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Dynamic modelling of a twin rotor multi-input multi-output system: a genetic algorithm optimisation approach

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ABSTRACT

This paper presents investigations into practical dynamic modelling of a twin rotor multi-input multi-output system (TRMS)[1] using genetic algorithms (GAs). The construction and operation of the TRMS in many aspects resembles that of a helicopter, with a significant cross-coupling between longitudinal and lateral directional motions. The TRMS permits both 1 and 2 degrees of freedom (DOF) motion, it can be considered as a static test rig for an air vehicle. The global search technique of GA is used to extract parameters of the model, based on one-step-ahead prediction, characterising the TRMS in hovering mode. The effectiveness of the modelling technique is validated and verified in terms of tracking, stability and ability of derived model in capturing the dynamics of the system. The model is validated thoroughly through a number of time and frequency domain tests and, therefore, is deemed to be reliable.

Keywords: genetic algorithms, parametric model, twin rotor system

I. INTRODUCTION

Rotary wing aerial vehicles, such as helicopters, have distinct advantages over conventional fixed wing aircraft on surveillance and inspection tasks, since they can take-off and land vertically in limited spaces and easily hover in places above a target. A scaled and simplified version of a practical helicopter, namely the twin rotor multi-input multi-output system (TRMS)[1], as shown in Figure 1, is often used as a laboratory platform for control experiments. Although the dynamics of the TRMS are simpler than those of a real helicopter, they retain the most important helicopter features such as couplings and strong nonlinearities. Due to size, cost, ease of operation and interfacing facilities with a personal computer, the TRMS has attracted many researchers and is being used as an interesting 'test rig' for aerodynamic modelling and control problems[2-5]. Several mathematical techniques, such as blade element approach, blade element momentum theory and general flow theory have been used to model different types of rotary wing systems.[6-10] Voorsluijs and Muldery[11] presented a modelling technique based on linear parameter varying method for a helicopter. Although the above modelling techniques prove to be effective but require clear knowledge of complex dynamic motions, aerodynamics, cross-coupling effects, structural dimensions of rotor systems and other components of the system, which seem extremely complicated to formulate and realise. Contrary to the above methods, black-box and some non-parametric modelling techniques are gaining considerable interest among researchers due to their simplicity.[12] These methods avoid aerodynamics/kinematics of the system and can derive models based on data collected at several input-output terminals. The success of such approaches depends on pre-experimentation techniques and the estimation or prediction technique, commonly known as algorithm.

Alam and Tokhi[2] presented dynamic modelling techniques for 1-DOF and 2-DOF motions of the TRMS using particle swarm optimisation algorithms. Adaptive filters have been used to model the TRMS where adaptive algorithms such as, least mean square (LMS), normalised LMS and recursive least square (RLS) were used to update the filter taps.[4-5] Adaptive algorithms such as LMS, NLMS, RLS and their variants require relatively fewer computations and are favoured for on-line applications despite the risk of getting trapped at local minima[13]. Genetic algorithms (GAs) are global stochastic search algorithms based on natural biological evolutions. Since their introduction by Holland[14] there has been widespread interest among scientists and engineers in the use of GAs in system modelling and controller design[15-16]. Although a large volume of work has been reported in recent years, little work has been reported in identifying rotary wing systems using GAs. Assuming only input–output measurements of unknown dynamic systems are available; this paper presents a practical modelling technique where GAs are employed to extract parametric model of the TRMS in hovering mode, essentially to determine the vibration modes of the system.

II. EXPERIMENTAL SET-UP

The experimental TRMS and its schematic diagram are shown in Figures 1 and 2 respectively. The TRMS rig consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal and vertical directions producing yaw and pitch movements, respectively. At both ends of the beam there are two rotors driven by two DC motors. The main rotor produces a lifting force allowing the beam to rise vertically (Pitch angle/movement), while, the tail rotor is used to make the beam turn left or right (Yaw angle/ movement). The measured signals are: position of the beam, that is, two position angles, and angular velocities of the rotors.

