Three-way decision of target threat decision making based on adaptive threshold algorithms

Bo Li[1,2] ✉, Linyu Tian[1], Yue Han[1], Daqing Chen[3]

[1]*School of Electronics and Information, Northwestern Polytechnical University, Shanxi, Xi'an, People's Republic of China*
[2]*CETC Key Laboratory of Data Link Technology, Shanxi, Xi'an, People's Republic of China*
[3]*School of Engineering, London South Bank University, London, UK*
✉ *E-mail: libo803@nwpu.edu.cn*

Abstract: In order to solve the problems of existing threat assessment algorithms, a threat assessment method based on intuitionistic fuzzy three-way decision method is proposed. Firstly, the indexes are described by theory of intuitionistic fuzzy sets on the basis of that the threat assessment index system is established. Then, TOPSIS method is used to calculate the target threat. Finally, the targets are classified by three-way decision method according to the result of target threat. It is proved that it is feasible to introduce three-way decision method into threat assessment. At the same time, an improved algorithm for calculating the threshold of three-way decision is proposed. Experiments show that the improved algorithm has stable results and good classification effect.

1 Introduction

At present, the research on threat assessment of air targets is mostly based on the selection and treatment of threat factors. While for the research of the results of threat assessment, two-way decision models are always used to make a choice. Decision makers need to make decisions immediately based on the current information. For targets whose threat value is higher than a certain threshold, they should adopt attack strategy, and for targets whose threat value is lower than that threshold, they should adopt the strategy of abandoning attack. This kind of decision-making result is either one or the other, so a two-way decision model may lead to a high-cost loss due to the lack of suitable information for making a precise decision.

Three-way decision is an uncertain theory put forward by Yao Yiyu in recent years [1]. Its core idea is to divide a unified set into three disjoint paired regions: positive domain, negative domain, and boundary domain. It provides an effective strategy and method for solving complex problems by formulating corresponding decision-making strategies for each region. After the emergence of the three-way decision, experts and scholars at home and abroad have conducted extensive research on it. The research on the three-way decision by combining intuitionistic fuzzy set and rough set is a hot topic at present. Therefore, this paper intends to use the three-way decision method based on intuitionistic fuzzy sets to study the air combat threat assessment and the threat assessment from a new perspective, hoping to provide new ideas and ways to solve the problems in this field.

2 Basic concepts

2.1 Three-way decision model based on intuitionistic fuzzy set

Let the set of event objects be U and x be any event object in U, satisfying rule $x \in U$. There are two kinds of decision state descriptions: membership description P and non-membership description N. The decision-maker can evaluate the event object according to the evaluation function composed of membership function and non-membership function, and determine the cost of different decisions according to the cost matrix of decision-making problem, as shown in Table 1.

Let the evaluation function be $F(\omega_x|x)$ and ω_x denote the decision-making action of event object x, and ω_P, ω_N, ω_B denote the decision-making action of event object when it is divided into positive domain, negative domain, and boundary domain. Positive field indicates that object x belongs to L, that is, accepting event object. Negative field denotes that object x does not belong to L, that is, reject event object. The boundary region indicates whether the uncertain object x belongs to L, i.e. the object of non-commitment or delayed decision-making events. $\mu(x)$ and $\upsilon(x)$ represent the membership degree and non-membership degree of event object x, respectively. If event object x belongs to L, the cost of evaluation function for executing decision-making action ω_P, ω_N, ω_B is λ_{PP}, λ_{NP}, λ_{BP}, respectively. If event object x does not belong to L, then the cost of evaluation function for executing decision-making action ω_P, ω_N, ω_B is λ_{PN}, λ_{NN}, λ_{BN}. Considering the significance of evaluation function in reality, it is reasonable to assume that the cost of evaluation function above satisfies two conditions: $\lambda_{PP} \leq \lambda_{BP} < \lambda_{NP}$, $\lambda_{NN} \leq \lambda_{BN} < \lambda_{PN}$.

According to the concept of intuitionistic fuzzy set, there are three kinds of decision-making actions in Table 1: $0 \leq \mu(x) + \upsilon(x) \leq 1$. The corresponding decision-making risk assessment is obtained, in which the evaluation function $F(\omega_x|x)$ is defined as the mathematical expectation of the risk of decision-making action ω_x.

