesson and I think it is great to add another dimension to mathematics lessons especially at KS3. Very useful when especially teaching statistic components to really bring to life where maths is used in the real world as well.

In future lessons I will be looking at different countries GDP and wealth and poverty figures and drawing scatter graphs comparing different things like GDP against literacy levels or average living age.

Tao (24.5.15)

Students were engaged in discussion about proportion. Most were able to perform calculations without a calculator – some were given permission to work with calculators. Work was recorded clearly in exercise books by many students. Students contributed well to discussion about the IBM time and motion study – not all recorded this in their exercise books. Students were shocked by the data I recorded on the time they spent working. This was reflected in their comments on the post-it notes. More time was allowed for activities and less time was given to teacher-led discussion! This meant that not all students recorded their work, even if they did discuss it verbally.

More challenging work was allowed for Task two – most students were already familiar with pie charts - could have focused more interpretation here.

Appendix 8 : Participants' lesson Plans

1. Aron's lesson Plan

1/2

The Learning Cycle Lesson Plan

Teacher: Mr. [REDACTED]	Date: 18/05/15	Period: 2	Class: 7b/Ma1	Room: 36
Lesson Title: Payday Loans	Lesson Context: Beginning of Unit on Percentages	Subject: Mathematics		
		Nat Curr Level Range/TMG Range: Exceeding		
Learning Objectives:		Learning Outcomes - include details of differentiated outcomes if applicable		
<p>To understand how to calculate compound interest</p> <p>To understand how compound interest can cause a sum of money to grow vastly in a short space of time</p> <p>To use mathematics as a tool for understanding the real world</p>		<p>All pupils will have used the formula for compound interest to calculate an amount with time</p> <p>Most pupils should have been able to compare different services with different interest rates</p> <p>Some pupils could have come up with other factors that may sway their choice of provider</p>		
Resources	Worksheets, PowerPoint, Green and Red pens, Mini Whiteboards,			
RA/Health and Safety				
Checklist for Teachers - Features of Quality First Teaching				
Re-connect/starter	X	Targeted questioning	X	Use of support staff
Clear learning objectives	X	Oral/written feedback given	X	Use of new technologies
Use of success criteria	X	BfL Systems in place	X	Key literacy incorporated
AfL review	X	Plenary/review	X	Numeracy incorporated
Prior attainment used to inform planning and activities	X	Group/pair work/interactivity	X	SMSC opportunities
Further details on any of above features:				

Time	Lesson Cycle	Learning Activities	Differentiation (paired activity, group work, independent learning)	Assessment for Learning
5 mins	Starter	Pupils find the percentages of different values	Final question unseen, group discussion encouraged	Questioning Oral feedback Recapping previous knowledge
10-15 mins	Demonstration and Application	Video of commercial for payday loans shown How to calculate compound interest shown Pupils copy down example MWB Questions Worksheet on compound interest	Class discussion encouraged Targeted questioning Extension work provided for higher ability pupils in order to deepen understanding	Questioning Oral feedback Individual work Discussion MWB quiz Mini Whiteboard questions Discussion Targeted questioning
5 mins 15 mins 5 mins	Demonstration and Application - Challenging Students	Task on different providers of loans introduced Teacher example Pupils work to choose a payday loans company and why Pupils discuss their choices as a class	Targeted questioning to check understanding Higher ability students will progress to different sections and different levels of understanding	Self-Marking Questioning Feedback Discussion
10 mins	Self/Pair Assessment	Pupils self assess	Higher LOs for higher ability pupils More iterate pupils encouraged to write a note about how the lesson went	Self/Peer Assessment

2. Edwin's lesson plan

Max cylinder

A soft drinks company has recently introduced a new type of can.

- The new cans have the same volume as the original ones – 330 ml.
- After experimenting with various options, they decided that the new cans would be taller and more slender than the original ones – the marketing department thought this gave them a distinctive modern look!



Making the can look good is a great idea, but what about the cost of materials needed to make it? In this activity, you are going to explore the following question:

A cylindrical can holds 330 ml. What combination of radius and height will minimise the amount of material needed?

The following points should help you to get started:

- The real shape of a can is actually quite complicated, with curves, welds, a ring-pull and so on. Ignore the finer details, and use a simple geometric shape to model the can.
- Can you find formulae for the volume and surface area of the can?
- You could use a table or spreadsheet to investigate various combinations of dimensions that give the required volume.
- Can you use algebra to describe the relationship between the height (or radius) of the can and its surface area?
- Can you draw a graph to show the relationship?

3. Fabia's lesson plan

1/2

LESSON PLAN				
Subject:		Mathematics	Date:	21/05/2015
Class/Year Group:		9.4	No in class:	15
Area of Focus:		Different Voting Systems	Length of lesson:	90 mins
Key Standards:		S1 High expectations S3 Subject and curriculum knowledge. S4 Plan and teach well structured lessons. S7 Learning environment S6 Assessment	Cross-curricular links:	<ul style="list-style-type: none">Government and PoliticsEnglishHistory
Learning Objectives			Learning Outcomes	
<ul style="list-style-type: none">To look at the election results.To understand the differences between different vote counting systems.To analyse our voting system.			ALL:	
Key Words <ul style="list-style-type: none">ConstituencyFirst past the postAlternative Vote			MOST:	
			SOME:	
Differentiation (SEN/G&T) & FSM			Assessment Opportunities	
Gifted and Talented: 0 SEND: 7 (Mary, Yasmin, Melissa, Shaymaa, Zara, Ayo, Arlinda) Statemented: 1 (Samid) FSM: 6			They will self assess after marking Task 1. There will be a plenary half way through where pupils will explain their voting systems to the class. There will be a plenary debate at the end of the lesson related to AV/First Past the Post. There will be a plenary quiz about politics.	
Resources				
Voting Ballot Slips Scaffolded Borda Counting sheets Worksheet for 'General Election in nearly 60 seconds' Task 1 sheet on maths questions				
Timing	Content/Activities			
5mins	Starter 'What is a constituency' – Discussion of key concept			
10mins	Task 1 – Watch 'General Election in nearly 60 seconds' and answer the questions on the worksheet. This will then be discussed and pupils will feedback their answers.			
10mins	Task 2 –'Election Maths' – pupils will engage with the election data for the constituency that the school is in and answer the questions on the worksheet. We will discuss what percentage of the electorate voted.			
5mins	Class Activity - We will discuss this years election results and look at a bar chart and at a table that shows the numerical data. Pupils need to put the parties in percentage order – from highest to lowest. They should notice that the order changes (noticeable UKIP).			

5mins	Discussion 1 – Pupils will discuss with each other what they know about how votes are counted.
5mins	Watch a video on AV vs First Past the Post. This is to introduce pupils to thinking critically about the different vote counting systems, before they try counting votes themselves.
10mins	Task 3 - Feedback on the video and give pupils a chance to read about their chosen vote counting method.
15mins	Task 4 – Pupils will now use their vote counting method to work out who would win. They should use this time to think critically about their allocated method. I have enough worksheets for all groups to try all the methods if they finish quickly.
10mins	Discussion 2 – Pupils will feedback on their results and think about what they have learnt from this activity.
10mins	(Optional) Discussion 3 and Task 5– From the previous discussion pupils will think about the advantages and disadvantages of AV and FPTP.
5mins	Plenary – Look at the question ‘Do you think that AV would have given more votes to Conservative or Labour in this years election?’ Mini Quiz to end

4. Jason's lesson plan

1/2

1

Probability in the Real World

LO: To apply knowledge of probability to investigate real world problems

2

<http://www.bbc.co.uk/news/election-2015>



- The conservative party won the election
- It took lots of people by surprise that they won



3

They have tasked their best mathematicians available to help them

You will be researching
"Auto-Recessive"
conditions and using your
knowledge of probability to
help the government

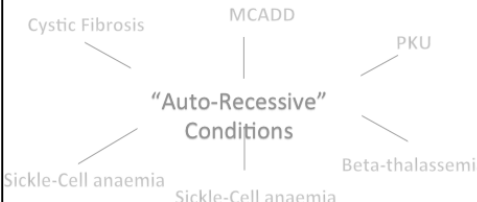
4

To complete this activity to the highest possible standard you will need to:

1. Make a report for the Health Minister relating to this topic
2. Pick a partner you will work well with
3. Pick a condition to research
4. Login to a computer in the breakout space
5. Find this power-point in Tdrive/Departments/Maths
6. Complete the 3 tasks described
7. Present your learning on a power-point file
8. Use the PLTs skills from I-Learn to help you in structure your time and work
9. Treat all information you deal with sensitively. If at any point you are concerned about something you find out let me know.

5

Pick a condition to build your investigation around



6

Hints
Spend no more than 15 minutes on this task

Task One – Research and Probability Expectation

Key Questions

- What is it?
- Can it be cured?
- Can it be treated?
- How?
- How many people suffer from it?
- What are the symptoms of the condition?
- Other relevant information?

What is the probability?

- How many people in the UK have the condition?
- What is the probability therefore of picking someone who has it?
- Was it the probability of a baby being born with the condition?
- Why might these be different?

Expectation

How many people would you expect to have your condition in:

- Your class
- Your school
- The borough of Enfield
- The whole of London

Hints
Expectation = Probability x number of trials
You will need to pick one probability to use.

Key Words

Symptoms – the signs of a disease. “The symptoms of the flu are high temperature and feeling sick”

Condition – broad term that includes all diseases, disorders, illnesses and injuries

7

Hints
Spend 30 minutes on this task

Task Two – Building your probability tree

Key Questions

- What does “Auto-Recessive Conditions” mean?
- How do they get passed on from parent to child?
- For a child to get the condition what do both parents need?
- If only one parent is a carrier can their child inherit the condition?

Hint: BBC Bitesize explains this clearly

What is the probability?

- Of a child inheriting your condition, when the Dad is a carrier but the Mum isn't?
- Of a child inheriting your condition, when both the Mum and Dad are carriers?
- Of having a healthy child if both parents are carriers?

Probability Tree

Design a probability tree for a family considering having two children. What is the probability:

- Both are healthy?
- Both have the condition?
- At least one child has the condition.

Extension
How does this change for a 3 child family? Or a 4?

Key Words

Inherit – Get from your mum or dad.
‘I inherited blue eyes from my dad’

Allele – different forms of a gene that make us who we are

Dominant – overpowers
Recessive – loses out to dominant

8

Hints
You can do these bits in any order

Task Three – Evaluating

Big Questions:

Using what you have found out about your condition

- Do you think everyone should be tested to find out if they are a carrier?
- If two future partners were both carriers, as a medical research what would your advise be to them?

Use information and probability you have found to support your decision.

Dominant Diseases

What is the difference between “autosomal conditions” (such as Huntingdon’s disease) and “auto-recessive conditions”?

What is the chance of a child having Huntingdon’s disease if both parents are carriers?

How does this change your probability trees?

Would your answers to the ‘Big Questions’ change?

Extension

Can you put together an estimate for the costs for the NHS to look after patients with your condition?

If the probability is reduced what effect would this have on future costs?

How else is probability used in medical research?

Key Words

5. Minervia's lesson plan

1/2

Asia Flight Plan

There are a lot of jobs to be done in organising your flight plan!

Job 1 - Calculate the bearing to fly on from each city to the next destination.
You should also draw the course on the map.

Job 2 - Measure the distance between the cities with a ruler, then work it out
in miles. The scale is on the map.

Job 3 - Calculate the flight time. The average speed of a Boeing 747 is around
500mph.

Job 4 - Fill in an estimated arrival time (based on how long the flight will take...)

Job 5 - Planes take around 30 minutes to be refuelled, cleaned and safety
checked for the next flight. You need to choose what time to set off for the
next destination. Remember that times are often rounded.