Figure 1: The twin rotor MIMO system.

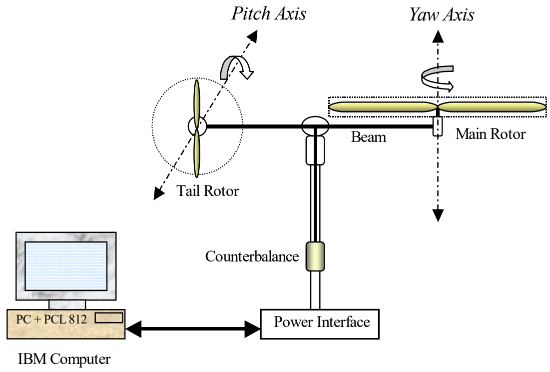


Figure 2: Schematic diagram of TRMS.

The control inputs are the supply voltages of the DC motors. A change in the voltage value results in a change in the rotational speed of the propeller, which in turn results in a change in the corresponding position of the beam. The system is interfaced with a personal computer through a data acquisition board, PCL-812PG[1]. The TRMS can be perceived as an unconventional and complex "air vehicle" with a flexible main body.

III. GENETIC ALGORITHMS

GA as a stochastic optimization algorithm is motivated by the mechanisms of natural selection and evolutionary genetics.[14], [17] The basic element processed by a GA is a string formed by concatenating sub-strings, each of which is a numeric coding of a parameter. Each string represents a point in the search space. Selection, crossover and mutation are the main operators of a GA. Selection directs the search of GA towards the best individual. In the process, strings with high fitness receive multiple copies in the next generation while strings with low fitness receive fewer copies or even none at all. Crossover can cause to exchange the properties of any two chromosomes via random decision in the mating pool and provides a mechanism to produce and match the desirable qualities through crossover. Although selection and crossover provide most of the power skills, the area of the solution will be limited. Mutation is a random alternation of a bit in the string and assists in keeping delivery in the population.[14], [17]

IV. DYNAMIC MODELLING OF TRMS WITH GA

System modelling is about building mathematical models for describing an observed system. In control engineering, system modelling is employed to determine a model of the system (plant). The process consists of two subtasks; 1) structural identification of the equations in the model M, and 2) parameter identification of the model's parameters $\hat{\theta}$. The modelling problem can be formulated as an optimisation task where the objective is to find a model and a set of parameters that minimise the prediction error between system output $y(t)$, i.e., the measured data, and the model output $\hat{y}(t,\hat{\theta})$ at each time step t. The basic schematic diagram for modelling is shown in Figure 3. The objective of a modelling experiment is to estimate a linear time-invariant (LTI) model of the 1-DOF model of the vertical subsystem of TRMS. The process involves several steps which are discussed below.

Preliminary Experimentation

The system was excited with a Gaussian random signal (GRS) in order to ensure that all system resonance modes are captured. The GRS signal level, ±0.2 volts, was selected so that it does not drive the TRMS out of its linear operating range. An input data of 8000 points and the corresponding system response thus recorded are shown in Figure 4. Out of 8000 data points the first 4000 were used for modelling and the rest 4000 for validating the model. Good excitation was achieved from 0-10 Hz that covers all the important rigid body and flexible modes of the system.[2-5]