The risk of accepting the object of the event:

$$F(P|x) = \lambda_{PP} \times \mu(x) + \lambda_{PN} \times \upsilon(x) \quad (1)$$

Table 1 Cost matrix for decision problems

Decision-making action	Membership description	Non-membership description
Acceptance decision:ω_P	λ_{PP}	λ_{PN}
Rejection decision:ω_N	λ_{NP}	λ_{NN}
Delayed decision:ω_B	λ_{BP}	λ_{BN}

The risk of rejecting the object of the event:

$$F(N|x) = \lambda_{\mathrm{NP}} \times \mu(x) + \lambda_{\mathrm{NN}} \times \upsilon(x) \qquad (2)$$

Risks of delayed decision-making event objects:

$$F(B|x) = \lambda_{\mathrm{BP}} \times \mu(x) + \lambda_{\mathrm{BN}} \times \upsilon(x) \qquad (3)$$

Decision-makers always choose the decision-making action with the least risk when evaluating the decision-making risk of the event objects. Therefore, three decision-making rules are obtained:

Acceptance of decision rules(P):if $F(P|x) \leq F(N|x)$ and $F(P|x) \leq F(B|x)$ are true, then there is $x \in \mathrm{POS}(X)$;
Rejection of Decision Rules(N):if $F(N|x) \leq F(P|x)$ and $F(N|x) \leq F(B|x)$ are true, then there is $x \in \mathrm{NEG}(X)$;
Delayed Decision Rules(B):if $F(B|x) \leq F(P|x)$ and $F(B|x) \leq F(N|x)$ are true, then there is $x \in \mathrm{BND}(X)$.

If

$$\alpha = \frac{(\lambda_{\mathrm{PN}} - \lambda_{\mathrm{BN}})}{(\lambda_{\mathrm{PN}} - \lambda_{\mathrm{BN}}) + (\lambda_{\mathrm{BP}} - \lambda_{\mathrm{PP}})}$$

$$\beta = \frac{(\lambda_{\mathrm{BN}} - \lambda_{\mathrm{NN}})}{(\lambda_{\mathrm{BN}} - \lambda_{\mathrm{NN}}) + (\lambda_{\mathrm{NP}} - \lambda_{\mathrm{BP}})} \qquad (4)$$

$$\gamma = \frac{(\lambda_{\mathrm{PN}} - \lambda_{\mathrm{NN}})}{(\lambda_{\mathrm{PN}} - \lambda_{\mathrm{NN}}) + (\lambda_{\mathrm{NP}} - \lambda_{\mathrm{PP}})}$$

By assuming $\lambda_{\mathrm{PP}} \leq \lambda_{\mathrm{BP}} < \lambda_{\mathrm{NP}}$ and $\lambda_{\mathrm{NN}} \leq \lambda_{\mathrm{BN}} < \lambda_{\mathrm{PN}}$, $\alpha \in (0, 1]$, $\beta \in [0, 1)$, $\gamma \in (0, 1)$ can be obtained, and $0 \leq \beta < \gamma < \alpha \leq 1$ can be obtained. Thus, decision rules can be simplified as follows:
From hypothesis a, b, c, D can be obtained, and E can be obtained. Thus, decision rules can be simplified as follows:

Acceptance of decision rules (P):if $\mu(x) \geq \alpha$ is true, there is $x \in \mathrm{POS}(X)$;
Rejection of decision rules (N):if $\mu(x) \leq \gamma$ is true, there is $x \in \mathrm{NEG}(X)$;
Delayed decision rules (B):if $\beta < \mu(x) < \alpha$ is true, there is $x \in \mathrm{BND}(X)$.