	Journey	Departure Time	Distance (miles)	Bearing	Flight Time (rounded)	Arrival Time
	Moscow -	15:30				
	Yakutsk	Wednesday				
	Yakutsk -					
	Tokyo					
	Tokyo -					
	Beijing					
	Beijing -					
	Bombay					
	Bombay -					
	Damascus					

Financial plan

You are now a financial engineer and need to work out the total cost for your flight plan.

Job 6 - The Boeing 747 consumes on average 11 tons of fuel an hour when in cruise, which is approximately 240 litres every minute. You need to calculate the amount of fuel needed for every journey, allowing 15% extra each time, in case of any problems.

Job 7 - The current cost of aviation fuel is 38 US cents per litre. You need to calculate how much the fuel will cost (in pounds Sterling), for your total journey. The current conversion rate is £1 = \$1.42.

Journey	Flight Time (in minutes)	Amount of fuel needed (in litres)	Cost in \$	Cost in £

So the total cost in £ is _____.

6. Rachel's lesson plan

Teacher: Ms Reid	Subject: Maths	Group: 9P4						
Date: 19 th January 2016	Support Staff: Ms Lau (deaf support for Halima)							
<p>Differentiation strategies (Use students' prior attainment – SEN, EAL, G&T, MPA, groupings)</p> <p>Highest attainers: Patricia, Cristina, Yauvan, Syeda, Anil. These students should be forming their own conclusions and coming up with questions independently.</p> <p>Lower attainers: Aisha, Fatima A., Abdul, Hammad, Sumayyah, Aliya, Aleks. These students are largely supported by stronger students around them. They should be following the prompt questions directly.</p> <p>Halima is deaf, but can hear with use of a radio aid. She is supported in class by another member of staff.</p>								
<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 40%; vertical-align: top;"> <p>Learning Objective: To criticise misleading graphs.</p> </td> <td style="width: 60%; vertical-align: top;"> <p>Context of lesson: Part of a unit on Statistics; students have previously covered all of the types of graphs which they will encounter during this lesson.</p> </td> </tr> <tr> <td style="vertical-align: top;"> <p>Key Words: Scale, axes, labels</p> </td> <td></td> </tr> <tr> <td colspan="2" style="vertical-align: top;"> <p>Success Criteria: All: To identify errors in bar charts. Most: To find reasons why a graph is misleading. Some: To analyse the best statement or type of graph to represent a situation.</p> </td> </tr> </table>			<p>Learning Objective: To criticise misleading graphs.</p>	<p>Context of lesson: Part of a unit on Statistics; students have previously covered all of the types of graphs which they will encounter during this lesson.</p>	<p>Key Words: Scale, axes, labels</p>		<p>Success Criteria: All: To identify errors in bar charts. Most: To find reasons why a graph is misleading. Some: To analyse the best statement or type of graph to represent a situation.</p>	
<p>Learning Objective: To criticise misleading graphs.</p>	<p>Context of lesson: Part of a unit on Statistics; students have previously covered all of the types of graphs which they will encounter during this lesson.</p>							
<p>Key Words: Scale, axes, labels</p>								
<p>Success Criteria: All: To identify errors in bar charts. Most: To find reasons why a graph is misleading. Some: To analyse the best statement or type of graph to represent a situation.</p>								
Learning Experiences								
<p>Do Now: Students should list/brainstorm the names of different graphs and charts they have encountered. This should include pie chart, bar chart, frequency polygon, scatter graph, stem and leaf, two-way tables...</p> <p>Main: Discuss together one example of a misleading graph (bar chart/pictogram crossover with unevenly spaced images), and the meaning of "misleading".</p> <p>Students will be given sugar paper in pairs. They need to create a poster answering questions about various misleading graphs that will be passed around in a carousel. Prompt questions will be on the board; students will have 3 min per graph before moving on.</p> <p>Students will then be chosen, at random, to present their findings on a particular graph to the class. The class will make notes on these presentations on a summary handout.</p> <p>Plenary: Post-it self-reflection. Students should write down one question they would use to interrogate the validity of graphs they find, and one statement regarding their progress towards the learning objective.</p>								

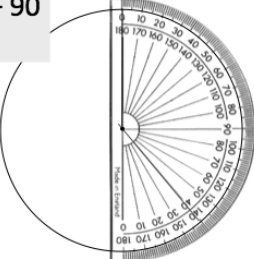
STARTER Drawing Pie Charts

Musical Instrument	
Guitar	35
Violin	10
Recorder	15
Drum	5
Keyboard	25
Total	90

$\text{Guitar} = 4^\circ \times 35 = 140^\circ$
 $\text{Violin} = 4^\circ \times 10 = 40^\circ$
 $\text{Recorder} = 4^\circ \times 15 = 60^\circ$
 $\text{Drum} = 4^\circ \times 5 = 20^\circ$
 $\text{Keyboard} = 4^\circ \times 25 = 100^\circ$

In a survey, people were asked to indicate which one of five musical instruments they played. The information is given in the table. Display the information in a pie chart.

$360 \div 90 = 4$



Title:
Pie Charts

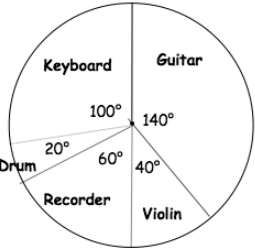
LOs:
1. Understand government public spending figures and represent the data on pie charts

Drawing Pie Charts

Musical Instrument	
Guitar	35
Violin	10
Recorder	15
Drum	5
Keyboard	25
Total	90

$\text{Guitar} = 4^\circ \times 35 = 140^\circ$
 $\text{Violin} = 4^\circ \times 10 = 40^\circ$
 $\text{Recorder} = 4^\circ \times 15 = 60^\circ$
 $\text{Drum} = 4^\circ \times 5 = 20^\circ$
 $\text{Keyboard} = 4^\circ \times 25 = 100^\circ$

In a survey, people were asked to indicate which one of five musical instruments they played. The information is given in the table. Display the information in a pie chart.

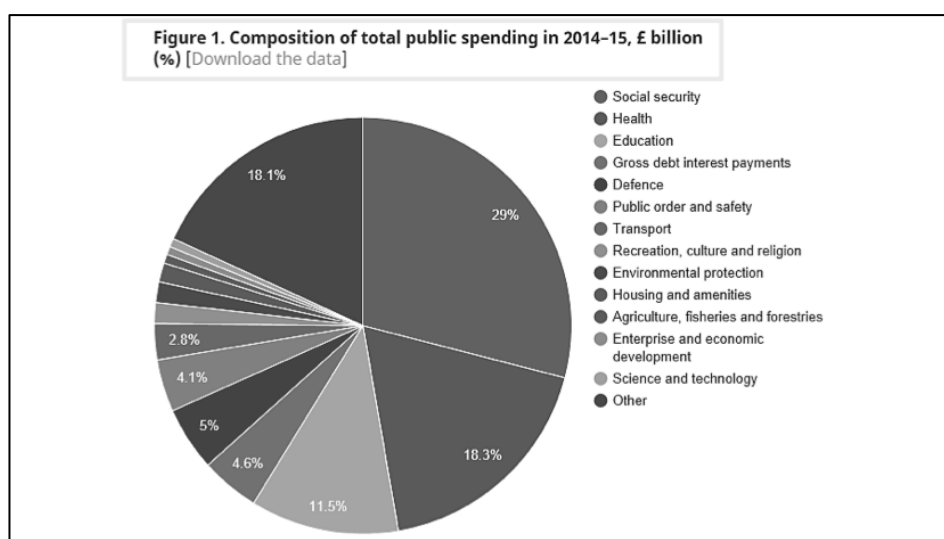
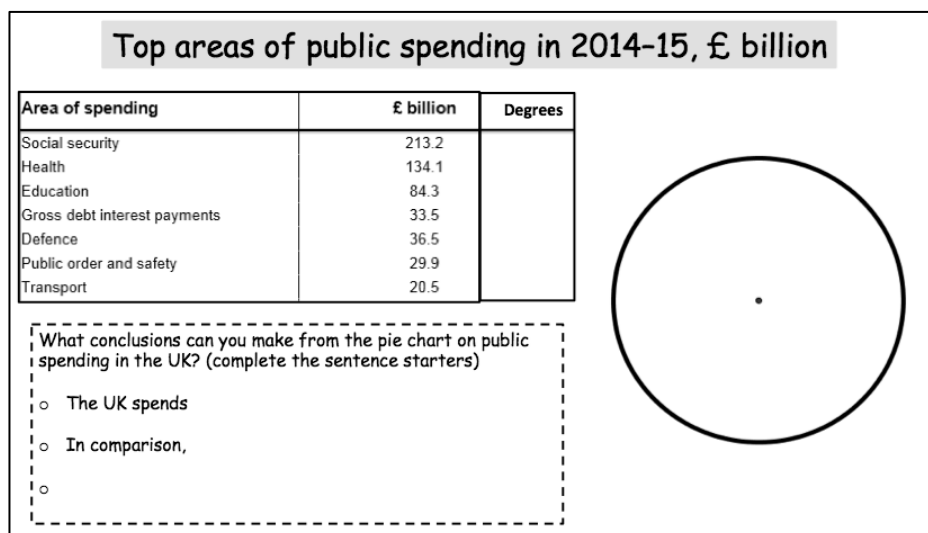
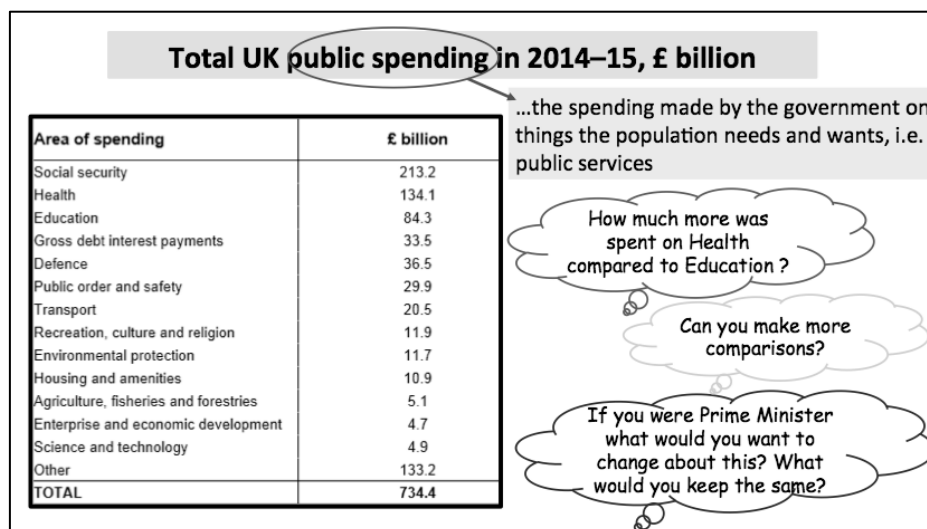


How does the government spend your public money?





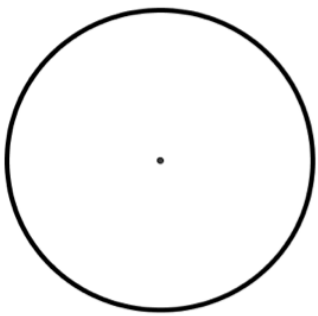
https://www.youtube.com/watch?v=ANkLfbx4VdI



What public services will you spend on if you were Prime Minister?

★ In your books, draw a Pie Chart labelling what you would spend on and how much with a £1000 budget!!

Area of spending	£	Degrees



Answer these questions on your pieces of paper:

1. What was the maths you learnt in your lesson today?
2. How did the maths relate to real world issues?
3. What did you learn about real world issues today?