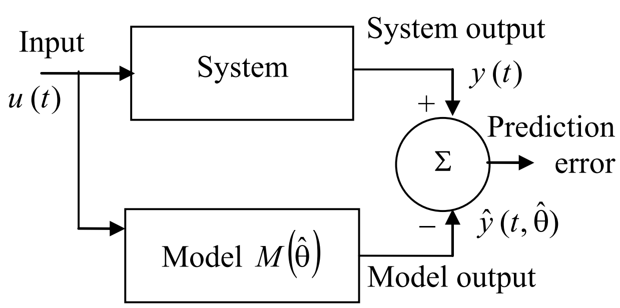


Figure 3: Basic schematic diagram for system identification

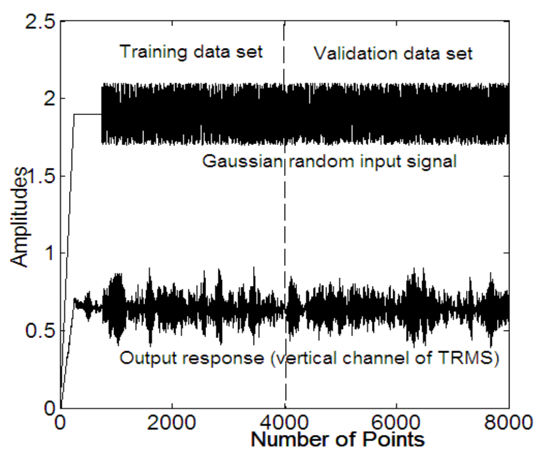


Figure 4: Gaussian random input signal and response of vertical channel of the TRMS in time domain

Structure formulation

Different types of model structures such as, auto regressive (AR), autoregressive exogenous (ARX), autoregressive moving average (ARMA) and autoregressive moving average with exogenous input (ARMAX) are reported in the literature for modelling.[12] Considering the simplicity, performance and computation costs, the ARMA structure was chosen. This is expressed as[12]:

$$\hat{y}(k) = -\sum_{i=1}^{N} a_i \times y(k-i) + \sum_{j=0}^{M} b_j \times u(k-j) + \eta(k) \tag{1}$$

where a_i, b_j are denominator and numerator polynomial coefficients, N and M are number of co-efficients in the denominator and numerator polynomials, y, u, $\hat{y}$, and η are measured output, input, predicted output and noise respectively. The order of the transfer function depends on N. Taking the values of N and M as 4 and 3 and neglecting η, equation (1) can be simplified as:

$$\hat{y}(k) = -a_1 y(k-1) - ... - a_4 y(k-4) + b_0 u(k-1) + ... + b_3 u(k-4) \tag{2}$$

In matrix form, the above equation can be written as:

$$\begin{aligned} \hat{y}(k) &= -[a_1, a_2, a_3, a_4][y(k-1), y(k-2), y(k-3), y(k-4)]^T \\ &+ [b_0, b_1, b_2, b_3][u(k-1), u(k-2), u(k-3), u(k-4)]^T \end{aligned} \tag{3}$$

Parameter encoding and GA optimisation process

The GA optimisation process begins with a randomly generated population called chromosome. An initial population of dimension 80×8×16 is created where number of individuals and parameters in each individual are 80 and 8 respectively. Each parameter is encoded as 16 bit Gray code which is logarithmically mapped into real number within a range of [-1, 1]. Each individual represents a solution where the first four elements are assigned to $b_0,...,b_3$ and the next four to $a_1,...,a_4$ as indicated in equation (3). The predicted output $\hat{y}$, at any sample instant, is calculated by putting actual input and output data in equation (3). Subsequent predicted outputs are calculated in a similar way with the same parameters while taking consecutive input and output data. The difference between the predicted and actual output is recorded as error, $e(k) = y(k) - \hat{y}(k)$, which in turn is used to form the objective function (*f(x)*). Here, sum of squared error is chosen as the objective function. This is given as:

$$f(x)=\sum_{i=1}^{n}(y(n)-\hat{y}(n))^2 \tag{4}$$

where $n=4000$. The stability of the derived model is the most basic objective to be satisfied. In case of a discrete-time transfer function, the maximum pole magnitude provides a simple and effective means of assessing the stability. In the modelling problem, stability is set as a constraint in the GA optimisation process. To extract a stable model, prior to calculating $\hat{y}$, the transfer function (discrete) is formed taking elements of each individual, then poles are calculated and if any pole falls outside the unit circle ($z-$plane) then that individual is penalised by adding a big number (penalty value) with its objective function, *f(x)*. This penalty value will reduce the probability of the individual being selected as parent for reproduction and on the contrary, favour stable solutions to be selected as parents and subsequently can guide the searching process towards regions of stable-solution.

