Elsevier Editorial System(tm) for European Journal of Operational Research
Manuscript Draft

Manuscript Number: EJOR-D-15-00396R2

Title: Sustainable Management of Fossil Fuels: A Dynamic Stochastic Optimization Approach with Jump-Diffusion

Article Type: Innovative Application of OR

Section/Category: (D) OR in natural resources

Keywords: OR in natural resources; stochastic dynamic optimization; jump-diffusion; simulations; sustainability.

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Manuscript Region of Origin: ITALY

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EJOR−D−15−00396R1

Response to the Reviewer 2

We would like to thank the Referees for their insightful and valuable comments that contributed to improve the paper.

We have taken in full consideration the comment of the Reviewer, and checked carefully the paper. Several typos and misprints have been removed.

Sustainable Management of Fossil Fuels: A Dynamic Stochastic Optimization Approach with Jump-Diffusion

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RESEARCH HIGHLIGHTS

- We optimize stock of crude oil to be employed in R&D and consumption.
- We introduce sustainability: a stock of oil is spared for future generations.
- We develop a stochastic optimal control model in presence of jump diffusion.
- Solution is obtained theoretically (dynamic programming) and by using simulations.
- Outcome: an environmental sustainability policy is economically optimal.

Sustainable Management of Fossil Fuels: A Dynamic Stochastic Optimization Approach with Jump-Diffusion

Rosella Castellano* Roy Cerqueti† Luca Spinesi‡

April 21, 2016

Abstract

This paper deals with a relevant aspect of energy modeling, i.e. fossil fuels management. The issue is faced by using purely operational research techniques, which are suitable in this context. In particular, a dynamic stochastic optimization model is developed to optimally determine use and stock of resources to be employed in consumption and investments, in a wide economic sense: human and physical capital, R&D, etc. It is assumed that a sustainability criterion drives the optimality rules, i.e. decisions are also grounded on the well-being of future generations. The policymaker maximizes an aggregated intergenerational expected utility under the dilemma of present consumption/conservation of natural resources for the future. In reference to standard environmental economic theory, jump-diffusion dynamics for the stock of natural resources and infinite time horizon are assumed. Extensive numerical experiments complete the analysis and contribute to determine fossil fuels management policies, showing that long-term investments make the difference for the well-being of present and future generations.

Keywords: OR in natural resources, stochastic dynamic optimization, jump-diffusion, simulations, sustainability.

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1 Introduction

Can Operations Research contribute to formalize sustainable economies for the exploitation of natural resources? The answer is undoubtedly positive and, in this regard, it is worth to mention some supporting references. Higgins et al. (2008) develop a multi-objective integer programming model for investments, aiming at maximizing environmental benefits under budget constraints. Munda (2009) describes the concept of sustainability in the context of resource management and argues that economic reasonings cannot be the only routes to follow for taking decisions.

This paper moves from this problem and deals with the development of a stochastic dynamic optimization model for identifying the optimal consumption and stock of fossil fuels (mainly, crude oil, natural gas, coal) to be employed in investment, under sustainability assumptions. The responsible management of natural resources must account for time evolution and uncertainty, which represent key features of the combined human-natural systems. Uncertainty has been previously considered for natural resources management by Quérou and Tidball (2010), and Batabyal and Beladi (2004). Quérou and Tidball (2010) consider a problem of resource extraction by developing a theoretical game with incomplete information, whose players are involved in repeated interactions. A characterization of the optimal consumption policy is also provided. Batabyal and Beladi (2004) focus on the likelihood that a particular resource will not collapse in the long run by maximizing the time restrictions of its use. While both approaches are useful for some types of natural resources management problems, such as fishery, hunting, and similar contexts, they are not applicable to problems involving fossil fuels, which are the focus of this paper. Indeed, in Quérou and Tidball (2010) a strictly positive rate of regeneration of the natural resource – unreasonable when thinking of fossil fuels since it requires millions of years – plays a key role in the analysis. In Batabyal and Beladi (2004) time restrictions imply no use of fossil fuels in some periods. From an economical-practical point of view, when fossil fuels are considered, this assumption seems to be quite unrealistic. The stochastic dynamic optimization approach for identifying the optimal use path of the stock of fossil fuels enables to consider more realistic problems, and then it complements both the approaches proposed by Quérou and Tidball (2010) and Batabyal and Beladi (2004).

When fossil fuels are considered, some genuine stochastic elements impact on the uncertain evolution of the system under consideration. It is well known that uncertainty might show up as unpredictable random shocks in the dynamic evolution of an ecosystem, either in the form of an ongoing stream of small fluctuations or as abrupt and substantial discrete occurrences. Here, we consider both types of random shocks[1].

Moreover, uncertain elements affect resource exploitation and management both directly and indirectly via their influence on economy-wide variables. Accounting for such an uncertainty requires an integrated model allowing feedback effects between natural resources, climate change, and the overall economic context. This paper considers a lab-equipment model in which a commodity, the natural resource, in this case, is used up both in consumption and investments (see, e.g., Acemoglu, 2009)[2].

Substantially, we aim at joining the two conflicting targets that an (ethic) policymaker should pursue: to act as homo œconomicus by maximizing the benefits from the use of fossil fuels; to be sustainable and save a stock of such resources for future generations (see Heal, 1998; Chichilnisky, 1996). The relevance of this issue lies in the strict connections between the availability of fossil fuels and the search for alternative sources of energy production. This is the real challenge of the century, being fossil fuels also responsible for global warming and both air and water pollution.