8. Tao's lesson plan

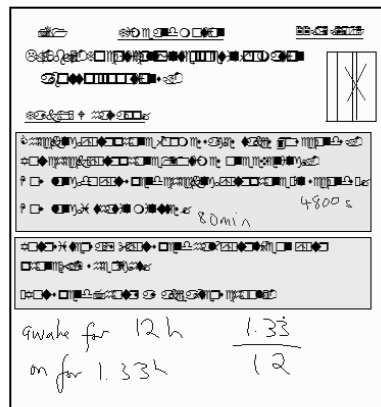
1/4

<div style="display: inline-block; width: 80px; height: 15px; background-color: black; margin-right: 5px;"></div> School 2014 - 2015		
Subject: Maths	Date: 22 May 2015	
Teacher: Tom Walsh	Time:	
Class: 7B/Ma1	Term: 2014/15 SuT 2	
Lesson title: Probability (1)		
<p>Lesson Objectives + BLP (reciprocity, reflectiveness, resourcefulness, resilience)</p> <p>Context SOW, specific issues with this group eg shared class/carousel/missed days</p> <p>To critically interpret information about proportions.</p> <p>(BLP: making links – use of pie charts for representation of real-life proportional information; reflecting – discussing ethics of time and motion studies)</p>		
<p>Classroom support</p> <p>No in-class support for this lesson.</p>		
Learning Activities	Resources	Learning Outcomes
Preparation: Ensure exercise books are available on desk near door. Protractors available and worksheets prepared.	IWB 360° protractors Worksheets	-
Homework: No homework due or set.	-	-
Starter (10-15 min): Task 1: practice with proportionality. Students to answer practical questions on fractions and multiplication in context. REMEMBER REGISTER AT THIS POINT!	IWB	- Discussion of proportion in real-life context. Contrasting natural language with mathematical vocabulary – e.g. “half your life”.
Development including mini-plenaries (35 - 40 min): Throughout, use three stop-watches to record time spent writing, discussing work and chatting off-topic. - (5 min max.) Recap of procedure for drawing a pie chart, demo on IWB. - (10 min) Task 2: worksheet on drawing pie charts using information about classroom	IWB 360° protractors Worksheets	- Reminder of formula and logic behind drawing pie charts. - Real-life: use of time and motion to determine “useful” time in a lesson or at work. Is this equitable? - Students should be able to

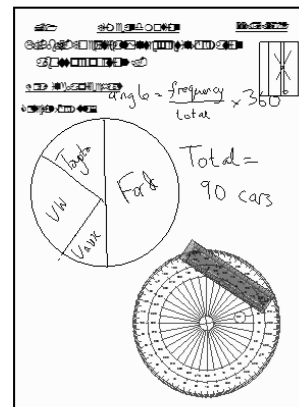
<p>activities.</p> <ul style="list-style-type: none"> - (10 min) Show table of time and motion data from IBM in 1960s. Task 3: discussion about purpose of data. - (10 min) Show excerpt of video about time and motion. Demonstrate that teacher has been recording time for "a group" during the lesson (who was it?). Students to prepare pie chart on findings. 		<p>draw and interpret pie charts as a representation of proportion.</p>
<p>Plenary (5 - 8 min):</p> <p>Students to complete post-it notes with comments on learning points from lesson and how it felt to be the subject of a time-and-motion study.</p>	Post-it notes	<ul style="list-style-type: none"> - Discussion of opinions about time and motion studies; reflection
<p>Clear-up (2 - 5 min):</p> <p>Books and post-its to be collected.</p>		

7B1 21.05.2015.notebook

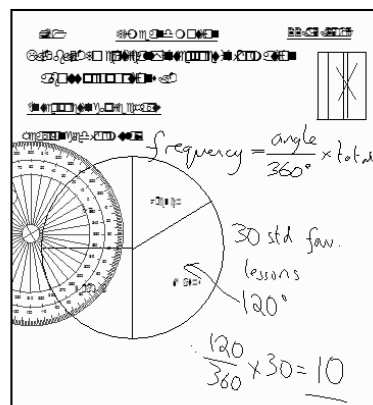
May 21, 2015



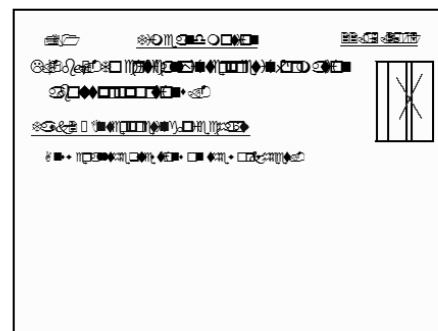
Sep 4-14:10



Sep 4-14:10



Sep 4-14:10



Sep 4-14:10

Drawers	Minutes
Open side drawer of desk	0.014
Open centre drawer	0.026
Close side drawer	0.015
Close centre drawer	0.027

Chair activity	Minutes
Get up from chair	0.039
Sit down in chair	0.033
Turn in swivel chair	0.009

Sep 4-14:10

Chair activity	Minutes
Get up from chair	0.039
Sit down in chair	0.033
Turn in swivel chair	0.009

Sep 4-14:10

7B1 21.05.2015.notebook

May 21, 2015

7B1 Time and motion 22/05/2015

L. Obj.- To critically interpret information about proportions.

Saving time?

HOW LONG CAN YOU WORK ON MAKING A ROUTINE TASK MORE EFFICIENT BEFORE YOU'RE SPENDING MORE TIME THAN YOU SAVE? (INCLUDE THE HOME)

HOW OFTEN YOU DO THE TASK


	50/d	5/d	1/d	WEEKLY	MONTHLY	YEARLY
1 SECOND	1 DAY	2 HOURS	30 MINUTES	4 MINUTES	1 MINUTE	5 SECONDS
5 SECONDS	1 DAY	12 HOURS	2 HOURS	21 MINUTES	5 MINUTES	25 SECONDS
30 SECONDS	4 WEEKS	3 DAYS	12 HOURS	2 HOURS	30 MINUTES	2 MINUTES
1 MINUTE	8 WEEKS	6 DAYS	1 DAY	4 HOURS	1 HOUR	5 MINUTES
5 MINUTES	4 MONTHS	30 DAYS	5 DAYS	21 HOURS	5 HOURS	25 MINUTES
30 MINUTES	2 YEARS	150 DAYS	1 DAY	1 DAY	2 HOURS	
1 HOUR	4 YEARS	300 DAYS	2 DAYS	2 DAYS	5 HOURS	
6 HOURS	24 YEARS	1800 DAYS	12 DAYS	12 DAYS	30 HOURS	
1 DAY	120 YEARS	9000 DAYS	60 DAYS	60 DAYS	180 DAYS	

Sep 4-14:10

7B1 Time and motion 22/05/2015

L. Obj.- To critically interpret information about proportions.

Time and motion studies



Time and motion (One Show BBC1)

Sep 4-14:10

7B1 Time and motion 22/05/2015

L. Obj.- To critically interpret information about proportions.

Time and motion in 7B1

i) Draw a pie chart to represent these data.

ii) How does this time and motion study on you make you feel?

Sep 4-14:10

7B1 Time and motion 22/05/2015

L. Obj.- To critically interpret information about proportions.

Your thoughts

What did you learn this lesson?

Sep 4-14:10

Appendix 9 : Evidence table

What are the mathematical beliefs of early career secondary mathematics teachers who are interested in teaching RWEI in their lessons?

Interview data.

Language from the interviews was used as identifiers using belief phrases from Ernest's outline of teacher's belief systems. These indicators were then used in the analysis to indicate teachers' aims and beliefs within the following framework of : authoritarian, utilitarian, progressive, socially aware.

Card Sort data.

The teacher's arrangements of the card sorts were also used to verify the interview evidence (see appendix)

Aron	
Evidence	Analysis
<p>Interview evidence</p> <p>'What is the number one for learning maths - I think its questioning and negotiating meaning. I think if you put yourself in the thought where mathematics is a language you can practice and do rote till the cows come home but you are not teaching them anything'.</p> <p>'You know if you are starting to do transformations at GCSE but your coordinates aren't very good then you are stuffed and the reason you are stuffed is because you are trying to do two things you find though at the same time but if your coordinates are second nature because you have done enough skill acquisition and practice then it allows you to do the harder maths' .</p> <p>'Whilst its important to transmit a body of pure mathematical knowledge it is not the goal of a KS3 or KS4 class'</p>	<p><i>Theory of learning maths</i> Aron's beliefs tended towards 'socially aware and progressive' although he did consider the utilitarian beliefs of 'skill acquisition and practice' to be more important than the maths centred belief of 'understanding and application being the key to progress'.</p> <p><i>Aims of maths education</i> Here Aron's beliefs tended much more towards 'authoritarian' and 'utilitarian' with less importance on the socially aware belief of 'critical awareness and democratic citizenship via maths' .</p>

<p>'I think that useful maths with an industry centred focus is definitely my number one priority'.</p> <p>'I think second to that is creativity and self-realization via mathematics. I think thatpriority is that kids are mathematically literate and that's quite a basic priority but its number one, number two is that once they are literate they are able to essentially express themselves through mathematics, that could be computer programming which I think would be so valuable for kids'.</p> <p>'The back to basics numeracy is the backbone of a useful industry centered focus, I think they definitely sit side by side for what I am trying to achieve, I would like kids to have critical maths but if they cant have basic numeracy and and mathematical literacy then critical maths is useless'.</p> <p>'Transmit a body of pure mathematical knowledge, they are not unrelated all these things, and a critical awareness of society via maths – I don't know if that is what I would do in my mathematics education I think that other subjects should speak numeracy, should speak number'.</p> <p>'Transmission and drill and no frills is important but not what I think is the most important'.</p> <p>'The issues with all of these is that, or the issue with this task, is that it depends on the pupil and I think and with higher ability pupils I much prefer to teach them by facilitating personal exploration.'</p> <p>'My ideal way that I would teach mathematics is through essentially</p>	<p><i>Theory of teaching maths</i> Aron believed the 'socially aware' and 'progressive' beliefs to be his priorities. In common with the other aims, he also believed an 'utilitarian' approach was important but not as much as the beliefs he had prioritised.</p> <p><i>Theory of maths</i> Aron believed maths to be a 'structured body of knowledge', a 'maths centered' belief. The other beliefs within the framework did not feature within his beliefs as a maths teacher.</p>
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discussion, personal exploration - I think that personal exploration follows from discussion and questioning. I think you cant just give someone a pen and paper and say go figure out the maths unless they are motivated or prompted by discussion’.	
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Fabia	
Evidence	Analysis
<p>‘Questioning and negotiating meaning in terms of the way that I teach for students understanding is a major part of my teaching it’s incredibly important for pupils to be questioned and pupils be able to question themselves’.</p> <p>‘I mean they are all important . Activity and exploration, I think they are important for learning but not used as much in schools’.</p> <p>‘Understanding application is key to progress, yes I do think that is important for me. This is more important than practice and rote because I feel that helps them focus their learning and what they have learnt is what they are modeling as opposed to practice and rote which can mean that they have learnt to copy a teachers description or example, that does not necessarily mean that they have learnt the maths but learnt a process’.</p> <p>‘Practice and rote, I do think that rote learning does have a place in most subjects. Certainly, I do think it helps in speed and fluency in Maths’</p>	<p><i>Theory of learning maths</i> Fabia prioritised a ‘socially aware’ approach although she also gave importance to the ‘maths centered’ belief that ‘understanding and application is key to progress’. The ‘utilitarian’ and ‘authoritarian’ beliefs were given lowest importance.</p> <p><i>Aims of maths education</i> Once again the ‘socially aware’ and ‘progressive’ beliefs were prioritised, although Fabia did believe that a ‘back to basics’ approach to numeracy was an important aim of maths. The ‘utilitarian’ and ‘maths centred’ beliefs were less important to her.</p>