2.2 Improved adaptive threshold algorithm

According to Bayesian Minimum Risk Decision-making Theory, every decision-making behaviour will have corresponding risk loss [2]. Choosing the behaviour with less risk to make decision can calculate the threshold pair of decision-making boundary (α, β). Aiming at the optimisation of decision-making risk loss, an optimisation problem related to risk loss is constructed, and an improved adaptive learning loss function value and threshold algorithm is proposed. The conditional probability of each sample is taken as the search space, and the appropriate probability value is found as the decision boundary threshold to minimise the decision risk based on the threshold.
Decision Rough Set chooses two state sets $\Omega = \{X, X\}$ and three action sets $A = \{a_P, a_B, a_N\}$ to describe the decision problem. The probability values of x_i belonging to X for each object are calculated by various theories and methods, marked as p_i. According to the three-way decision rough set model, normal rules $p_i \geq \alpha$ are used for objects x_i, negative rules are used for objects x_j satisfying condition $p_j \leq \beta$, and boundary rules are used for objects p_k of $\beta < p_k < \alpha$. Assuming $\lambda_{\mathrm{PP}} = \lambda_{\mathrm{NN}} = 0$, the total risk loss caused by partitioning each object of the whole decision table is as follows:

$$R = \sum_{x_i \in \mathrm{POS}(X)} \lambda_{\mathrm{PN}}(1 - p_i) + \sum_{x_j \in \mathrm{NEG}(X)} \lambda_{\mathrm{NP}} p_j$$
$$+ \sum_{x_k \in \mathrm{BND}(X)} [\lambda_{\mathrm{BN}}(1 - p_k) + \lambda_{\mathrm{BP}} p_k] \qquad (5)$$

According to Bayesian decision-making theory, the smaller the total risk loss is, the better. The problem of optimizing decision-making risk loss is as follows: (see (6)) Among them, ε is a penalty factor to avoid dividing too many samples into boundary areas.
The three thresholds (α, β, γ) are calculated from six loss function values. Assuming $\lambda_{\mathrm{PP}} = \lambda_{\mathrm{NN}} = 0$, the remaining four loss function values can be deduced from formula (4) in reverse, expressed by thresholds (α, β, γ) and λ_{PN} as follows:

$$\lambda_{\mathrm{NP}} = \frac{1 - \gamma}{\gamma} \lambda_{\mathrm{PN}}$$

$$\lambda_{\mathrm{BN}} = \frac{\beta(\alpha - \gamma)}{\gamma(\alpha - \beta)} \lambda_{\mathrm{PN}} \qquad (7)$$

$$\lambda_{\mathrm{BP}} = \frac{(1 - \alpha)(\gamma - \beta)}{\gamma(\alpha - \beta)} \lambda_{\mathrm{PN}}$$

Assuming $\lambda_{\mathrm{PN}} = 1$, the final optimization problem can be expressed as:

$$\min \sum_{p_i \geq \alpha} (1 - p_i) + \sum_{p_j \leq \beta} \frac{1 - \gamma}{\gamma} p_j$$
$$+ \varepsilon \sum_{\beta < p_k < \alpha} \left[\frac{\beta(\alpha - \gamma)}{\gamma(\alpha - \beta)}(1 - p_k) + \frac{(1 - \alpha)(\gamma - \beta)}{\gamma(\alpha - \beta)} p_k \right] \qquad (8)$$
$$s.t. \quad 0 \leq \beta < \gamma < \alpha \leq 1, \varepsilon \geq 1$$

When solving the threshold, the search space is limited to the set of probability values of all objects. The concrete idea is as follows:

(1) Firstly, the initial threshold value is set, and the total loss R_X is calculated and recorded as Min_R.
(2) All objects are sorted according to the size of the probability value, the big one is before the small one.
(3) The threshold (α, β, γ) is replaced by the probability value of the target object. In order, all threshold combinations satisfying condition $\alpha > \gamma > \beta$ are taken out in turn, and the thresholds are re-assigned to $(\alpha', \beta', \gamma')$.
(4) Recalculate the total risk loss R_X' of the sample set under the new threshold. If condition $R_{X'}' < \mathrm{Min}_R$ is satisfied, the threshold (α, β, γ) is updated to $(\alpha', \beta', \gamma')$, otherwise the threshold remains unchanged.
(5) Determine whether all possible threshold combinations satisfying the $\alpha > \gamma > \beta$ condition have been traversed completely. If so, execute step (6), otherwise, go to step (3) until all combinations have been traversed.
(6) The final threshold (α, β, γ) is the required result.

The flow chart is shown in Fig. 1.