The term sustainability has to be here intended as a concept invoked for guaranteeing the consideration of future generations which comes from the increasing alarm about anthropogenic climate changes. However, such a definition often fails to be effective due to its vague implications.
The Brundtland Commission proposed a generally accepted definition of sustainable development: it is (quoting) the *development that meets the needs of the present without compromising the ability of future generations to meet their own needs* (see also Goodland, 1995; Krysiak and Krysiak, 2006; Krysiak, 2009 and references therein). The ethical appeal of this statement is grounded on the requests of actions taken today to allow future generations to be treated fairly. If we agree that sustainability requires that a certain amount of goods should remain available in the long run, the key issue is to build a measure that allows evaluating whether a generation leaves enough to the future. As present and future generations need to be considered together, the discount factor of consumption and well-being of future generations play a key role. In this respect, Nordhaus (2007) and Weitzman (2007) suggest to

[1]Other main sources of uncertainty consist in the lack of understanding of the key natural and economic parameters. See Tsur and Zemel (2014) for a review related to various forms of uncertainty.

[2]The need for an integrated framework led to the development of the so-called Integrated Assessment Models (IAMs). As integrated models tend to be analytically intractable, they call for the use of numerical analysis. Even if IAMs provide the key tool in the study of resource management and climate change since the foundation of the Intergovernmental Panel on Climate Change (IPCC) (Clarke et al., 2009), they are not safe from criticisms (see Farmer et al. 2015).

apply the current interest rate and thus take 4% (or even 6%) over the next century, determining a discount factor of 0.985 per year. Stern (2007, 2008) indicates that the only acceptable justification for discounting future well-being is the risk-aversion for the possibility that future generations might not exist, for which the corresponding discount factor is 0.999 per year. Even if discounting future consumptions is a commonly accepted procedure, particularly in the macroeconomic literature, many scholars such as Sidgewick (1907), Pigou (1920), Ramsey (1928) and Harrod (1948) objected that it is unacceptable to treat adversely future generations. This concern about future generations has given rise to the literature on ranking utility streams. Unfortunately, its key result is the difficulty of aggregating each generation's well-being into a social welfare function that is sensible to the interest of each, and treats all generations equally (Diamond, 1965) [3]. Here, the crucial assumption is the measurability and comparability of well-being across generations but, to our knowledge, economic theory does not yet provide a way to construct such well-being indexes from individual's choices. A different approach concerns recent advances in social choice theory. In particular, Fleurbaey and Maniquet (2005), collecting and extending previous works, propose a compelling framework for studying resource distribution problems and the aggregation of individual's welfare in terms of social ordering functions, associating a complete ranking of feasible alternatives to each problem. A similar contamination from social choice theory to intergenerational equity can be found in Asheim et al. (2010). They study the problem of selecting an appealing intergenerational distribution of a single good for each specification of the time-invariant production technology. In particular, they show that a planner concerned with procedural and redistributive equity should select sustainable consumption paths.

Under a theoretical point of view, we adopt a stochastic dynamic optimization approach and solve it through dynamic programming and numerical analysis. In particular, we develop and solve an aggregated intergenerational expected utility maximization problem by selecting the optimal consumption and utilized stock of resource. The problem is constrained by the random dynamics of the available quantity of the resource, which are assumed to follow a jump-diffusion process. The resulting jump-diffusion stochastic optimal control problem is not trivial and includes several aspects which need peculiar attention. The adopted strategy of joining the pure theoretical analysis of the corner problems – the so-called dictatorship cases, see Section 3 for details – and the numerical procedures for the general case – see Section

[3]More recent contributions provide rather negative results: Svensson (1980); Basu and Mitra (2003); Zame (2007); Lauwers (2010) and Zuber and Asheim (2012).

4 – meets the requirement of being scientifically rigorous and also affordable under a mere practical point of view. The techniques used for the theoretical study of the model – i.e. the dynamic programming principle and the resulting Hamilton-Jacobi-Bellman equation in a jump-diffusion setting – are widely used in the field of applied dynamic optimization theory. In this respect, we address the reader to the recent contributions of Josa-Fombellida and Rincon-Zapatero (2012), Ren and Wu (2013), Huang et al. (2016) and – for an overview of theory and applications – to Øksendal and Sulem (2007). The numerical analysis complements and completes the analysis (see e.g. Castellano and Cerqueti, 2012, 2014).

The stochastic processes used to model the dynamics of the stock of the natural resource formalize the sources of randomness to which it is submitted: continuous-time normal flows – captured by a Brownian Motion – and extraordinary events of random size, occurring at random times, described through suitable point processes. The available stock of resource admits an absorbing state, which is related to the exhaustion of the resource. Needless to say that the target of being sustainable is not reached when the absorbing state is achieved. The presence of such a barrier represents a further constraint of the optimization problem. The time-horizon is assumed to be infinite since the time-span of the problem must include current and future generations. In this respect, generations can overlap and are fully contained in bounded time intervals.

Three main contributions are provided in this paper. Firstly, we propose a novel stochastic model which is a generalization of the deterministic set-up proposed by Chichilnisky. This leads to a more realistic and reasonable framework that can be effectively faced by adopting operational research techniques, grounded in stochastic optimal control theory with jump diffusions. Secondly, we avoid the application of a discount factor for intergenerational utility and rather propose a suitably chosen weight function[4]. In particular, the weight function considers the consumption of the natural resource, and it is assumed to be lower for higher values of such consumption. This, jointly with the use of the natural resource for investments in a broad economic sense – human and physical capital, purposeful Research and Development (R&D) – allows a sustainability criterion to be accounted for, while avoiding ethical and technical difficulties due to the use of a discount factor as in current literature. Finally, in describing the dynamics of the natural resource, we consider unpredictable random shocks in the form

[4]This technicality has been already adopted by Chichilnisky (1996), but the Chichilnisky's weight inevitably leads to underweight the utilities of the future generations.