<p>If we are talking about the broad overall aims of maths education should be more (more motivating for the pupil about the discovery of maths and how that helps them live in the real world</p> <p>‘Useful maths with an industry centered focus’. I just feel that will turn so many children off maths, like if you aim maths education towards you should be able to become something that uses maths in your job, I feel it will put so many pupils off maths’.</p> <p>‘And for the same sort of reason I don’t think that the aim is to just have a body of mathematical knowledge, its to be able to use maths and think about maths not just know a set of mathematical facts’.</p> <p>‘I feel that one of the aims of maths is the ability to use a mathematical mind, to be creative and to have self realisation and that is helped by the processes that you are learning’.</p> <p>So I think that discussion and questioning for me is a central part of my teaching and how I think , how I believe about teaching anything but especially maths because actually every other subject that they do is so centred around discussion and one of the things I find about maths is that pupils don’t expect to have to talk about it or to reason or to explain their answer.</p> <p>Transmission and drill, like the practice and rote, with teaching I do think it has a place but I don’t value it as highly. In an exam focused exam system, it is necessary.</p> <p>I feel it is a collection of facts and rules, it is socially constructed and</p>	<p><i>Theory of teaching maths</i> Fabia believed that a ‘maths centred’ approach and a ‘socially aware approach’ were both equal priorities in this aerea. The progressive belief of ‘ facilitating personal exploration’ was also a higher priority within her beliefs, but the ‘authoritarian’ and ‘utilitarian’ beliefs were of lower importance to her.</p> <p><i>Theory of maths</i> Fabia had no strong belief in this area and thought all the belief systems had the same importance.</p>
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<p>should be taught in a way that they realise that there is pure knowledge in there and it can be personalized. But I think it (maths) is a specific way of working and using your brain and the process, mathematics is about processing and how you get to answer and how you tackle a problem and for me that is a really central part of mathematics.</p>	
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Rachel	
Evidence	Analysis
<p>I think practicing gets a bit of a bad rap. I think there's some things that, you do just need to have a little bit of time to get your head around. It's repressive in terms of, if we did that every lesson, then they'd get bored, and they just-- motivation would go out the window, engagement will go plummeting, but I do think it has its place.</p> <p>Activity and exploration are central. I think they're important. I wouldn't necessarily have said central. I guess like I said before about facilitating personal exploration. It is it is a really nice way of doing it, but, I find it pedagogically quite hard.</p> <p>Questioning and negotiating meaning. Yes. I agree with that actually. I think definitely, discussion means something very different in Math, as opposed to other subjects particularly. But no, I think questioning and discussion has its - it definitely has its place.</p> <p>Then understanding and application is key to progress, definitely. Its that kind</p>	<p><i>Theory of learning mathematics</i> Rachel considered all these theories to be important although</p> <p><i>Aims of mathematics education</i> Rachel's beliefs were maths centered. She emphasised the importance of</p>

<p>of, you know, you've got to do it, you got to practice it yourself.</p> <p>I really like the pure stuff, but I'm aware that that only appeals to a certain type of brain. But of the children I teach there's probably fewer than 10 that would appreciate that. I think it's really important that it's there, for their sake. actually because Math is so very different once you get further up. I think with a lot of other subjects, you do get, kind of a feel for what it looks like A level and for University. You don't really get that with Math. Yes, I think it is important to have some pure stuff.</p> <p>The critical awareness in society and the numeracy stuff. I think that's what most students would expect out of Math. I think what most parents would want for their children, and what Math GCSE is meant to stand for really. Use of math so that ind cen focus. Is that looking at things like, Trigonometry being useful for engineering, things like that?</p> <p>I quite like that there are some applications for things. I'm very much kind of why would bother applying. It's fun for its own sake, but actually there's quite a lot of nice things. Again, it's quite specialized. Again, I would probably downgrade that. It's a nice idea, but in a class of 30, probably not always applicable. Creativity and self realization, I think it's because I'm not very good at the creative side myself. I find that quite hard to teach. We're trying to do a lot more investigations at the moment, and I just find it very hard to kind of, give a structure without dictating what needs to be done. I think actually, there's a lot of Mathematics</p>	<p>mathematics being a pure body of knowledge. With regards to the other areas, such as critical wareness of scioety via mathematics, numeracy and an industry centered focus, Rachel recognized that these might be expected by students but that mathematics could be 'fun for its own sake'.</p> <p><i>Theory of teaching mathematics</i> With regards to the 'theory of teaching mathematics' Rachel did not prioritise any particular theory, rather related different theories to different learning contexts. However, with regards to her practice, Rachel said she frequently</p>
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<p>creativity is just kind of, playing around numbers, just problem solving, just having a bit of fun.</p> <p>Yes. I was showing you my further Math script, and tuckers self refreential formula. I don't know if you've come across that. It's basically a graph that plots the same formula. It's so exciting, and actually it's that kind of thing that I really love and they really love, but most people would just think it's a bit hard. I think that's quite nice, but I just-- again, it only appeals to certain students and given that we're operating under schemes of work, quite often, I find it hard to actually find where can I put these in.</p> <p>If I could get away with no frills, then my life would be significantly easier. But a lot my children just can't cope with that. I think this is the kind of thing you can get away with more mature students. It's kind of almost lecture style but you need the maturity and you need an attention span. You need an ability to actually understand what's going on as you go. That works for very few students of this age, I think.</p> <p>'Facilitating personal exploration'. I think that's quite interesting. I really like the idea of it but I find it very, very difficult to do in a class of 30. There needs to be real depth of understanding of the pedagogical side of what hints the children need, what are the right questions and the right phrasings.</p> <p>Discussion and Questioning. I do that quite a lot. Questioning is really useful because you can't open up a brain and see what's going on in there. It's kind of the next best thing we can do. Questioning, again, it is quite hard because you have to find the right</p>	<p>engaged with discussion and questioning.</p> <p><i>Theory of mathematics</i></p> <p>Rachel placed importance on the theory of mathematics being an 'unquestioned body of knowledge' a 'structured body of knowledge' and 'a collection of facts and rules. She did mention that she felt these facts and rules were related and that although mathematics was a 'structured body of pure knowledge' there are also aspects of application and reasoning. Rachel did not agree progressive or socially aware aspects of the theory of mathematics, rather her beliefs reflected a 'authoritarian', 'utilitarian' and 'maths centered' systems.</p>
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<p>question to get the answer you're looking for.</p> <p>Motivation through work relevance. Actually this is one of my targets at the moment. I'm conscious that I don't do it very much because I see so much in the kind of pure sense. I'm content with that. I forget that people aren't, but no, I do think it is quite important for students to realise.</p> <p>Explain motivate pass on structure of knowledge. I think it is important, but I'd be wary of which classes I would do that with. Probably, higher sets I would do that with a little bit more, because they're kind of capable of seeing the bigger picture.</p> <p>"Unquestioned body of useful know," that's an interesting choice of words. I suppose because Math is very absolute, is very axiomatic, once it is true, it is always true. Supposing that sense is unquestioned. There's nothing in there about, linking things together, because I think that's one of the most exciting things about Math. Yes, it is a collection of actual facts and rules, but actually they will relate to each other. Actually, yes, it is facts and rules, but actually it's also, how do you apply those and how do they all relate to each other. It's the relationships I think, the most exciting things.</p> <p>'Mathematics is a personalized activity'. I would disagree with that. 'Socially constructed practice' I suppose a lot of the ways that we see Math in everyday is social constructed by, I think it is still there. Even if we didn't use the names we had for them, we'd still want to get use a lot of the concepts that we have. I</p>	
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<p>would disagree with this. A structured body of pure knowledge". I agree with that, that's not all there is to it, there's the links and there's the applications and the reasoning between all. I might stick a post-it on that section and just say-- kind of links in relationships between things. Not just applying but analysing and applying. That would be my caveat for those three (referring to 'A collection of facts and rules', 'Unquestioned body of useful knowledge' and 'Structured body of pure knowlwdge'.</p>	
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Minervia	
Evidence	Analysis
<p>'Practice and rote / Skills, acquisition, and practice, and I think this just could be there on the same level. Questioning and negotiating meaning is essential. Yes, this makes me think of the activities you made us do in the, always true sometimes, but it was never true. I think these are actually really quite good for students, because they make them think about -- it's discussion and questioning as well'.</p> <p>I'll just put this one with the back-to-basics. You might see that these two are quite good together. Activity and exploration are central, definite. It should be central'</p> <p>'I would definitely put this one as the first one; creativity and self-realization for mathematics. I like to motivate. It's not just giving on the knowledge, it's all about giving the passion as well for the</p>	<p><i>Theory of learning maths</i> Minervia's beliefs tended towards maths centered as well as progressive and socially aware. Although Minervia did not dismiss 'Practice and Rote' and 'Skill accusation and practice' these were given less priority with an 'Authoritarian' and 'Utilitarian' beliefs featuring lower down in her hierarchy of beliefs.</p> <p><i>Aims of maths education</i> Minervia's choices here clearly reflected 'Progressive' and 'Socially Aware' beliefs. Indeed the she dismissed the 'Utilitarian' and 'Maths centered' approaches. The 'Authoritarian' belief of back to basics numeracy did feature at</p>

<p>subject, and not just the plain knowledge’.</p> <p>‘Yes. Making sure I equip them with the basics in numeracy for them to succeed in life and be able to do the basic things in the supermarket or whenever they’re going sell or things like that’.</p> <p>‘</p> <p>Transmit body of pure mathematical about useful math with industry-centered focus. I will just put this one there for now’.</p> <p>‘Discussion and questioning, these two go together, understanding an application that keeps progress; together. Motivate, pass on the structure of knowledge is much better than just transmit body of pure mathematical knowledge; because this is quite static, whereas this is implying that you pass on your passion for the subject as well’.</p> <p>‘I don't know. I'm a very vigorous person. I really like this, but again, I think this doesn't really imply that you would still teach creativity; explain, motivate, pass on structure of knowledge, because you want to be passing on this structure, and you want to be passing on the rigorousness, but that doesn't really imply that you're being creative and maybe we can just put it there. It just completes each other. What does it mean transmission and drills? No frills, as in no creativity?’</p> <p>‘Yes. I suppose to an extent, some people would see that as if you think back to things like when we did Skemp, and the idea of instructional learning in a very transmission-based and this drills and frills --</p>	<p>the lowest point of the hierarchy as she felt that it would equip children to succeed in life and engage with everyday tasks such as shopping.</p> <p><i>Theory of teaching Mathematics</i> Here, too, Minervia dismissed the ‘Authoritarian’ and ‘Utilitarian’ beliefs and prioritised the beliefs reflecting a ‘Maths centred’, ‘Progressive’ and ‘Socially aware’ approach. She related back to Skemp’s paper to explain that creativity was important in maths and that ‘transmission and drill’ implied a lack of creativity and that facilitating personal exploration and creativity were important aspects of mathematics teaching.</p> <p><i>Theory of mathematics</i> Minervia believed Maths to be a socially constructed body of knowledge and a</p>
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<p>'Facilitate personal exploration so that the kids would be encouraged to actually work out from there. That's quite good. Yes, personal exploration. Yes. There was this TED Talk actually from a math teacher, an American math teacher. He was saying all about how we should teach the students instead of looking at a math problem, and finding the steps for themselves'.</p> <p>'Socially constructed body of knowledge - I'll definitely pick this one Unquestioned body of useful knowledge and a collection of facts and rules. I totally disagree' (also disagreed with structured body of pure knowledge)</p> <p>'Structured body of pure knowledge. It sounds a bit like, that in a question of body of useful knowledge and you can't really touch it and model it'.</p>	<p>personalized activity, so reflecting a 'Progressive' and 'socially aware' belief.</p> <p>She dismissed the idea that maths could be a 'unquestioned body of pure knowledge' or a collection of rules and facts' and that if maths was a 'structured body of pure knowledge' it could not be 'modelled'. As such these did not feature in her beliefs.</p>
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Edwyn	
Evidence	Analysis
<p>'Theory of Mathematics education. A collection of facts and rules, I'm going to put that right up there (top of the hierarchy). I think that structured body of pure knowledge is a - I would say probably another similar way of saying the same thing, maybe a more poetic way, essentially it's a collection of facts of rules. The pure knowledge, I like that because I do agree that maths is knowledge in it's simplest form'.</p>	<p><i>Theory of mathematics Education</i> Edwyn had a definite hierarchy of beliefs in relation to the theory of mathematics education. His choices clearly prioritized an 'authoritarian' approach with maths centred, progressive and socially aware running down his hierarchy in that order. With regards to the 'utilitarian' belief he placed this slightly outside his hierarchy as he felt it could be interpreted in different ways. In one</p>