3 Threat assessment index system

3.1 Construction of target threat assessment index system

(1) Air combat capability factor
Air combat capability mainly includes target type and target jamming ability [3]. The corresponding relationship between target air combat capability and intuitionistic fuzzy set is shown in Table 2.
(2) Target Situation Information

$$\min \sum_{p_i \geq \alpha} \lambda_{\mathrm{PN}}(1 - p_i) + \sum_{p_j \leq \beta} \lambda_{\mathrm{NP}} p_j + \varepsilon \sum_{\beta < p_k < \alpha} [\lambda_{\mathrm{BN}}(1 - p_k) + \lambda_{\mathrm{BP}} p_k]$$
$$s.t. \quad 0 \leq \beta < \gamma < \alpha \leq 1, \varepsilon \geq 1 \qquad (6)$$
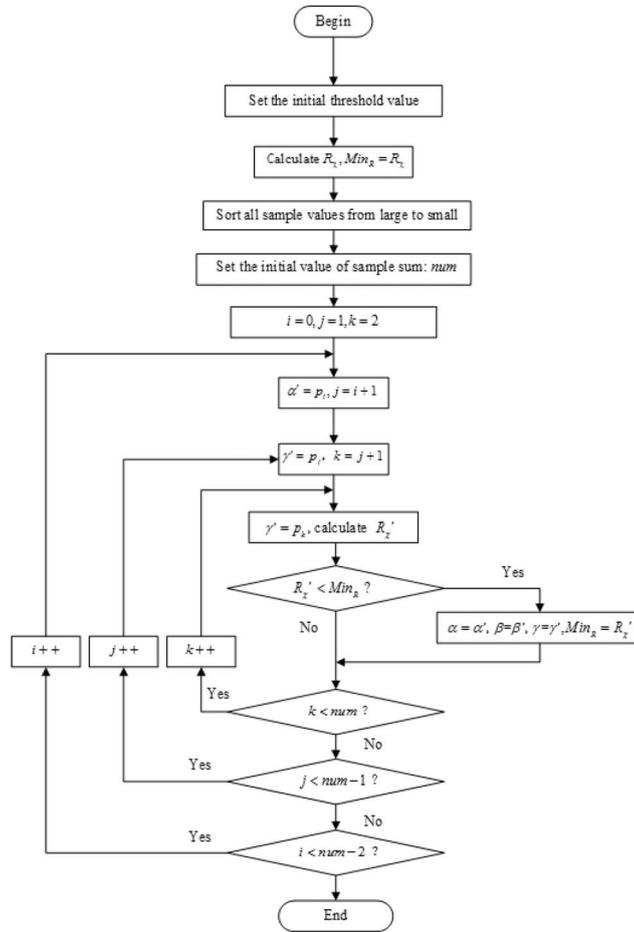
Fig. 1 *Flow chart of improved adaptive threshold algorithm for three-way decision*

Table 2 Relationship between target air combat capability and intuitionistic fuzzy sets

Fuzzy language	Membership degree	Non-membership degree
high	0.90	0.05
higher	0.75	0.10
normal	0.55	0.25
lower	0.50	0.25
low	0.50	0.20

Target situation information includes the target speed, the angle between the target and the speed direction of the aircraft, and the distance between the target and the aircraft. It can be expressed by interval index. Interval-type indicators are divided into benefit-type indicators and cost-type indicators. For benefit-based indicators, there is:

$$[b_{ij}^L, b_{ij}^U] = \left[\frac{a_{ij}^L}{\max_{1 \le k \le m} (a_{kj}^U)}, \frac{a_{ij}^U}{\max_{1 \le k \le m} (a_{kj}^U)} \right] \quad (9)$$

Similarly, for cost-based indicators, there is:

$$[b_{ij}^L, b_{ij}^U] = \left[\frac{\max_{1 \le k \le m} (a_{kj}^L)}{a_{ij}^U}, \frac{\max_{1 \le k \le m} (a_{kj}^L)}{a_{ij}^L} \right] \quad (10)$$

Normalized interval numbers are transformed into intuitionistic fuzzy numbers, whose membership degree and non-membership degree are defined as

$$\begin{cases} \mu_{ij} = \gamma \dfrac{b_{ij}^U}{\max_{1 \le k \le m} (b_{kj}^U)} \\ v_{ij} = 1 - \mu_{ij} - \dfrac{b_{ij}^U - b_{ij}^L}{b_{ij}^U + b_{ij}^L} \end{cases} \quad (11)$$

Among them, $\gamma \in [0.5, 1]$ is the optimistic index.