of abrupt and substantial discrete occurrences, with random times and sizes. This allows the existing literature on resource management problems to be extended. Indeed, in some resource management problems, the planning horizon is either given exogenously or is a decision variable which can be determined for any extraction policy. In either case, its incorporation within the management problem involves no uncertainty. A more realistic approach calls for considering an uncertain completion date. The completion date – as a random variable whose realizations mark the depletion of the resource – was initially examined through an unknown initial stock (Kemp, 1976). An extension of the term depletion allowed to consider cases in which the resource could become obsolete – such as the uncertain arrival of a backstop substitute (Dasgupta and Heal 1974, Dasgupta and Stiglitz 1981), and to account for uncertainty in political (e.g., wars) or economical (e.g., technological breakthrough) events. By introducing a stochastic jump size for the stock of resource which allows considering all such cases, we extend the current literature. This enriches the existing analysis because shocks at random times might not necessarily lead to resource depletion.

Under a macroscopic perspective, this paper can be adequately considered in line with the literature dealing with resource management through stochastic optimal control theory. In this respect, the interested reader is addressed to classical references like the overviews of Clark (1974, 1976) and the Nobel Laureate Smith (1977)'s contribution. For more recent reviews, the reader can refer to the monograph of Sethi and Thompson (2000) and the Handbook of Biørndal et al. (2007).

The rest of the paper is organized as follows: Section 2 outlines the optimization model we deal with; in Section 3 the solution of the model, with a specific reference to the theoretical analysis of corner cases (dictatorship of either present or future) are derived; Section 4 faces the general case of combination between the well-being of present and future generations by numerical analysis; the last Section concludes.

2 The model

As in a standard economic set-up, we consider the existence of a benevolent social planner who maximizes the intergenerational well-being (see, e.g., Mas Colell et al., 1995). Before entering into the details of the benevolent social planner analysis, the stochastic dynamic law of the natural resource is described. To keep the model as simple as possible and without losing generality, we consider only one fossil fuel.

All the introduced stochastic processes belong to a filtered probability

space $\{\Omega, \mathcal{F}, \mathbb{F} := \{\mathcal{F}_s\}_{s>0}, \mathbb{P}\}$.
For modelling the available stock of fossil fuel, we follow the approach proposed by Olsen and Shortle (1996), Willassen (1998) and Motoh (2004) and introduce jump-diffusion dynamics $\{N_s\}_{s>0}$, whose formalization and explanation is reported below[5]. Specifically, the available stock of fossil fuel can be then formalized through an $\mathbb{F}$-adapted process $\{M_s\}_{s>0}$ as follows:

$$M_t(\omega) = \begin{cases} N_t(\omega), & \text{for } \omega \in \Omega \,|\, t < \vartheta(\omega); \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where ϑ is the exit time capturing the exhaustion of resource, and it is defined as:

$$\vartheta = \inf \{t \geq 0 \,|\, N_t \leq 0\}. \tag{2}$$

Remark 1. *By (1) and (2), the random time ϑ is associated to the absorbing barrier 0 in equation (1). In fact, if one takes $\omega \in \Omega$ such that $M_t(\omega) = 0$, then $M_{t+u}(\omega) = 0$, for each $t, u > 0$. This practically means that depletion is an irreversible status for the resource.*

Let us describe the details.
The stochastic process for the resource, $\{N_s\}_{s>0}$, obeys the following evolution law:

$$N_t = n + \int_0^t (\mu N_s - c_s + \Lambda(R_s) - R_s)\, \mathrm{d}s + \int_0^t \sigma N_s \mathrm{d}W_s + \sum_{i=1}^{+\infty} \zeta_i \mathbf{1}_{\{\tau_i \leq t\}}, \tag{3}$$

where: $N_0 = n > 0$; $t > 0$ is time; $\mu, \sigma \in (0, +\infty)$ are the natural growth rate of the available fossil fuel and its instantaneous volatility, respectively; $\{W_s\}_{s>0}$ is a standard 1-dimensional Brownian motion; c_t, R_t are extracted by $\mathbb{F}$-adapted stochastic processes with support $[0, +\infty)$, and represent consumption and utilized flow of resource for investments at time t, respectively. We denote their starting points as $c_0 = c$ and $R_0 = R$, respectively. It is worth noting that the stochastic process $\{R_s\}_{s>0}$ may well indicate different forms of investments, such as those in physical capital, human capital, and R&D, etc. We refer to *investments* only for the sake of simplicity.
The term Λ describes the size of nonnegative increments of the stock which can be obtained only by investing a positive quantity of the resource. More formally:

$$\Lambda : [0, +\infty) \to [0, +\infty) \qquad : \qquad R_t \mapsto \Lambda(R_t), \qquad \forall\, t > 0,$$

[5]For different perspectives in the stock of resource modeling, see the disturbed Markovian approach of Mitra and Roy (2007), and the point process framework of Cerqueti (2014).