<p>'Unquestioned body of useful knowledge could be true, could not true depending on how you look at it. The collection of facts, that's what we're trying to do when we prove theorems and we should prove it's absolutely true so that there is no question of doubt about it. However, I think definitely math should be questioned, I think whenever I learned the maths I was asking 'why'. So this one is - I'm going to put that slightly outside because that can be interpreted differently'.</p> <p>'Personalized activity and socially constructed practice, you do maths by yourself and then you do maths in groups. I think it's a personalized activity and then brought into groups. You try it by yourself, you do it by yourself, and then you bring into groups'.</p> <p>'I think I'm going to put – and it's very different to when I started teaching, but I would put back to basics numeracy right at the top because I think before I get on to any of these, they need to have this level of numeracy. You can't really do anything without this, so yes, put that right at the top. Creativity and self realization via mathematics. As much as I wouldn't - I don't think the aims of maths education in this country is creativity. As much as I believe, you can have that. I don't think that's necessarily an aim. I'd put that at the bottom'.</p> <p>'The transmit body of pure mathematical knowledge, useful math with an industry centered focus and critical awareness of society by maths. I would really tie all of these three together - transmit body and pure mind useful. I would tie the industry-focused one as well with the transmit body for maths knowledge and</p>	<p>sense he felt that when theorems are proved in maths they are absolutely true, hence unquestioned. However, he also felt that it was important for pupils to question the maths they were learning.</p> <p><i>Aims of Maths Education</i> Although Edwyn prioritised the authoritarian belief of 'Back to Basics Numeracy' he gave the remaining beliefs equal status in his hierarchy.</p> <p><i>Theory of teaching Mathematics</i> Edwyn's choices here prioritized a 'maths centred' approach with 'authoritarian' and 'utilitarian' following in the hierarchy. He felt that there were fewer opportunities to facilitate personal exploration and that,</p>
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<p>the critical awareness of society via maths. Because I think, I would try to fit in as much as I can all of three of these in my lessons. I couldn't really say that for the creativity, but when I see an opportunity I would try and fit these in'.</p> <p>'Motivate through work relevance and that links to what we were talking about before. I've got my own personal bias, I didn't enjoy that at school. I know there will be some people who will find that more motivating if it's related to work'.</p> <p>'Then facilitate personal inspiration, and then I've left discussion at the bottom. I'll leave it on (facilitate personal exploration)because opportunities for it are probably rarer than these (pointing to the other cards)'.</p> <p>'Discussion I'm going to put down below, because - we talked about in theory of maths, we talked about when you can have mathematical conversations and definitely in my classes, the students can work together to get them discuss their answers. I think discussion probably works a lot better - it's a little easier in other subjects. I'll do. Questioning, I think is very important. I'm going to put questioning and discussing answers is good (<i>new card created</i>)'.</p> <p>'Explain, motivate, and pass on structured of knowledge, that's definitely what I'm trying to do more of my teaching is now is do maybe one example and then pass it over to the students quickest there, so a lot of time they might struggle up a bit more with it but I think they're gaining more from actually having that independence and trying to doing themselves'.</p>	<p>although pupils can have mathematical conversations and work together to discuss their answer, discussion works better in other subjects. However, Edwyn did believe that pupils questioning and discussing answers was important and, as such, created his own card, 'Questioning and discussing answers' , which he placed second in his hierarchy.</p> <p><i>Theory of learning mathematics</i> Edwyn's choice here reflected the 'socially aware' beliefs as he prioritized 'Questioning and negotiating meaning is essential'. He placed equal importance on 'Activity and exploration are central', 'Skill acquisition and practice' and 'Practice and rote'. 'Understanding and application as key to progress', the</p>
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<p>'Transmission drills and no frills , I think that definitely has it's place. That's probably been the easiest one to agree with all, all five of them. I don't necessarily see all of these as mutually exclusive. For example the practice and rote one, I think is an unfair representation. Skill accusation and practice maybe would be examples in the textbook and they are changing, in that they're not just asking you to do the same thing .'</p> <p>'I'm going to put questioning probably up high, because - and negotiating means - because I think that's actually really when there's a lot of learning going on and checking''.</p> <p>'I think these three (Activity and exploration are central, Skill acquisition and practice, Practice and rote) are the actual doing activities and I put them all as equal because, I don't think they are mutually exclusive. Then understanding and application is key to progress'.</p> <p>'I'm just trying to think about different sets. I think it's probably very much true for a bottom set possibly if you got higher ability class they don't necessarily need to practice the application to progress. I leave this one'.</p>	<p>'Maths Centered' belief, was placed at the bottom.</p>
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Jason	
Evidence	Analysis
<p>‘Useful maths with an industry centred focus’, I don’t think that is so important....we live in a world that’s changing as well so we could teach them some industry centred maths if we wanted them to, however it is not necessarily going to be applicable in four five years time plus not doubt the industry themselves can do it’.</p> <p>‘I’d say transmitting, though I’m not sure I’d use the word transmit. I like the ‘pure mathematical knowledge’ and that I would still include that in other areas of applied maths anyway, is quite important’.</p> <p>‘Creativity and self realisation, I think if you can work something out on your own, not necessarily undirected, you can get the pupils to direct their own studying, then that will in turn make them more intelligent therefore more capable of studying, not only maths, but other subjects’.</p> <p>‘Numeracy is essential but it shouldn’t be at the top of the list because a lot of people can master that quite early on and you’ve got to look at what you build on from that. I think critical awareness is more important, we live in a world where the newspapers lead us into different ways of thinking and we need to be really careful when they do that so pupils can be really aware of statistics and percentages and how that kind of data is gathered and used that will make them more capable of making their own decisions when they are older’.</p>	<p><i>Aims of maths education</i> Jason prioritised the ‘maths centred’ approach and this beliefs also tended towards ‘progressive’ and ‘socially aware’. In relation to the ‘maths centred’ belief he explained that it was not so much the ‘transmission’ of pure mathematical knowledge but the ‘pure mathematical knowledge’ in itself that was important. Although ‘critical awareness of society via maths’ was placed third in his hierarchy, Jason felt that it was important as ‘we live in a world where the newspapers lead us into different ways of thinking and we need to be really careful when they do that so pupils can be really aware of statistics and percentages and how that kind of data is gathered and used that will make them more capable of making their own decisions when they are older’.</p> <p><i>Theory of Learning mathematics</i> Here too, Jason prioritised the ‘maths centered’ approach and although the ‘progressive’ and ‘socially aware’</p>

'I said maths is about making them more intelligent but as a teacher you do have that role of...for instance a lot of my Year 11s right now have aspirations that they want to go onto college, some of them have football coach type aspirations some have picked a school where they can go to do plumbing and they need to be able to get their C in maths in order to do that. My responsibility is not to make them understand the maths in that situation, my responsibility is to make sure they have the opportunity to be able to go to that college – so it is a hoop jump'.

'Activity and exploration essential I would put that towards the bottom. As I said in the last one, creativity and self realization is important but I think the a lot of the mistake with maths educations is to think that that need to happen in a constructivist and indirect way and I don't think that's an effective way to teach what is essentially a large body of knowledge'.

‘Practice and rote is interesting I believe that practice is incredibly important and I think if they work hard, if they are taught well and they work hard they can do well at maths . I am going to put that below activity because of the word rote. But I would put ‘skill accusation and practice ‘ higher because that sounds like they are actually getting it and they are not just memorizing and that is more effective. You do something rote they will get it and they will repeat it but you’ll be lucky if they can do it six months later’.

‘Understanding and application is key to progress, I think if that is talking about applying maths to the real world then going on what I said I don’t think that is so important but in terms of understanding something and being able to apply it to a different context in terms

beliefs were prioritised over the 'authoritarian' approach of 'practice and rote' the 'utilitarian' belief was second in his hierarchy.

Theory of Mathematics

Jason was quick to dismiss the 'authoritarian' belief that maths is a collection of rules and facts. He also dismissed the 'utilitarian' belief of 'unquestioned body of useful knowledge'. He prioritised the 'socially aware' approach and, in line with his other beliefs, identified the 'maths centered belief' as second in his hierarchy.

<p>of the body of pure mathematical knowledge. I do see as important so I'm going to put that at the top'.</p> <p>'Questioning and negotiated meaning as essential I think that this is something that is probably more important for those higher ability, higher attaining students. I'd expect all my students to question. So put that in there above activity and exploration'.</p> <p>'Theory of Maths. I think this is where, not having studied maths at degree level, my opinions are less strong and less clear to me. However, it's obvious that a 'collection of facts and rules' is not true, I would go as far as to say that is not what maths is'.</p> <p>'A personalised activity, I don't really like the sound of that – I suppose it could be personalized in the sense that you might have your own way'.</p> <p>'Unquestioned body of useful knowledge. Well that's just silly because I know people who are doing PhDs and if it's unquestioned, then it's not going anywhere'.</p> <p>'Socially constructed practice. So people have thought up what it is whereas a lot of mathematicians would believe that there is a naturalness to maths. Whenever things get difficult I think like a historian. I am going to put socially constructed practice at the top'.</p> <p>'Theory of teaching mathematics. I think that's an interesting one to think about, the work relevance. Pupils can be really motivated particularly with things like interest rates and compound interest they find it absolutely fascinating'.</p>	<p><i>Theory of teaching mathematics.</i></p> <p>Jason prioritised the 'maths centred' approach in his beliefs. He also tied this in with the 'socially aware' belief of discussion and questioning. Jason explained that in a lot of his lessons he passed on a structure of knowledge but this was also followed with meaningful discussion. Although Jason did not completely believe in the progressive approach of 'facilitating personal exploration'.</p>
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'Transmission drill and no frills, it's interesting maybe I am rejecting this out of hand but some of my lessons are very much transmission drill and no frill, I'm not going to lie about that but it's not necessarily the way I would want them to be but I don't think that's what maths should be'.

'A lot of my lessons are more like this, explain motivate and pass on structure of knowledge. We look at something and we discuss it, we do some well meaningful AfL to understand who knows what and then we go into some practice of it, maybe something that might resemble drills but that's only because we have had that opportunity to get an understanding of it. So in that sense questioning and also that wider questioning, questions on the board challenging questions'.

'Facilitate personal exploration. Constructivism is a mistake by the maths educational body. You want pupils to be active and their brains to be making connections and thinking because that's what going to help them learn and as a profession we made a mistake by thinking that the best way to do that was to give them a task and let them get on with it and in fact what is more important and more effective is to give them some of the information and ask them the right questions and definitely don't involve discovering the rule for yourself'.