After evaluating all individuals in the objective domain, fit individuals are selected based on stochastic universal sampling selection technique to form the mating pool.[18] The number of individuals in the mating pool depends on 'generation gap', for example, if generation gap is set at 90% and the number of individuals in the population is set as 80, then 80% of the total individuals, i.e., 72 are selected for mating. Genetic operators such as crossover, mutation and reinsertion are applied to form the new population for the next generation.[17] The crossover rate and mutation rate are set as 0.8 and 0.0062 respectively. Matlab[18] coding and genetic algorithms toolbox[19] have been used to implement the modelling technique and the optimisation process. The maximum number of generations was set to 200. The algorithm achieved the best sum-squared error level of 0.0014429 in the 200th generation. Figure 5 shows the predicted error between the model output and the actual output of the system at the end of maximum generation whereas the algorithm convergence is shown in Figure 6. It is evident from the convergence curve that the objective function gradually reduces as the algorithm proceeds and the rate of convergence is high at the beginning.

V. RESULTS: MODEL FORMULATION AND VALIDATION

Eight parameter values: $b_0,\ldots,b_3$ and $a_1,\ldots,a_4$ are obtained at the end of maximum generation of the GA process. Using these values in equation (3), the discrete transfer function for vertical channel at a sampling time of 0.1sec thus formed is given as:

$$\frac{y(z)}{u(z)}=\frac{-0.0005001z^3+0.005997z^2+0.0113z+0.0142}{z^4-0.8777z^3-0.3962z^2-0.3153z+0.7069} \tag{5}$$

Using Matlab[18] functions, this discrete transfer function can be converted into continuous form (s-domain) as:

$$\frac{y(s)}{u(s)} = \frac{-0.08927s^3 + 2.249s^2 - 45.57s + 595.1}{s^4 + 3.469s^3 + 519.6s^2 + 35.95s + 2189} \tag{6}$$

where u(s) represents the main rotor input (volt) and y(s) represents pitch angle (radians). The derived model was validated with a separate input-output data set and the actual and one-step-ahead predicted outputs are shown in Figure 7. Time domain tracking reveals that the predicted output follows the actual output very well.