such that $\Lambda(0) = 0$ and Λ is strictly increasing.
The term $\Lambda(R_t)$ is a measure of the profitability of employing at time t the quantity of resource R_t to eventually obtain a positive increment in the stock, $\Lambda(R_t) - R_t > 0$, such as the discovery of a new resource field. Note that, the difference $\Lambda(R_t) - R_t$ can also represent a change in efficiency in the use of the already exploited natural resource. A positive value indicates an improvement of the efficiency in the utilization of the natural resource, such as improvements due to successful investments in R&D, human capital, education, etc. A negative increment indicates a net usage of the resource with either low or no improvement in efficiency, i.e. to fix ideas unsuccessful R&D investments.
The point process $\{(\tau_i, \zeta_i)\}_{i\in\mathbb{N}}$ represents times and sizes of random shocks. The τ's are $\mathbb{F}$-adapted stopping times. Moreover, fixed $i \in \mathbb{N}$ the support of τ_i is $[0, +\infty)$ while ζ_i can have only a negative support. Indeed, any positive jump is either captured by Λ or already included in N_t[6].
More specifically, the point process $\{(\tau_i, \zeta_i)\}_{i\in\mathbb{N}}$ is assumed to evolve according to a Lévy process $\{\Gamma_s\}_{s>0}$, such that a jump at time t is described by $\Delta\Gamma_t = \Gamma_t - \Gamma_{t-}$. Such assumption leads to the independence between ζ_k and $\{(\tau_i, \zeta_i)\}_{i<k}$. Moreover, given the features of random jumps in the specific context of natural resource evolution, we also assume that ζ_k is independent of $\tau_k - \tau_{k-1}$. In doing so, we implicitly assume that the distribution of ζ_k is independent of time. We denote it as $p(dz)$.
The random sequence $\{\tau_i\}_{i\in\mathbb{N}}$ is assumed to follow a Poisson process with intensity λ. The higher the value of λ and less rare are the negative jumps in the stock of resource.
For each $t \geq 0$, of particular interest in our framework is the quantity

$$Y(t) := \sum_{i=1}^{+\infty} \zeta_i \mathbf{1}_{\{\tau_i \leq t\}}.$$

A classical result assures that, given a Borel set $\mathcal{B} \subseteq \mathbb{R}$, the Levy measure ν of $Y(t)$ is $\nu(\mathcal{B}) = \lambda p(\mathcal{B})$ (see Protter, 2003, Theorem 1.35).
In the following we will assume that:

$$\int_{-\infty}^{0} (1 \wedge |z|)\nu(dz) < +\infty, \tag{4}$$

which means that jumps have finite variations.

[6]We are reasonably assuming that an impulsive increase in the stock of available resource can be obtained only by implementing investment policies. This implies the absence of positive random jumps.

Remark 2. *N_t represents the available stock of fossil fuel, at time t. When N_t reaches or falls below 0, then the resource is exhausted and cannot be replenished through investments. Indeed, the function Λ provides a positive contribution only by employing a positive quantity of resource R_t. Moreover, $N_t = 0$ implies $c_t = R_t = 0$ (one cannot consume or employ for investment an exhausted resource). Exhaustion occurs at exit time ϑ, which represents a natural bound to random times τ's associated to shocks (the quantity cannot fall below zero), i.e. $\tau_i(\omega) \leq \vartheta(\omega)$, for each $\omega \in \Omega$ and $i \in \mathbb{N}$.*

Let us consider now a benevolent social planner who maximizes the intergenerational well-being. To this aim, an instantaneous utility function is introduced

$$U : [0, +\infty)^2 \to \mathbb{R} \qquad : \qquad (c_t, M_t) \mapsto U(c_t, M_t) \qquad \forall\, t > 0.$$

Function U is assumed to be increasing w.r.t. consumption c_t and available stock M_t. Moreover, since the exhausted resource does not generate utility and consumption, we also assume that $U(0, 0) = 0$. In general, without entering into technical details, the function U is assumed to *well-behave*, i.e. it satisfies all the regularity conditions required for the statement of theoretical results.

Differently from the standard social planner's problem, the instantaneous utility at time t is a function of the available stock M_t, and not only of final consumption c_t. This assumption formalizes the idea of a policymaker implementing an optimal sustainable path for the economic system (see, among others, Heal, 1998; Chichilnisky, 1996). Finally, it is worth recalling that because of the adopted social planner approach, market prices do not exist by definition of the economic problem at hand. Yet, as usual in such a set-up, market prices can be shown to coincide with shadow prices, as represented by multipliers in the dynamic optimization problem. By definition, such shadow prices embody the economic implications of stock changes, such as increasing extraction costs as the resource dwindles and price of scarcity, when a nonrenewable resource is nearing depletion (see e.g. Mas Colell et al., 1995). The optimized objective (value) function becomes:

$$\begin{aligned} V(n) \quad = \quad & \sup_{\{c_s\}_{s>0}, \{R_s\}_{s>0} \in \mathcal{A}} \mathbb{E}_n \Big[\alpha \int_0^{+\infty} U(c_t, M_t) \Delta(c_t) \mathrm{d}t + \\ & + (1 - \alpha) \lim_{t \to +\infty} U(c_t, M_t) \Big], \end{aligned} \tag{5}$$

where $\mathbb{E}_n$ is the usual expected value operator given $M_0 = n$, $\mathcal{A}$ is the admissible region containing the $\mathbb{F}$-adapted stochastic processes with nonnegative support, $\alpha \in [0,1]$ is the relative weight of the present on the future, and Δ is a weight function defined as:

$$\Delta : [0,+\infty) \to [0,+\infty) \qquad : \qquad c_t \mapsto \Delta(c_t), \forall\, t > 0.$$

As for the utility function U, we assume that Δ well-behaves.

The inclusion of a time-dependent weight function meets the purpose of removing the discount factor, which seems to be inappropriate in that it reduces the importance of future generations (see Section 1 and Chichilnisky, 1996).

Function Δ is assumed to decrease w.r.t. consumption. Such assumption formalizes the evidence that the over-consumption of the resource is penalized by a policymaker implementing sustainable policies. Moreover, we assume that:

$$\left| \int_0^{+\infty} U(c_t, M_t)\Delta(c_t)\mathrm{d}t \right| < +\infty. \tag{6}$$

Condition (6) formalizes the summability of the first integral in the value function V in (5). This is an unavoidable requirement to let the problem be mathematically tractable.