Target speed belongs to benefit interval index, and its membership degree and non-membership degree are calculated by reference formula (9). The target distance and the angle between the target and the aircraft belong to the cost-type interval index, and its membership degree and non-membership degree are calculated by reference formula (10).

3.2 Target attribute weight

Definition 6 Let the universe of intuitionistic fuzzy sets be $X = \{x_1, x_2, \ldots, x_n\}$, $E(A)$ is Intuitional Fuzzy Entropy of A, which is called Information Entropy for short. $E(A)$ is determined by Formula (12):

$$E(A) = -\frac{1}{n} \sum_{i=1}^{n} \left\{ \begin{array}{l} \left[\mu_A(x_i) + \dfrac{\pi_A(x_i)}{2} \right] \log_2 \left[\mu_A(x_i) + \dfrac{\pi_A(x_i)}{2} \right] \\ + \left[v_A(x_i) + \dfrac{\pi_A(x_i)}{2} \right] \log_2 \left[v_A(x_i) + \dfrac{\pi_A(x_i)}{2} \right] \end{array} \right\} \quad (12)$$

The corresponding weight of attribute C_j is:

$$\omega_j = \frac{1 - E_j}{n - \sum_{j=1}^{n} E_j} \quad (13)$$

Table 3 Target information matrix

Target	Type	Jamming ability	Speed, m/s	Distance, km	Angle, degree
T1	high	high	[890, 910]	[99, 100]	[0.9, 1.1]
T2	high	higher	[410, 430]	[299, 300]	[5.9, 6.1]
…	…	…	…	…	…
T20	lower	higher	[30, 50]	[119, 120]	[17.9, 18.1]

Table 4 Target attribute weight

	Type	Jamming ability	Speed	Distance	Angle
Weight	0.12434	0.21468	0.30383	0.26595	0.09121

Table 5 Target weighted intuitionistic fuzzy matrix of targets

	Type	Jamming ability	Speed	Distance	Angle
T1	⟨0.249,0.689⟩	⟨0.390,0.526⟩	⟨0.244,0.750⟩	⟨0.159,0.840⟩	⟨0.137,0.811⟩
T2	⟨0.249,0.689⟩	⟨0.257,0.610⟩	⟨0.097,0.894⟩	⟨0.045,0.955⟩	⟨0.012,0.986⟩
…	…	…	…	…	…
T20	⟨0.083,0.842⟩	⟨0.257,0.610⟩	⟨0.010,0.904⟩	⟨0.127,0.873⟩	⟨0.004,0.996⟩

The corresponding attribute weight of the target T_i is:

$$\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_n) \qquad (14)$$

4 Target threat assessment algorithms

Based on the operation rules of intuitionistic fuzzy sets, the weights of target attributes are calculated, and the threat assessment of targets is ranked according to the idea of TOPSIS method [4]. The steps of the algorithm are as follows [5, 6]:

(1) Determine the target attribute matrix according to the threat assessment index system. $\boldsymbol{H} = (h_{ij})_{m \times n}$.
(2) The target attribute matrix is processed to get the target intuitionistic fuzzy matrix $\boldsymbol{F} = (f_{ij})_{m \times n}$, $f_{ij} = \langle \mu_{ij}, v_{ij} \rangle$.
(3) According to the intuitionistic fuzzy entropy, the weight of target attributes $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is determined.
(4) The weighted intuitionistic fuzzy decision matrix $\boldsymbol{R} = (r_{ij})_{m \times n}$ is determined by the target attribute weight ω and the target intuitionistic fuzzy matrix $\boldsymbol{F}$, and $r_{ij} = \langle \alpha_{ij}, \beta_{ij} \rangle$.
(5) Calculate the ideal and negative ideal solutions of the weighted intuitionistic fuzzy decision matrix.

$$R^+ = (\langle \alpha_1^+, \beta_1^+ \rangle, \langle \alpha_2^+, \beta_2^+ \rangle, \ldots, \langle \alpha_n^+, \beta_n^+ \rangle) \qquad (15)$$

$$R^- = (\langle \alpha_1^-, \beta_1^- \rangle, \langle \alpha_2^-, \beta_2^- \rangle, \ldots, \langle \alpha_n^-, \beta_n^- \rangle) \qquad (16)$$