Tao	
Evidence	Analysis
<p>1. That personalized activity, that's how I learned it when I was at school. Spending time thinking about it for myself, maybe talking to somebody else about it. Structure body of pure knowledge, I think that's closer to how I see Maths as a whole. It's kind of this way, this entity of information is nice. That's why I think it's nice, because it's structured. It's a structured thing, so I like that. Then in terms of how I ended up learning Maths, that's socially constructed thing. Collection of facts and rules. I'm not going to start, I'm just going to put that right at the top because I think that's how students tend to see it.</p> <p>2. I'm going to start with the aims of Math Education, In terms of what I'd really like to be doing, I'd say is that creativity and self-realization via mathematics and I think that comes in more the older the students get, and if you are giving them more some investigative tasks. That's what I'd like to be doing. This critical awareness of society via maths, I think when I use that it tends to be as context for probably transmitting mathematical knowledge and this creativity and then in terms of useful maths with an industry center focus, I'm not sure that I spent much time [laughs] actually doing that, because I never learn Maths that way and I've never seen it taught that way.</p> <p>3. Practice and rote, practically some students tend to see it that way and it's obviously, and it has it's place. However, skills acquisition practices similar idea,</p>	<p><i>Theory of mathematics</i></p> <p>With regards to the theory of mathematics, Tao felt that puoils saw it as a collection of facts and figured, an unquestioned body of useful knowledge. However, his own belief was that it was structured body of pure knowledge and a personalized activity, although he did agree that it could be a socially constructed practice and he had experienced that himself.</p> <p>Here Tao's beliefs tended more towards maths – centred apparoach.</p> <p><i>Aims of mathematics</i></p> <p>With regards to the aims of mathematics education explained that he saw creativity and self realisation via mathematics as something he would like to be doing. He felt that he used critical awareness of society via mathematics in the context of transmitting mathematical knowledge also identified this with creativity. Tao identified industry-focused maths as something he did not spend time on. Tao's belief with regards to the aim of maths was 'progressive' with elements of a 'maths centered' .</p> <p><i>Theory of learning mathematics</i></p> <p>Here too, Tao's beliefs were 'maths centered' . Although he thought the 'uttilitarian' beliefs of 'skill acquisition and practice' were important he placed more emphasis on 'progressive' and 'socially aware' belief systems.</p>

<p>obviously it's a little bit more the idea of learning skills. That's how I see it. Understanding and application to progress, that's the sort of building we really do.</p> <p>Activity and exploration as central. There is not so much scope for this, in terms of the actual schemes of work and in terms of the curriculum. But, obviously, this is absolutely important because if they're to understand and apply, they have to have done the work for themselves, and to have that exploration. Then in terms of how it sticks once they've learned it, once they've acquire those skills, this is the important thing and it's to say that, that links how I learn Maths in the first place. That idea of having to discuss what's going on and I think some students really like that. They like to say, "So, what about this", and it's not necessarily into ability or to like Maths set, whether they like that or not.</p> <p>4. I think this 'explain, motivate pass on structure of knowledge' is closer to why I actually do most of the times. In terms of facilitating personal exploration, sometimes I do that, but I don't do as much as I would like to.</p> <p>Motivate through work experience. I don't spend a huge amount of time doing that, but when I do, it tends to be as context for, something else so for example, everything I've done, the course where that was based on climate change, so that was relevant to use stats in that particular field</p>	<p><i>Theory of teaching mathematics</i></p> <p>Similar to his beliefs about the 'Theory of learning mathematics' Tao's beliefs here were 'maths centred'. With regards to the other beliefs, he placed more more emphasis on 'progressive' and 'socially aware' belief systems.</p>
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Santana	
Evidence	Analysis
<p>I think that obviously crucial to it is having core numeracy skills. I think that that's like the most important thing we probably try to get as a base. If we think of everyone, you want everyone to be schooled with some basic numeracy skills. Then it's transmitting a body of pure mathematical knowledge in a way. They need to get the knowledge so they can get their qualifications as well. I wouldn't say industry center focus is important to me. Obviously I haven't come from a maths background or anything. I don't think that's the aim of mathematics education, to some extent, that might come later if students then decide they want to go into that.</p> <p>I think creativity is really important and a critical awareness of society. I put them together. These two are the 'back to basics' of numeracy and getting the maths, like the basics. Then if we get this creativity and getting them to think critically about maths and everything then that's where we would like them to get to.</p> <p>I would say structured body of pure knowledge because obviously there are methods, rules and formulas we follow but I wouldn't really agree with a lot of these. I wouldn't say it's necessarily unquestioned body of useful knowledge, because you can still think about why</p>	<p><i>Aims of maths education</i></p> <p>Santana prioritised the 'authoritarian' belief of 'back to basics numeracy' as she felt that everyone needed to be schooled with some basic numeracy skills. She also felt that a 'math centred' approach was important. She disregarded the 'utilitarian' approach and, although prioritising a 'back to basics' approach, she felt that the 'progressive and 'socially aware' approaches were also very important.</p> <p><i>Theory of Maths</i></p> <p>Santana had a 'maths centred' belief in relation to the theory of mathematics. She believed in the 'authoritarian' belief of maths being 'a collection of facts and rules'. She dismissed the other beliefs but added her own card 'developing logical mind'.</p>

<p>they work. Even if we know certain things are fact.</p> <p>One thing I always think of maths -- maths is a different way of thinking, a logical way of thinking, which you might not get that developed in other aspects. I guess you can say a collection of facts and rules, and then you're also developing your logical mind, which you might not get to in other areas of education or in life.</p> <p>I think understanding an application is key to progress. It's important for acquisition and practice, because you need that first and then I think question and meaning is so important.</p> <p>I guess the way the theory that I would look at, like is more like concrete, pictorial and abstract and obviously we've been doing that. More and more I'm starting to appreciate that and how important it is. I'll write CPA, but I guess that links to the activity and exploration. Pictorial, abstract and that links to question and meaning, so we're not just representing maths as an abstract thing.</p> <p>It's about explaining, motivating and passing on the knowledge. Then transmission and drill I guess, you do just need them. This changes with their age group as well, so maybe I'll put it lower but I think with everyone it's about motivating and passing on knowledge and discussion and questioning. I don't think work relevance is -- I don't ever really do that in my practice, or don't really facilitate personal exploration that much. There'll</p>	<p>Theory of Learning mathematics</p> <p>Here Santana discarded the 'authoritarian' belief of 'practice and rote' and prioritised 'utilitarian' and 'maths centred' beliefs. Santana also discussed how the school has been looking at learning mathematics using a 'concrete, pictorial and abstract' approach and she felt this tied in with her more 'socially aware and 'progressive' beliefs.</p> <p>Theory of teaching maths</p> <p>In relation to the theory of teaching mathematics, Santana discarded the 'utilitarian' belief of 'motivating through work relevance' and the 'progressive' belief of 'facilitating personal exploration'. She prioritised the 'maths centred' belief as well as the 'socially aware belief'. She felt there was a place for 'transmission and drill, no frills' approach but placed these lower than her other beliefs.</p>
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be bits in my questioning that might do that but not in itself. I'm going to get rid of it too.	
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How did the sample of teachers situate RWEI in their classrooms?

Indicators

1. The curriculum area.
2. The RWEI

Data

1. In the post-lesson feedback teachers discuss how the lesson was motivated both in terms of mathematics content and RWEI. They also discuss aspects of **RWEI** they plan to situate into future lessons
2. My own lesson observation notes describe the lesson, this includes reference to how the teachers motivates the lesson and its content both in terms of **curriculum content** and **RWEI**.
3. The teachers Lesson Plans will also be part of the analysis as it is likely to make reference to the two indicators. This data will further indicate which **areas of the curriculum** are addressed how the **RWEI** is situated into maths lessons.

In the analysis I have made reference to the source of the evidence, which is numbered on the second column.

Aron	
Evidence	Analysis
<p>1. Objective Lesson on understanding compound interest and how compound interest can cause a sum of money to grow vastly in a short space of time.</p> <p>2. National Curriculum Define percentage as ‘number of parts per hundred’, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%</p> <p>3, RWEI To use mathematics as a tool for understanding the real world, in this case using the example of loan companies and interest rates.</p> <p>4. Aron’s evaluation: By motivating the pupils with the real world problem of borrowing/saving money at a particular interest rate I</p>	<p>Aron situated the RWEI of loan companies into a lesson which addressed percentages through the context of compound interest. (1)(2)</p> <p>He was motivated to use maths as a tool for the real world as he was concerned that the school did not prepare them to tackle real world problems (4). By studying interest rates Aron was able to discuss the ideas of borrowing and saving money and how interest rates could work in your favour (5). The lesson had clear objectives, that pupils should know how to calculate compound interest and how compound interest can cause a sum of money to grow vastly in a short space of time. In the context of compound interest, Aron also wanted to address how mathematics can be used as a tool for understanding the real world. (6)</p> <p>The lesson started with Aron setting four non contextual questions about the</p>

<p>was able to engage them in solving problems which lay at the edge of many of their abilities.</p> <p>Planning the lesson and making sure my focus was on both conveying the real world truths as well as the maths was a far more pleasant experience for me as I am often troubled by how little school can prepare pupils to effectively tackle real world problems. This lesson gave the pupils a tool they can use to start thinking about the benefits of saving vs the problems of credit.</p> <p>Additionally their comments to me after the lesson was how much they enjoyed it, especially in talking about the maths in a qualitative way, i.e. what mathematical conditions make a loan 'bad' versus 'good'.</p> <p>5. Observer notes</p> <p>6. Lesson plan</p>	<p>percentage of some numbers.</p> <p>This was followed by Aron briefly explaining that 'we will use percentages to gain skills and avoid making bad decisions'. A video clip for 'Kwik Cash', a Payday loan company, was shown to the pupils and this was followed by brief discussion about loans. This mainly centered around how loans can be useful, such as mortgages, and also how compound interest can work to your advantage, when you are saving money in a bank, and to your disadvantage, when you are borrowing money. This was demonstrated by showing an example of someone investing an initial amount of £7300 at compound interest of 3.2% and how much this would be after 3 years. Much of what followed had a focus on the mathematical methods to calculate compound interest and questions with an emphasis on repeated practice of exercises. This was followed by a mini whiteboard activity to assess learning. (6)</p> <p>Aron's then used examples Payday Loan Companies to teach pupils how to work with percentages greater than 100%. Following this he introduced a worksheet (Appendix 1) titled loans investigation in which pupils had to calculate the repayment amount for each company (6). This was followed by a discussion about which of these loans would be the best or the worst. For example, would a high interest loan over a short period of time be better than a low interest loan over a longer period of time? Hence through this part of the lesson Aron addressed his third objective 'how mathematics can be used as a tool for understanding the real world'(5).</p> <p>A real case study followed, this involved using the loan company Wonga as an example and putting forward the scenario that if you can borrow £400 for 1 month at 292% APR. 'How much do</p>
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	<p>you have to pay back at the end of the month'? (4)(6)</p> <p>The 292% APR interest was surprising to many pupils but when broken down to a month there was more of a discussion on whether this was an acceptable amount to charge. Although the idea of APR was not discussed in detail it still gave pupils an idea of how much the monthly interest was and the further interest if the loan was not paid off (5).</p> <p>Although there is no evidence in the interviews or lesson plans as to why this RWEI is particularly relevant, the fact that the proportion of students, in this school, who are known to be eligible for the pupil premium is nearly double the national average suggests that some parents might use the services of Payday loan companies (5).</p> <p>Aron felt that the mathematical content had challenged the pupils and that he 'was able to engage them (the pupils) in solving problems which lay at the edge of many of their abilities'. (4)</p>
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Fabia	
Evidence	Analysis
<p>1. Objective Collating and organising data and ordering percentages and decimals in order to look at election results and understand the differences between different counting systems.</p> <p>2. National Curriculum Order positive and negative integers, decimals and fractions; compare two quantities using percentages; construct and interpret appropriate tables, charts, and diagrams, including</p>	<p>Fabia situated the RWEI of the election system in a lesson which addressed ordering percentages/decimals and organising and collating data (2).</p> <p>She felt that this was an opportunity for the pupils to 'do maths that stretched them' (4). The lesson was particularly relevant as it took place a few weeks after the General Election in the UK.</p>