By definition of exit time ϑ in (2), the limit term of the value function (5) disappears when $P(\vartheta < +\infty) = 1$.

3 The solution of the problem

The adopted solution strategy is grounded on the evidence that the optimal consumption-use of the resource should be a balanced choice between the conflicting targets of the well-being of current generations and future sustainability.

The optimization problem is first considered in the corner cases: $\alpha = 0$ and $\alpha = 1$. This leads to two subproblems which are treated separately,from a theoretical perspective. Subsequently, the original optimization problem is rewritten to combine all the information derived in the study of the subproblems. The latter case is analyzed through numerical analysis.

Under the constraint given by the state equation (3), the subproblems are the following:

$$\sup_{\{c_s\}_{s>0},\{R_s\}_{s>0}\in\mathcal{A}} \mathbb{E}_n \left[\int_0^{+\infty} U(c_t, M_t)\Delta(c_t)\mathrm{d}t \right]; \tag{7}$$

$$\sup_{\{c_s\}_{s>0},\{R_s\}_{s>0}\in\mathcal{A}} \mathbb{E}_n \left[\lim_{t\to+\infty} U(c_t, M_t) \right]. \tag{8}$$

Problem (7) comes out from the definition of the value function V in (5) with $\alpha = 1$, while (8) is the corner case characterized by $\alpha = 0$. The former describes the *dictatorship of present*, while the latter formalizes the *dictatorship of future*.

3.1 Dictatorship of the future

The solution of problem (8) is grounded on the so-called *green golden rule*, and formalizes the evidence that no positive consumption can be maintained forever. Hence, the optimal consumption is $\{c_t^\star\}_{t>0} \equiv 0$, and problem (8) can be rewritten as follows:

$$\sup_{\{R_s\}_{s>0}} \mathbb{E}_n \left[\lim_{t\to+\infty} U(0, M_t) \right]. \tag{9}$$

Proposition 3. *Assume Λ twice differentiable and convex. Then there exists a unique solution $\{R_s^\star\}_{s>0}$ of problem (9) defined as follows:*

$$R_t^\star = \begin{cases} J(1), & \textit{if } \Lambda(R_t) - R_t > 0; \\ 0, & \textit{otherwise.} \end{cases}, \qquad \forall\, t > 0 \tag{10}$$

where J is the inverse of the function Λ'.

Proof. Since, for each $t > 0$, function U is increasing w.r.t. M_t, then formula (9) is equivalent to the following problem:

$$\sup_{\{R_s\}_{s>0}} \mathbb{E}_n \left[\lim_{t\to+\infty} M_t \right]. \tag{11}$$

Equation (3) and the monotonic property of the expected value operator assure that the solution of (11) is given by the process $\{R_s^\star\}_{s>0}$ defined as:

$$R_t^\star = \max\left\{\operatorname{argmax}_{R_t\in[0,+\infty)}\{\Lambda(R_t) - R_t\}, 0\right\}, \qquad \forall\, t > 0. \tag{12}$$

By applying first order conditions and under regularity assumptions on Λ, we obtain the thesis. □

Proposition 3 provides a theoretical support to the most reasonable optimal strategy. Indeed, sustainability may be achieved by implementing, at each time, the most profitable rule: to use the *best* quantity of resource when it is worthy ($R_t^\star = J(1)$, when $\Lambda(R_t) - R_t > 0$), and do not use fossil fuel when it is not worthy ($R_t^\star = 0$, when $\Lambda(R_t) - R_t \le 0$).
Furthermore, starting from the optimal utilized stock of fossil fuel given by (10), we derive that $\Lambda(R_t) = 0$ implies $R_t^\star = 0$, for each $t > 0$.

It is worth noting that this result recalls the deterministic set up of Chichilnisky (1996) which is a subcase of the stochastic framework presented here.

3.2 Dictatorship of the present

The removal of the term associated to time going to infinity in the objective function leads to the remarkable reduction of the complexity of the model, which becomes a standard stochastic optimal control problem in a context of jump-diffusion.
Since $(\{R_s\}_{s>0}, \{c_s\}_{s>0})$ is a couple of Markov controls, then the generator of the diffusion Levy process $\{N_s\}_{s>0}$ is:

$$\begin{aligned} A_{R,c}\phi(n) &= [\mu n - c + \Lambda(R) - R]\,\phi'(n) + \frac{\sigma^2 n^2}{2}\phi''(n) + \\ &\quad + \int_{-\infty}^{0} \{\phi(n+z) - \phi(n)\}\,\nu(dz). \end{aligned} \tag{13}$$

We proceed by adopting a dynamic programming approach, and derive the Hamilton Jacobi Bellman equation (HJB):

Theorem 4 (HJB)**.** *Assume that $V \in C^2(\mathbb{R})$. Then:*

$$\sup_{(R,c)\in[0,+\infty)^2} [U(c,n)\Delta(c) + A_{R,c}V(n)] = 0. \tag{14}$$

We do not report here a formal proof for Theorem 4, and address the reader to Øksendal and Sulem (Chapter 3, 2007), to the recent contributions of Castellano and Cerqueti (2012), and Kharroubi and Pham (2015).
The twice differentiability of the value function is guaranteed by the regularity assumptions on functions U and Δ (for some details on this, see Ceci and Gerardi, 2010).
The optimal $\{R^\star_s\}_{s>0}$ is as in (10). For what concerns the optimal consumption, from Theorem 4 we have:

$$\bar{c}(n) = \mathrm{argmax}_{c\in[0,+\infty)} \{U(c,n)\Delta(c) - cV'(n)\}\,. \tag{15}$$