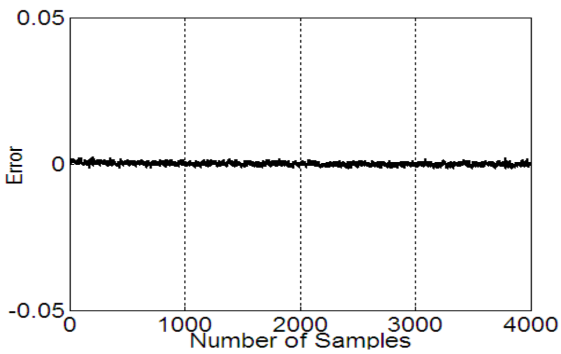


Figure 5: Error between actual and predicted output

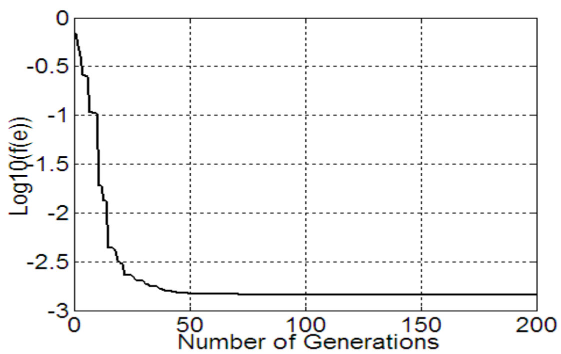
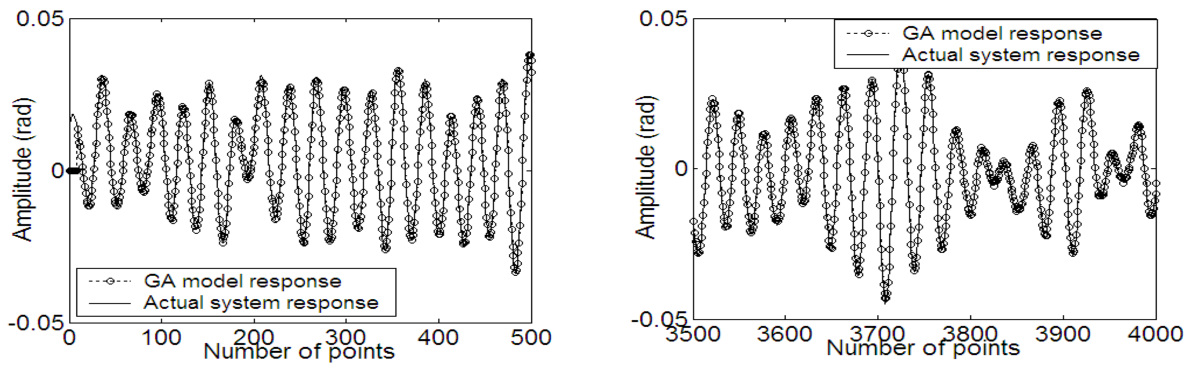


Figure 6: Convergence of GA optimisation process



(a) First 500 samples

(b) Last 500 samples

Figure 7: Actual output and GA predicted output of vertical channel of the TRMS

The frequency domain plot (Figure 8) of the predicted and actual outputs indicates that the model has successfully captured the system dynamics, especially the main dominant modes at the low frequency region. The main vibration mode, as found from the GA simulated output was at 0.3515 Hz. The stability of the model was monitored using its pole-zero diagram. It was observed that all the poles remained inside the unit circle (Figure 9). Although, there were some zeros outside the unit circle which indicates the system has non-minimum phase behaviour.