<p>frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data</p> <p>3. RWEI</p> <ul style="list-style-type: none"> • To look at the election results. • To understand the differences between different vote counting systems. • To analyse our voting system. <p>4. Fabia's Evaluation</p> <p>Pupils' prior knowledge of politics in the UK was not strong, and so I had to adapt to make sure that pupils understood all the key terms. In particular they found it difficult to grasp what a constituency and a seat were. It was really interesting to see pupils who usually really struggle in Maths getting settled and approaching activities more quickly and confidently. This may have been because pupils didn't see the work as 'Maths'.</p> <p>Both my class and I really enjoyed the lesson, and it gave me a chance to see different strengths and areas for improvement in my pupils, whilst still giving them opportunities to do maths that stretched them - ordering percentages/decimals; organising and collating data.</p> <p>My intention was to allow pupils the chance to engage with the election data, and show them the data in different ways (eg percentage of vote share vs number of seats) and then show them how the way in which we count the votes plays a part in this. I feel that pupils did learn a lot in the lesson, and are more critical in the way they can think about voting as a result of this lesson.</p>	<p>Fabia choose to situate RWEI in the maths lesson as she felt that 'pupils prior knowledge of UK politics was not strong' and it was her intention to allow pupils to engage with election data and show them that how we count the votes plays a part in the final result (4).</p> <p>At the start of the lesson Fabia discussed the UK voting system with reference to constituencies and comparing the 2010 London constituency map with that of the election in 2015. As such pupils were interpreting data represented pictorially (5).</p> <p>Fabia also showed a 1 minute 'Parliament UK' video which explained the first past the post voting system in the UK. Pupils were also given a set of questions, which they had to answer during the video. These questions related to the voting system and had no mathematical content (5)(6).</p> <p>Following a discussion of the recent election results in the school's constituency, Fabia went on to introduce a more critical discussion as pupils had to calculate how many people actually voted in the UK given that the electorate was 46,425,386 and 66.1% actually of the electorate actually voted . Discussions centred around why people might not vote and who should be allowed to vote. (5)(6)</p> <p>Fabia then presented a bar chart of the recent election results, showing the number of seats each party had, this was then broken down into raw votes and displayed on a table. This gave rise to many pupils saying that the voting system was unfair as elections were decided by seats and not raw votes. (5)</p>
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<p>5. Observer notes</p> <p>6. Lesson plan</p>	<p>The pupils then worked out the raw votes as percentages and placed them in order. This resulted in some parties who had very few seats ending up as significant parties when looking at raw votes. Indeed, UKIP was the ninth most popular party when considering seats but the third most popular party when looking at raw votes. As such, the potential success of UKIP, when counting raw votes, led many pupils to review their ideas about the voting system. The system of counting raw votes, which they originally thought was 'fair', was not a system they now agreed with (5).</p> <p>Fabia then followed this with a practical activity taken from a book 'Human Rights in the Curriculum – Mathematics,(2004)'. Fabia explained five different voting systems to the class and they had to discuss which was 'fair'. Pupils worked in five groups, each working with one of the voting systems. They were given 11 filled ballot papers from fictitious voters and had to decide which candidate would win under their voting system. (5)</p> <p>The activity further emphasised that different voting systems could elect different candidates, something the pupils had started to consider after looking at the UK elections. Some pupils found the mathematics challenging but were nevertheless engaged with the task. (5)</p>
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Rachel	
Evidence	Analysis
<p>1. Objective Identify key features of pie charts and bar charts to be able to interrogate the charts and identify misleading presentation of data.</p> <p>2. National Curriculum Construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data.</p> <p>3. RWEI To be able to identify misleading graphs.</p> <p>4. Rachel's Evaluation My objectives for the lesson were for students to criticise misleading graphs: to identify key features of pie/bar charts, answer prompt questions to determine the validity of statements which appeared to be supported by graphs, and (as a possible extension) to come up with their own questions which could be used to "interrogate" graphs. I had intended for students to investigate in small groups and then give them a chance to develop their presentation and explanation skills, but this didn't work as the students were not able to match up the hint questions from the board with the graphs presented to them on paper. I would change this in future by having each hint question printed alongside the relevant graph and erroneous statement.</p>	<p>Rachel situated the RWEI of misleading graphs in a lesson which addressed identifying the key features of bar charts and pie charts and to be able to interrogate the charts. (1)(2)</p> <p>She wanted students to be able to spot when graphs or statistics in newspapers (and other media) are manipulated to support statements which aren't necessarily true. (4)</p> <p>Rachel started the lesson by displaying a misleading pictogram of sales of cat products from 2000 to 2004. The pictogram used pictures of cats and they had not been equally spaced out on the chart. Rachel stated that in 2004 the most cat products had been sold and asked the class if this was true or false. She then discussed the meaning of the word 'misleading'. (5)(6)</p> <p>Rachel led a class discussion about how graphs could be misleading. Following this she gave out 6 graphs, consisting of pie charts and bar charts, and asked the students to answer true/ false statements for each one and say whether, and why, they were misleading. (5)(6)</p> <p>Based on the earlier discussion she gave the following prompts:</p> <ul style="list-style-type: none"> • What's the scale on the axes? • What is the actual increase/decrease? • How many people are represented? • Are labels present and correct?

<p>I wanted students to be able to spot when graphs or statistics in newspapers (and other media) are manipulated to support statements which aren't necessarily true. Although the examples I used had only one taken from real life, they were all intended to be the kind of "tricks" that are commonly employed. I think some of them have come away with a greater awareness of the importance of reading/scrutinising axes and scale in detail, and a few have picked up the fact that pie charts (in isolation) only tell you proportions, not amounts.</p> <p>5. Observer notes</p> <p>6. Lesson plan</p>	<p>She also emphasized that pupils should also create their own additional 'checklist' questions. (5)(6)</p> <p>Although only one of the six charts were 'real', they were all intended to include the type of misleading data "tricks" that are commonly employed. (4)</p> <p>For the most part pupils were engaged with the task and discussions on tables revealed how surprised pupils were when they discovered how bar charts and pie charts can be deceiving. Rachel felt that the pupils had come away with a greater awareness of the importance of reading/scrutinising axes and scale in detail, and a few had picked up the fact that pie charts, in isolation, only tell you proportions, not amounts. (4)</p>
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Minervia	
Evidence	Analysis
<p>1.Lesson Objective Flight plans. Come up with a flight timetable as well as a financial plan for it by using their skills and knowledge on bearings, maps and scales, percentages, and converting between units of measures (money, time, etc.).</p> <p>2. RWEI The specific real world equity objective was to look at the financials of flight plans and the amount of fuel used in journeys, discussing the environmental impact of plane journeys and which countries are taking more flights.</p> <p>1.National Curriculum Interpret and use bearings.</p>	<p>Minervia situated the RWEI of the environmental impact of flights in a lesson which developed the pupils' knowledge of scale factors and bearings through the use of flight maps. The pupils then calculated the cost of the flights and the fuel required for each journey (1)(4)</p> <p>She wanted pupils to independently choose skills and methods to get to the answers and then lead a discussion about the environmental issues related to flights.</p> <p>Minervia started the lesson with a brief reminder on working with bearings and scale factor. Pupils were split into groups and every group was given a map of a part of the world and a related worksheet to organize flight plans (3)(4).</p>

<p>Using scale factors, scale drawings and maps.</p> <p>Convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts.</p> <p>2.Minervia's Evaluation</p> <p>The objectives were for pupils to come up with a flight timetable as well as a financial plan for it by using their skills and knowledge on bearings, maps and scales, percentages, and converting between units of measures (money, time, etc.). From a mathematical point of view, the aim was for pupils to independently choose which skill and method to use. The flight plan involved finding the amount of fuel needed for the flights. I wanted to discuss the environmental costs of plane journeys with the pupils. Pupils seemed to enjoy the lesson and were successful working with the map in working out the flight plans and financial plans of the planes. It was interesting to see them use different mathematical skills working on this.</p> <p>They were interested in the discussion about the environment and how some countries had more flights than others. They had not really thought about this before. I think it would have been better to leave more time for the discussion.</p> <p>3.Observer notes</p> <p>4. Lesson plan</p>	<p>(4) The worksheet had a flight plan for a part of the world and pupils had tasks relating to this. For example the Asia Flight Plan worksheet had five flights, Moscow-Yakutsk, Yakutsk-Tokyo, Tokyo-Beijing, Beijing-Bombay, Bombay-Damascus. The pupils had to fill up a table for each journey based on the following tasks:</p> <ol style="list-style-type: none"> 1. Calculate the bearing to fly from each city to the next destination. You should also draw the course on the map. 2. Measure the distance between the cities with a ruler, then work out the miles. The scale is on the map. 3. Calculate the flight time, The average speed of a Boeing 747 is around 500mph. 4. Fill in an estimated arrival time (based how long the flight will take) 5. Planes take around 30 minutes to be refuelled, cleaned and safety checked for the next flight. You need to choose what time to set off for the next destination. Remember that the times are often rounded. <p>Task 6 and 7 were on the following worksheet which put the pupils in the role of a financial engineer who needed to work out the cost for each flight and fill up a table based on the following tasks:</p> <ol style="list-style-type: none"> 6. The Boeing 747 consumes on average 11 tons of fuel an hour when in cruise, which is approximately 240 litres every minute. You need to calculate the amount of fuel needed for every journey, allowing 15% extra each time, in case of any problems. 7. The current cost of aviation fuel is 38 US cents per litre. You need to calculate how much the fuel will cost (in pounds sterling) for your total journey. The current conversion rate is £1 = \$1.42
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	<p>Following this Minervia led a discussion on the environmental impact of flights and which countries tend to have more flights. Pupils were interested in the discussion and , for the most part, participated well. Minervia felt that pupils had developed particular maths skill through the lesson and had used the findings to gain an understanding of the environmental impact of flights. With about 10 minutes of the lesson given to the discussion, Minervia felt that there could have been more time set aside for this. (2)(3)</p>
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Edwyn	
Evidence	Analysis
<p>1.Objective What combination of radius and height will minimize the amount of material needed to make a cylindrical can which holds 330ml?</p> <p>2.National Curriculum substitute numerical values into formulae and expressions, including scientific formulae model situations or procedures by translating them into algebraic expressions or formulae and by using graphs calculate surface areas and volumes of spheres, pyramids, cones and composite solids</p> <p>derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)</p>	<p>Edwyn situated an environmental RWEI in the context of a mathematical investigation which asked pupils to find the minimum surface area (or material) required to make a cylinder which holds 330ml of drink. (1)</p> <p>Edwyn started by introducing the problem to the pupils in the context of a soft drinks company who recently introduced a new type of can which are taller and more slender than the original ones in order to give them a more distinctive look. The question was that making the can look different could be a good idea but how much material would be required to make the new can? Work out what would be the optimum height and radius? With regards to optimum height and radius, Edwyn discussed the cost to the company and to the environment of using more material for the can. (5)(6)</p>