The optimal dynamics obey to the following jump-diffusion equation:

$$\bar{N}_t = n + \int_0^t \left(\mu\bar{N}_u - \bar{c}(\bar{N}_u) + \Lambda(R^\star_u) - R^\star_u\right)\mathrm{d}u + \int_0^t \sigma\bar{N}_u \mathrm{d}W_u + \sum_{i=1}^{+\infty} \zeta_i \mathbf{1}_{\{\tau_i \le t\}}, \tag{16}$$

which leads, by (1), to:

$$\bar{M}_t(\omega) = \begin{cases} \bar{N}_t(\omega), & \text{for } \omega \in \Omega \,|\, t < \vartheta(\omega); \\ 0, & \text{otherwise,} \end{cases}$$

and then the optimal consumption is:

$$c^\star_t = \bar{c}(\bar{M}_t), \qquad \forall\, t > 0. \tag{17}$$

Dynamics $\{\bar{N}_s\}_{s>0}$ satisfy the closed loop equation (16) with initial value $\bar{N}_0 = n$, and represent the optimal trajectory of the control problem, which leads to the optimal amount of resource.

4 The non-dictatorship case

In this section we study, via numerical analysis, the non-dictatorship case characterized by $0 < \alpha < 1$. It is important to remark here that the two corner cases discussed in the previous section share the same optimal amount of utilized resource in investments. In some sense, this finding is trivial since, for each $t > 0$, R_t positively affects N_t – and consequently the utility function when it provides a positive outcome – while it penalizes N_t in the case of negative outcomes. Accordingly, the optimal strategy $R_t^\star$ found in (12) also applies to the general non-dictatorship case, so that this case reduces to search for the optimal consumption, with $R_t = R_t^\star$ for each $t > 0$.

In the following, the building blocks of the numerical procedure[7] used to compute the optimal controls, $\{c_s^\star\}_{s>0}$, of the general theoretical problem described by the value function (5), are presented, and the results of extensive numerical simulations are also discussed.

4.1 Numerical Procedure

The numerical procedure consists of three building blocks: the former one, namely A, allows the generation of random jumps $\{(\tau_i, \zeta_i)\}_{i\in\mathbb{N}}$,; the second one, i.e. B, develops the simulations of the state equation (3); the latter one, i.e. the building block C, is implemented to optimize the objective function (5) with respect to $\{c_s\}_{s>0}$, being $R_t = R_t^\star$ as in (10), for each $t > 0$. In order to simulate the paths of the available stock of resource, the jump diffusion in (3) is discretized assuming a time interval $\Delta t = 1$ (a year):

$$N_{t+\Delta t} = N_t + [\mu N_t - c_t + \Lambda(R_t^\star) - R_t^\star]\,\Delta t + \sigma N_t \sqrt{\Delta t}\varepsilon_t + \sum_{i=1}^{I} \zeta_i \mathbf{1}_{\{t<\tau_i\leq t+\Delta t\}}, \tag{18}$$

where: $\{\varepsilon_s\}_{s\geq 0}$ is a stochastic process of i.i.d. random variables with standard normal distribution $N(0,1)$; random times $\{\tau_i\}_{i\in\mathbb{N}}$ obey a Poisson

[7] We remark that the time complexity of the numerical algorithm is of a polynomial type on the parameters, as it is evident by looking at the pseudo-code below, where the construction of the algorithm is illustrated. Hence, we have no dimensional problems to handle in our numerical experiments.

process with intensity $\lambda = 0.0135$, and $\zeta_i \sim N(-30, 0.2)$ is the size of the i-th jump, for each $i \in \mathbb{N}$. The parameter I is the truncation term of the infinite series of jumps, and is assumed $I = 10,000$. It is also assumed: $\mu = 0.005$ and $\sigma = 0.015$.

The procedure for simulating (18) is split in two parts. At the first stage, via building block A, we deal with random jumps at random times. Its pseudo-code is described in the following.

- **Building Block A** *Simulation of random jumps at random times* $\{(\tau_i, \zeta_i)\}_{i\in\mathbb{N}}$.

  $A.1$ set[8] $\tau_0 = 0$; $\lambda = 0.0135$; $\mu_\zeta - 30$; $\sigma_\zeta = 0.2$, $T = 30$, $I = 10,000$;

  $A.2$ set $i = 0$;

  $A.3$ generate a random variable H_{i+1} from the exponential distribution with mean $1/\lambda$;

  $A.4$ set $\tau_{i+1} = \tau_i + H_{i+1}$;

  $A.5$ generate a random variable ζ_{i+1} from the normal distribution $N(\mu_\zeta, \sigma_\zeta)$;

  $A.6$ set $i = i + 1$. If $\tau_{i+1} > T$ or $i = I + 1$, stop. Otherwise, go to step $A.3$.

Building block B uses the sequence of jumps obtained with block A and completes the procedure for simulating the stochastic dynamics of the natural resource, $\{N_s\}_{s>0}$. Here, for each $t = 1, \ldots, T$, the control process c_t is assumed to be taken from the discretized interval $[0, 3]$ with discretization step 0.1, so that the admissible region contains 31^T vectors[9] of length T. The initial value of the control variable is fixed at $c_0 = 0$.

As highlighted in the theoretical model, the difference $[\Lambda(R_t) - R_t]$ measures the profitability of investing a certain quantity of resource R_t at time t (see formula (12)). In this respect, for each $t = 1, \ldots, T$, it is assumed $\Lambda(R_t) = (1 + i_t)R_t$, where $\{i_s\}_{s>0}$ is a stochastic process of i.i.d. random variables from the uniform distribution $U(a, b)$, with $a < 0 < b$. For each $t = 1, \ldots, T$, function Λ maps R_t into a quantity which depends on the realization of the random rate of return i_t, and the optimal rule is to use a certain quantity of natural resource when it is worthy ($R_t^\star = J(1)$, if

[8] We have also considered a different value of the time-horizon, i.e. $T = 60$.