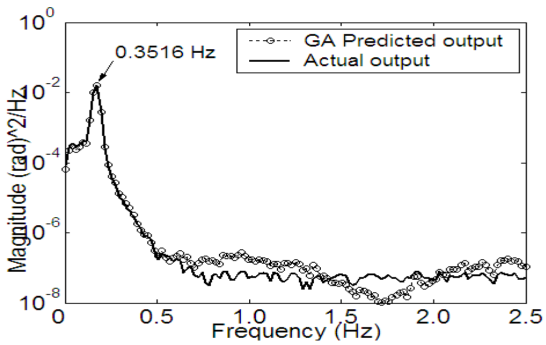


Figure 8: Power spectral density of the actual and predicted outputs

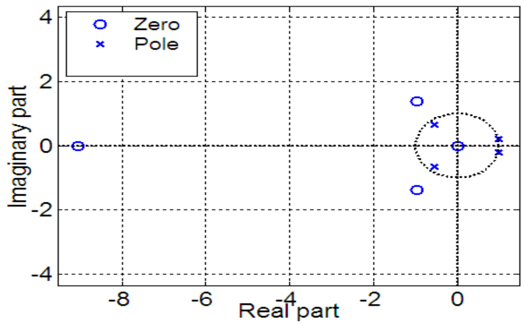


Figure 9: Pole-zero diagram

Correlation tests

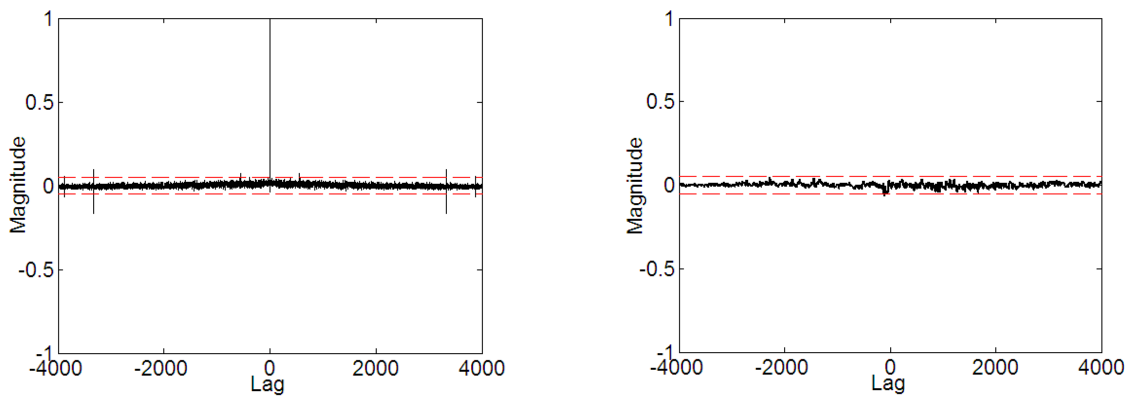
If the model is adequate, the residuals, $\varepsilon(t)$, should contain no information about the past residuals or dynamics of the system. It can be verified by showing that the residuals are uncorrelated with all linear and nonlinear combinations of past inputs and outputs. Billings and his co-workers have recommended a number of auto-correlation and cross-correlation tests expressed as[20-21]:

$$
\begin{aligned}
\phi_{\varepsilon\varepsilon}(\tau) &= E[\varepsilon(t-\tau)\varepsilon(t)] = \delta(\tau) \\
\phi_{u\varepsilon}(\tau) &= E[u(t-\tau)\varepsilon(t)] = 0 \qquad \forall\tau \\
\phi_{u^2\varepsilon}(\tau) &= E[(u^2(t-\tau)-\overline{u}^2(t))\varepsilon(t)] = 0 \qquad \forall\tau \\
\phi_{u^2\varepsilon^2}(\tau) &= E[(u^2(t-\tau)-\overline{u}^2(t))\varepsilon^2(t)] = 0 \qquad \forall\tau \\
\phi_{\varepsilon(\varepsilon u)}(\tau) &= E[\varepsilon(t)\varepsilon(t-1-\tau)u(t-1-\tau)] = 0 \qquad \tau \geq 0
\end{aligned}
\tag{7}
$$

where $\phi_{u\varepsilon}(\tau)$ indicates the cross-correlation function between $u(t)$ and $\varepsilon(t)$, $\varepsilon u(t) = \varepsilon(t+1)u(t+1)$, $\delta(\tau) =$ an impulse function. In practice, normalised correlations are computed. The correlations will never be exactly zero for all lags and the 95% confidence bands defined as $1.96/\sqrt{N}$ are used to indicate if the estimated correlations are significant or not, where N is the data length. However, for linear time invariant models, the first two correlation tests of equation (7) are applicable[20] which are shown in Figure 10. It is noted that the first two correlation functions are within the 95% confidence bands indicating that the model is adequate.

VI. CONCLUSION

This investigation has witnessed the development of dynamic modelling of the TRMS in hovering mode. The success of this approach depends on two factors: 1) suitable selection of model structure and 2) GA with its global search ability so as to estimate the parameters of the model accurately. The model structure specifies the order and form of the differential equation(s) which describe system dynamics. GA has been successfully used to derive a model for vertical channel of the TRMS. The modelling problem was formulated as a multi-modal minimisation problem with 8-dimensional search space. It is evident from the results that the GA can extract stable and satisfactory model for vertical channel of the TRMS which is a good approximation of the nonlinear sub-system in the vicinity of the operating point, mainly, to capture the dominant modes of the corresponding channel. Both time and frequency domain analyses were utilised to investigate and establish confidence in the obtained model. All results have clearly revealed that the effectiveness of the modelling approach and the GA optimisation in characterising dynamic systems.



(a) Auto-correlation of residuals *(b) Cross-correlation of input and residuals*

Figure 10: Correlation tests

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