$\alpha_j^+ = \max_{1 \le i \le m} \alpha_{ij}$, $\qquad \beta_j^+ = \min_{1 \le i \le m} \beta_{ij}$,
$\alpha_j^- = \min_{1 \le i \le m} \alpha_{ij}$, $\beta_j^- = \max_{1 \le i \le m} \beta_{ij}$, $j = 1, 2, \ldots, n$
(6) Calculate the distance from each object to the ideal solution and the negative ideal solution

Taking $R_i = (\langle \alpha_{i1}, \beta_{i1} \rangle, \langle \alpha_{i2}, \beta_{i2} \rangle, \ldots, \langle \alpha_{in}, \beta_{in} \rangle)$ as the vector representing the i-th objective, the formulas for calculating the distance S_i^+ from the target i to the ideal solution and the distance S_i^- from the target i to the negative ideal solution are as follows [7, 8]:

$$S_i^+ = d_2(R_i, R^+)$$
$$= \sqrt{\frac{1}{2n} \sum_{j=1}^{n} [(\alpha_{ij} - \alpha_j^+)^2 + (\beta_{ij} - \beta_j^+)^2 + (\pi_{ij} - \pi_j^+)^2]} \qquad (17)$$

$$S_i^- = d_2(R_i, R^-)$$
$$= \sqrt{\frac{1}{2n} \sum_{j=1}^{n} [(\alpha_{ij} - \alpha_j^-)^2 + (\beta_{ij} - \beta_j^-)^2 + (\pi_{ij} - \pi_j^-)^2]} \qquad (18)$$

(7) Computing the relative closeness of each scheme

$$W_i = \frac{S_i^-}{S_i^- + S_i^+} \qquad (19)$$

(8) Solution of Threshold Pairs
Taking target threat degree as input, the threshold pair of threat assessment problem is calculated according to the improved adaptive threshold algorithm proposed above.
(9) Dividing the threat area of the target according to the threshold value.

For the target whose threat degree is greater than threshold α, it is divided into threatening areas; for the target whose threat degree is $< \beta$, it is divided into non-threatening areas; and for the target whose threat degree is between α and β, it is divided into potential threatening areas.

5 Simulation analysis

Twenty targets are simulated, and the target information is shown in Table 3 (for space reasons, only part of the data is shown).
The intuitionistic fuzzy decision matrix of each target attribute is determined according to step (2). Determine the weight of the target attribute according to step (3), as shown in Table 4.
The objective weighted intuitionistic fuzzy decision matrix is determined according to step (4), as shown in Table 5.
The positive and negative ideal solution targets are calculated, and the distance between each objective and the positive ideal solution is calculated. According to step (7), the relative pasting progress of each objective and positive ideal solution is calculated, as shown in Table 6.
According to step (8), the threshold of the three decision-making branches is calculated, and the initial threshold value is set to $\alpha = 0.7$, $\beta = 0.3$, $\gamma = 0.4$. The existing algorithm and the improved self-adaptive algorithm are used to calculate the decision threshold. The results of the threshold are recorded in Tables 7 and 8. According to the results of the threshold, the threat areas of the target are divided, and the results are recorded in Tables 9 and 10.
When the penalty factor in the boundary area increases from 1.1 to 1.15, the target in the potential threatened area changes from 19 to 0 and the target in the non-threatened area changes from 1 to 20. It shows two points: (i) With the increase (or decrease) of the penalty factor in the boundary region, there may be a sudden change in the result of target region division. (ii) Existing

Table 6 Target threat

Target	Threat	Target	Threat	Target	Threat	Target	Threat
T1	0.636	T6	0.495	T11	0.304	T16	0.210
T2	0.321	T7	0.406	T12	0.498	T17	0.311
T3	0.298	T8	0.275	T13	0.513	T18	0.490
T4	0.368	T9	0.605	T14	0.348	T19	0.334
T5	0.637	T10	0.318	T15	0.431	T20	0.255

algorithms may get extreme values. When the threat factor is 1.15–4, the threshold value, the difference between them is <0.001. At this time, the result of target region division is very unreliable, and almost no samples are divided into boundary regions.