[9] The quantity 31^T comes out from taking, at each time $t = 1, \ldots, T$, a value $c_t = 0.0, 0.1, 0.2, \ldots, 2.9, 3.0$. Hence, at each time t, the quantity c_t can assume 31 different values. Thus, by combining all the available c_t over $t = 1, \ldots, T$, 31^T different vectors of length T are obtained.

$(1 + i_t)R_t - R_t > 0)$ and do not use it when it is not worthy ($R^\star_t = 0$, when $(1 + i_t)(R_t) - R_t \leq 0$).

To analyze the behavior of the value function in reference to the effectiveness of investments, three scenarios $\{i_s\}_{s>0}$ are extracted from the uniform distribution with different supports: $U(-1, 1)$ captures a fair effectiveness of the investments; $U(-0.5, 2)$ and $U(-2, 0.5)$ imply optimistic and pessimistic views, respectively. The initial value i_0 is set to 0.

As discussed in Section 2, we remark that, when the process $\{N_s\}_{s>0}$ reaches (or falls below) 0, the reserve is exhausted. So, the natural bound of $N_t = 0$, for $t > 0$, is an absorbing barrier for the process $\{M_s\}_{s>0}$. In the following, the pseudo-code of building block B is described.

- **Building Block B** *Simulation of the state equation (3).*

  $B.1$ set[10] $\Delta t = 1$; $T = 30$; $n = 400$, $a = -1$; $b = 1$, $\mu = 0.005$; $\sigma = 0.015$; $K = 10,000$, $D = 4$, $h = 0.1$;

  $B.2$ construct the admissible region of controls as a matrix with 31^T rows and T columns

  $$C \equiv \prod_{t=1}^{T} \{0, 0 + h, \ldots, 0 + 30h\},$$

  and $c_{t,m}$ is the generic element of the set C, for $t = 1, \ldots, T$ and $m = 1, \ldots, 31^T$;

  $B.3$ generate a T-dimensional random vector $\{i_s\}_{s=1}^T$ of i.i.d. random variables $U(a, b)$ and set $c_{0,m} = 0; i_0 = 0$; $R^\star_0 = 0$, for each $m = 1, \ldots, M$;

  $B.4$ set $\Lambda_t = 1 + i_t$, for each $t = 0, \ldots, T$;

  $B.5$ if $i_t > 0$ set $R^\star_t = D$ otherwise $R^\star_t = 0$;

  $B.6$ generate a random matrix E whose generic element $(\epsilon_{k,t})$ is a random draw from $N(0, 1)$;

  $B.7$ set $m = 1$;

  $B.8$ set $k = 1$;

  $B.9$ set $t = 1$;

[10] To discuss a wider range of cases, the following values were assigned to the parameters: $n \in \{0, 0 + h, 50 + h, ..., 50 + 19h\}$ with $h = 50$; $T \in \{30, 60\}$; $(a, b) \in \{(-1, 1); (-0.5, 2); (-2, 0.5)\}$.

$B.10$ set

$$N_t^{m,k} = N_{t-1}^{m,k} + \left[\mu N_{t-1}^{m,k} - c_{t-1,m} + \Lambda_{t-1} R_{t-1}^{\star} - R_{t-1}^{\star}\right] \Delta t + N_{t-1}^{m,k} \sigma \sqrt{\Delta t} \cdot \epsilon_{k,t} + \sum_{i=1}^{I} \zeta_i \mathbf{1}_{\{t-1<\tau_i \le t\}};$$

$B.11$ if $N_t^{m,k} \le 0$, set $M_t^{m,k} = 0$ and go to step $B.12$. Otherwise, set $M_t^{m,k} = N_t^{m,k}$ and go to step $B.13$;

$B.12$ set $t = t+1$. If $t = T+1$, go to step $B.14$. Otherwise, set $M_t^{m,k} = 0$ and remain in $B.12$;

$B.13$ set $t = t+1$. If $t = T+1$, go to step $B.14$. Otherwise, go to step $B.10$;

$B.14$ set $k = k+1$. If $k = K+1$, go to step $B.15$. Otherwise, go to step $B.9$;

$B.15$ set $m = m+1$. If $m = M+1$, stop. Otherwise, go to step $B.8$.

To select the optimal path for the controls $\{c_s^{\star}\}_{s>0}$ which maximize the value function given in (5), a grid search procedure is implemented. This is described in the pseudo-code of building block C. We remark that for each $t = 1, \dots, T$, the control variable c_t is assumed to be taken from the discretized interval $[0,3]$ with discretization step 0.1, and initial value fixed at $c_0 = 0$ (see building block B).

Furthermore, the utility function in (5) is assumed to be of Cobb-Douglas type: $U(c_t, M_t) = c_t^{\beta_c} M_t^{1-\beta_c}$, for each $t > 0$. The weight function Δ, used to overcome criticisms posed by the discount factor, is assumed to be time-independent and penalizing the overconsumption of resource. Hence, a power law $\Delta(c) = c^{-\gamma}$, with rate of decay $\gamma > 1$ is considered. Recall that the optimal state variable – the optimal stock of available resource – will be denoted as $\{\overline{M}_s\}_{s>0}$. It is obtained from equation (3) and definition (1), by replacing c_t and R_t with the optimal $c_t^{\star}$ and $R_t^{\star}$, for each $t > 0$.

In the following, the pseudo code of the algorithm implemented to identify the optimal path of the consumption, $\{c_s^{\star}\}_{s>0}$, which solve the aggregated intergenerational expected utility maximization problem, is presented.