<p>3.RWEI Designing a 330ml can with with the minimum amount of materials needed, hence least wastage of materials.</p> <p>4.Edwyn's Evaluation My objectives for the lesson were for students to work out the optimum radius and height for a 330 ml can. I wanted the pupils to be confident about using formula for the surface area and volume of a cylinder and be able to use formulae in an excel spreadsheet to find the optimum dimensions. I wanted pupils to get involved in elements of problem solving and also to look at cost of choosing a particular design both financially and with regards to the environment.</p> <p>We worked together to find how to change the radius and height while keeping the capacity the same. I had to help with placing formulas in excel.</p> <p>The students did very well and it was interesting to hear the range of comments with some commenting on the maths, such as finding the dimensions of different cylinders, while some really started thinking about the bigger picture. For example, when asked what he had learnt a pupil explained that he had learnt formulas and methods of how to work out the surface area and volume of cylinders and how a change in the shape could affect the world environmentally.</p> <p>Overall I felt pupils developed mathematically by collaborating on a problem solving task and using excel. I had intended for the pupils to be challenged with the maths but also see how maths can be used to bring attention to real life problems. As they</p>	<p>Edwyn let the pupils work towards a formula to find the surface area of the can when the radius and height were changed, with the restriction that the volume had to remain at 330ml. Some pupils found this difficult and were given help. (6)</p> <p>Edwyn then discussed how these formulas could be used in an excel spreadsheet and what the advantages would be of using a spreadsheet. Pupils worked in pairs on laptops and entered the formulas and different radii.</p> <p>Edwyn also demonstrated how the results could be plotted using excel. Although not all the pupils were able to get to the stage of plotting the graph on their laptop, Edwyn did demonstrate this in order to show how the minimum point of the graph was the optimum radius for a 330ml can. (5)</p> <p>Pupils then discussed different shapes of cans they had seen in shops and which companies were the most cost efficient and also had least impact on the environment. (5)</p> <p>Edwyn felt pupils engaged well with the investigative task and also understood how spreadsheets could be used to solve mathematical problems. Edwyn was also pleased to see pupils had engaged in discussing real world issues based on the results from the investigation. (4)</p>
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<p>don't often engage in these type of lesson it was quite revealing for the pupils to realize that they could comment on world issues using this type of investigation.</p> <p>5. Observer notes</p> <p>6. Lesson plan</p>	
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Jason	
Evidence	Analysis
<p>1.Objective Probability in the real world</p> <p>2.National Curriculum</p> <p>3.RWEI (lesson from 2015) How will Conservatives handle NHS better? How can mathematicians help the NHS? Increasing aspirations of pupils in deprived areas.</p> <p>4.Jason' Evaluation</p> <p>Overall I was surprised how much they got from it. At the start they really bought into the idea and were excited by what they had to do. The depth of the task did meant that some of them lost focus towards the end, but the same can will always be expected in a 100 minute lesson!</p> <p>What I though was particularly beneficial was how the relevance would increase their aspirations. This is something that I will be able to refer back to in future lessons – which may not contain real world equity issues.</p>	<p>Jason' objective was to apply probability to the real world by looking at the interrelationship between maths, government, medicine and science. He situated the RWEI of decisions that the NHS have to make in a lesson in which addressed experimental probability, estimating for different sample sizes and drawing tree diagrams. (1)</p> <p>The lesson took place in 2015, soon after the general election. Jason started the lesson with a powerpoint summarising the outcome of the recent election. This was followed by a slide with pictures of hospital wards and tabloid headlines about the NHS, such as 'Now hospitals run out of beds' and 'New NHS scandal as 37,000 jobs go. ' He explained that the government are looking at how to manage the NHS better and the pupils have been tasked as the best mathematicians available to help them. (5)(6)</p> <p>Jason had put five 'auto recessive' conditions on the board (Cystic fibrosis, MCADD, PKU, Sickle-cell anemia and Beta-thalassemia). Pupils worked in pairs, firstly choosing a condition and then engaging with three tasks which Jason had set on computers. (5)(6)</p>

<p>I think the most suitable balance would be having one lesson a half term or topic like this, with other more content/knowledge heavy lessons. Where learning is perhaps faster and progress is consolidated.</p> <p>We covered the interrelationship between maths, government, medicine and science. Looked at different auto-recessive conditions. The end of the powerpoint would have covered them evaluating this learning to work out whether the government should test everyone for whether they are carrier, and whether parents who are carriers should have children.</p> <p>It was interesting for the pupils to realise the sort of decisions governments have to make and how maths plays an important part in this.</p> <p>5. Observer notes</p> <p>6. Lesson plan</p>	<p>Task 1</p> <p><i>Key questions</i> : What is the condition? Can it be cured? Can it be treated? How many people suffer from it? What are the symptoms of the condition?</p> <p><i>What is the probability?</i>: How many people in the UK have the condition? What is the probability therefore of picking someone who has it? Was it the probability of a baby being born with the condition Why might these be different?</p> <p><i>Expectation: How many people would you expect to have your condition in:</i> a. Your class b. Your school c. The borough of Enfield d. The whole of London.(6)</p> <p>Task 2:</p> <p>Key Questions: What does “Auto-Recessive Conditions” mean?</p> <p>How do they get passed on from parent to child?</p> <p>For a child to get the condition what do both parents need?</p> <p>If only one parent is a carrier can their child inherit the condition?</p> <p><u>What is the probability?</u></p> <p>a.Of a child inheriting your condition, when the Dad is a carrier but the Mum isn’t?</p> <p>b. Of a child inheriting your condition, when both the Mum and Dad are carriers?</p> <p>c.Of having a healthy child if both parents are carriers?</p> <p><u>Probability Tree</u></p> <p>Design a probability tree for a family considering having two children.</p> <p>What is the probability:</p> <p>a.Both are healthy?</p> <p>b.Both have the condition?</p> <p>c.At least one child has the condition. (6)</p> <p>Task 3:</p> <p><u>Big Questions:</u></p> <p>Using what you have found out about your condition</p>
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	<p>a.Do you think everyone should be tested to find out if they are a carrier?</p> <p>b.If two future partners were both carriers, as a medical research what would your advise be to them? (6)</p> <p>Use information and probability you have found to support your decision.</p> <p><u>Dominant Diseases</u></p> <p>What is the difference between “autosomal conditions” (such as Huntingdon’s disease) and “auto-recessive conditions?</p> <p>What is the chance of a child having Huntingdon’s disease if both parents are carriers?</p> <p>How does this change your probability trees?</p> <p>Would your answers to the ‘Big Questions’ change?</p> <p><u>Extension</u></p> <p>Can you put together an estimate for the costs for the NHS to look after patients with your condition?</p> <p>If the probability is reduced what effect would this have on future costs?</p> <p>How else is probability used in medical research? (6)</p> <p>The pupils worked through the tasks and were required to make a powerpoint of their findings, some were selected to present to the class. Pupils were interested interested in the probability of children carrying these conditions given different starting points of both parents being carriers and one parent being a carrier.</p> <p>Following pupils’ presentation of their findings, there were follow on questions for discussion :</p> <ol style="list-style-type: none"> 9. Should the government test everyone for these conditions? 10. If both parents are carriers, should they have children?
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	<p>This also brought about a discussion about if some people had the recourses to carry out private tests whereas others could only be tested if the government thought it was financially viable. (5)</p> <p>Jason felt the pupils had developed an understanding o the types of decisions which have to be made by governments. Further, looking into these ideas resulted in pupils having higher aspirations. (4)</p>
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Tao	
Evidence	Analysis
<p>1.Objective To critically interpret information about proportions.</p> <p>2.National Curriculum Construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data</p> <p>3.RWEI Critically interpret information about proportions in the context of time and motion studies.</p> <p>4.Tao's Evaluation Students were engaged in discussion about proportion. Most were able to perform calculations without a calculator – some were given permission to work with. Work was recorded clearly in exercise books by many students. Students contributed</p>	<p>The context of Yao's lesson was to critically interpret information about proportions in the context of time and motion studies. Using pie charts for the representation of real-life proportional information and by reflecting on and discussing the ethics of time and motion studies.</p> <p>Yao started the lesson by asking the pupils to practise drawing and interpreting pie charts. This was done using data about pupils' favourite subjects and different makes of cars owned by people who were surveyed. He then set them a worksheet on drawing pie charts and bar charts using classroom data. (5)(6)</p> <p>After 10 minutes, the pupils were stopped and shown a table of a time and motion study from IBM in the 60s and Yao explained what a time and motion study was. He then displayed data from a time and motion study from the 60s which analysed people's chair activity (5)(6)</p> <p>Based on this he asked the class to answer the following questions:</p>

<p>well to discussion about the IBM time and motion study – not all recorded this in their exercise books. Students were shocked by the data I recorded on the time they spent working. This was reflected in their comments on the post-it notes. More time was allowed for activities and less time was given to teacher-led discussion! This meant that not all students recorded their work, even if they did discuss it verbally.</p> <p>More challenging work was allowed for Task 2 – most students were already familiar with pie charts - could have focused more interpretation here.</p> <p>5. Observer notes</p> <p>6. Lesson plan</p>	<p>i) Why might someone want to know this information?</p> <p>ii) If you get up from your chair 25 times in a day, how long in minutes do you spend getting up?</p> <p>iii) If you work for 8 hours, what fraction of the time do you spend getting out of your chair?</p> <p>iv) How might someone collect this information now? How might they have collected it in 1960?</p> <p>This was followed by a short clip from the BBC 1 One Show explaining what time and motion studies were and why they were so controversial. The video clip explained that unions were against the idea, whereas management was in favour of it. Yao started a discussion about whether time and motion studies were equitable, and revealed that as the pupils had been working today he had used three stopwatches to record the time one of the groups of pupils spent on writing, discussing and chatting off task. He put up the data and pupils were asked to draw a pie chart to represent the findings and also asked if they could guess which group he had focussed on for the study.</p> <p>(5)(6)</p> <p>Yao commented that:</p> <p>Students contributed well to discussion about the IBM time and motion study.</p> <p>‘Students were shocked by the data I recorded on the time they spent working. This was reflected in their comments on the post-it notes. Although most pupils’ feedback was that they had learnt how to draw and interpret pie charts, many commented on how it was ‘creepy’ or ‘weird’ to be watched while working.’</p> <p>(4)</p>
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Santana	
Evidence	Analysis
<p>1.Objective Understand government public spending figures and represent the data on pie charts.</p> <p>2.National Curriculum Develop use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics. Construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data</p> <p>3.RWEI There was a focus on government public spending, with questions to get the students to think about their own opinions on where they think public spending should be allocated.</p> <p>4.Santana's Evaluation I wanted the students to understand why we use Pie Charts and how to construct them. There was a focus on government public spending, with questions to get the students to think about their own opinions on where they think public spending should be allocated.</p> <p>Mostly, I feel the maths objectives</p>	<p>Santana situated the RWEI of government spending in a lesson where pupils interpreted problem relating to financial mathematics and constructed pie charts. (1)(2)(6)</p> <p>Santana started the lesson by reminding pupils how to construct pie charts. She used fictitious data based on which musical instruments people play. (5)(6)</p> <p>She then discussed how pie charts could be used in real life and, after some discussion with the class, went on to ask 'How does the government spend your public money?', 'Does someone tell them how to spend it?'. (5)(6)</p> <p>Santana then showed a two minute Newsbeat video about 'How the government spends your money'. It mentioned that last year the government raised £648,000,000 through taxes and then discussed how this was spent. It ended by mentioning that the government had to borrow £84bn last year and that interest had to be paid on this loan. (5)(6)</p> <p>Santana then put up a table showing a breakdown on how much was spent on different public services in 2014-15. Based on this data she started to ask questions such as 'How much more was spent on health compared to education?'. Discussions were interesting, such as:</p> <p>Pupil: They spend more on education than they do on defence. Teacher: Don't you think that's important? Pupil: But they don't teach us anything! Teacher: Then maybe that's why it need more money.</p>

<p>were covered and the following lesson when the students were asked to complete a Pie Chart for the starter they were able to do this successfully, as well as in the recent assessment. It was interesting, the students found it difficult to articulate and think about their views on the equity issue, but with prompting and time they started to get into discussions. It was useful giving them time to discuss with partners where they perhaps felt more comfortable.</p> <p>I enjoyed the lesson and I think it is great to add another dimension to mathematics lessons especially at KS3. Very useful when especially teaching statistic components to really bring to life where maths is used in the real world as well. It was interesting to hear how engaged the pupils were in the discussions about public spending.</p> <p>5. Observer notes</p> <p>6. Lesson plan</p>	<p>In another discussion a pupil mentioned that too much was spent on recreation, to which another answered that it promotes health. Other pupils mentioned that the environment is important and not enough is spent on environment.</p> <p>Pupils were then asked to draw a pie chart of the government's main areas of spending. This followed another discussion about 'If you were Prime minister what things would you change and what would you keep the same?'</p> <p>Once again there were interesting comment here such as 'I would spend more on health and technology. Looking at how technology could be used to improve health'. (5)</p> <p>Santana then asked pupils, 'What public services would you spend on if you were Prime Minister?'. Pupils were given a £1000 pound budget and had to construct their own pie charts to show how this money would be spent on public services. (5)</p> <p>Santana enjoyed teaching the lesson and thought it brought to life where maths is used in the real world. She commented that the pupils were successfully able to construct and interpret pie charts and it was interesting to hear how they engaged in discussions about public spending. (4)</p>
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