Tables 8 and 10 show that when the penalty factor varies from 1.5 to 4, the threshold α decreases gradually with the increase of the penalty factor in the boundary domain, while the threshold β does not change significantly, and the threshold γ decreases gradually. With the increase of the penalty factor in the boundary area, the scope of the boundary area decreases gradually, and the number of targets in the potential threat area decreases gradually, and transforms into the threatened area and the non-threatened area.

Comparing the simulation results of the two algorithms, we find that there are some problems in the existing algorithms: the algorithm is unstable, with the change of penalty factor, the results of the algorithm have mutations, and it is difficult to determine the appropriate penalty factor value. The improved algorithm is simulated and analysed. The results show that the improved adaptive threshold algorithm proposed in this paper is stable, has good classification effect, and can overcome the shortcomings of the existing algorithm. When the penalty factor is within a certain range, the threshold solution results can maintain stability and ensure the reliability of the algorithm.

6 Conclusion

In order to solve the problems of existing threat assessment algorithms, this paper introduces three-way decision theory into the threat assessment of air combat decision-making and proposes a threat assessment method based on intuitionistic fuzzy three-way decision. The simulation proves the feasibility of introducing three-way decision theory into the threat assessment problem. Aiming at the shortcomings of the existing threshold solving algorithm, an improved adaptive threshold algorithm for three-way decision is proposed. The simulation proves that the threshold algorithm proposed in this paper is stable and has good classification effect. It can overcome the shortcomings of existing algorithms. When the penalty factor takes a certain range, the solution result of the threshold can be kept stable and the reliability of the algorithm is guaranteed.

7 Acknowledgements

Table 7 Threshold calculation results with different penalty factors under existing algorithms

Penalty value	α	γ	β
1.05–1.1	0.7	0.63735	0.20950
1.15–4	0.63799	0.63767	0.63735

Table 8 Threshold calculation results with different penalty factors under improved algorithm

Penalty value	α	γ	β
1.5–2.5	0.63664	0.63656	0.39858
3–3.25	0.63656	0.63336	0.39858
3.5–4	0.40616	0.40237	0.39858

Table 9 Three-way decision results with different penalty factor in existing algorithms

Penalty value	Threatened areas	Potential threat areas	Non-threat areas
1.05–1.1	0	19	1
1.15–4	0	0	20

Table 10 Three-way decision results with different penalty factors under improved algorithm

Penalty value	Threatened areas	Potential threat areas	Non-threat areas
1.5–2.5	1	8	11
3–3.25	2	7	11
3.5–4	9	0	11

8 References

[1] Yao, Y.Y.: 'Three-way decisions with probabilistic rough sets', *Inf. Sci.*, 2010, **180**, pp. 341–353

[2] Jia, X., Li, W., Shang, L., *et al.*: 'An adaptive learning parameters algorithm in three-way decision-theoretic rough set model', *Acta Electron. Sin.*, 2011, **39**, (11), pp. 2520–2525

[3] Zhang, A.-L., Li, Z., Zhang, Y., *et al.*: 'Research of early warning satellite threat assessment based on threat degree function', *J. Ordnance Equipment Eng.*, 2016, **37**, (1), pp. 66–69

[4] Fu, T., Wang, J.: 'Threat assessment of aerial targets in air-defense', *Command Control Simul.*, 2016, **38**, (3), pp. 63–69

[5] Zhang, H.-Y., Wang, D.-D., Gou, Q.: 'Method for attacking target assignment and priority sequencing based on threat assessment', *Command Inf. Syst. Technol.*, 2018, **9**, (6), pp. 55–60

[6] Chang, F., Zhang, X., Lin, Y.: 'Research and decision for multi-target threat sequencing in air defense combat', *J. Ordnance Equipment Eng.*, 2016, **37**, (2), pp. 159–161, 168

[7] Sun, Q., Li, Z., Chang, Y.: 'Multi-types airplane threat assessment based on combat power field', *Syst. Eng. Electron.*, 2018, **40**, (9), pp. 1993–1999

[8] Zhang, H., Xie, J., Ge, J., *et al.*: 'Intuitionistic fuzzy set threat assessment based on improved TOPSIS and multiple times fusion', *Syst. Eng. Electron.*, 2018, **40**, (10), pp. 2263–2269