- **Building Block C** *Solving the aggregated intergenerational expected utility maximization problem by selecting the optimal path for consumption.*

$C.1$ set $\alpha = 0.5$; $\beta_c = 0.5$; $\gamma = 2.1$;

$C.2$ set $m = 1$;

$C.3$ set $k = 1$;

$C.4$ set

$$V_{k,m} = \alpha \cdot \sum_{t=0}^{T} \left[(c_{m,t})^{\beta_c} (M_t^{k,m})^{1-\beta_c} \right] \cdot (c_{m,t})^{-\gamma} + (1-\alpha) \cdot \left[(c_{m,T})^{\beta_c} (M_T^{k,m})^{1-\beta_c} \right];$$

$C.5$ set $k = k+1$. If $k = K+1$, go to $C.6$. Otherwise, go to step $C.4$;

$C.6$ set $V^m = \frac{1}{K} \sum_{k=1}^{K} V_{k,m}$;

$C.7$ set $m = m+1$. If $m = M+1$, go to $C.8$. Otherwise, go to step $C.3$.

$C.8$ select $\overline{m}$ such that:

$$V^{\overline{m}} = \max_{m=1,\ldots,M} V^m;$$

The optimal consumption is obtained in three cases: $\alpha \in \{0.1, 0.5, 0.9\}$. This allows considering the possibility that a policymaker may be present ($\alpha = 0.9$), future ($\alpha = 0.1$), or fair oriented ($\alpha = 0.5$) from an intergenerational point of view. Furthermore, different values for the weight of consumption in the Cobb-Douglas utility function are considered, i.e. $\beta_c \in \{0.75, 0.5, 0.25\}$. Finally, two different values of γ, namely $\gamma \in \{2.1, 10\}$, for the weight function $\Delta(c) = c^{-\gamma}$, which penalizes the overconsumption of resource, are compared.

Running the procedure, we find the optimal controls, $c_t^\star = 0.1$, for each $t = 1, \ldots, T$, each initial value n and the entire set of selected values assigned to the parameters. Of course, the optimal consumption cannot be null, being the utility function null when consumption is zero. However, the $c^\star$'s reach the smallest available nonzero value. This result is in line with the definition of process $\{N_s\}_{s>0}$, whose t-th term contains the aggregated penalization due to consumption up to time t.

Starting from this, a large enough time horizon – as $T = 30$ in our specific numerical experiments – highlights the role of consumption in reducing the value of the aggregated intergenerational utility. To fix ideas, Figure 1 shows a collection of possible scenarios for the dynamics of the stock of resource, $\{\overline{M}_s\}_{s>0}$, in the case: time horizon $T = 30$, initial value $n = 400$ and fair effectiveness of the investments $i_t \sim U[-1, 1]$, for $t = 1, \ldots, T$.

[Insert Figure 1 about here]

Caption: simulated set of scenarios for the stock of resource, computed with the optimal controls and time horizon $T = 30$

Figure 2 shows, at the achievement of the time horizon, the survival probabilities of the stock of resource. They are computed as the ratio between the number of scenarios for which $\bar{M}_T > 0$ over the total number of scenarios $K = 10,000$. In particular, fixed the set of initial values $n \in \{0, 0+h, 50+h, ..., 50+19h\}$ with $h = 50$, two cases are presented: $T = 30$ and $T = 60$ years.

[Insert Figure 2 about here]

Caption: a) survival probabilities of the stock of resource as a function of the initial value n, computed at different time horizons; b) relative increments of survival probabilities.

Figure 2a shows two almost parallel curves for time horizons $T = 30$ and $T = 60$. As expected, the survival probabilities of the resource grow with its initial value and, for a fixed initial value of the natural resource, n, are higher when $T = 30$. It can be observed that the distance between the two curves decreases as n increases. More details can be gathered by looking at the relative increments of survival probabilities in Figure 2b. Their behavior is analogous to that of a stationary series when n is small and $T = 60$ while it becomes more regular – relative increments decrease – for $T = 30$ and higher values of n. The regularity is due to the upper bound of the probability, leading to a horizontally asymptotic shape of the survival probabilities.
The relationship between survival probabilities in the cases $T = 30$ and $T = 60$ are particularly meaningful. It suggests that the initial endowment of resource in the $T = 60$ years case should be more than 15 times greater than that of $T = 30$ years, to observe the same survival probabilities. This implies that, even in the case of optimal (low) consumption, doubling the prospective time horizon of a social planner leads to a significant increment (more than 15 times) in the stock of resource to avoid its exhaustion.

In Figure 3, the differences between the value functions, computed in the optimistic view, $i_t \sim U[-0.5, 2]$ for $t = 1, \dots, T$, in the pessimistic case, $i_t \sim U\ [-2, 0.5]$ for $t = 1, \dots, T$, and in the fair case, $i_t \sim U[-1, 1]$ for $t = 1, \dots, T$, for different levels of α and n are reported. It is also assumed: $\gamma = 10$ and $\beta_c = 0.5$.

[Insert Figure 3 about here]

Caption: difference between the value functions computed for different levels of α and initial values n, with parameters $\gamma = 10$ and $\beta_c = 0.5$.

Figure 3 shows that, for all the considered α and n, the optimistic scenario of investments leads to great benefits in terms of well-being, compared with the fair scenario. This is particularly relevant for $\alpha = 0.9$ and small values of n. Results stress the relevance of investments for the well-being of both present and future generations. Indeed, the optimistic scenario, in all the three cases, determines very large increments of the value function for low initial values, while for higher initial values its contribution is always positive and almost constant. Particularly interesting is also the evidence occurring in the case of pessimistic scenario. Here, a decrease in the value function is detected in all the cases, yet it is much lower than the increase in utility observed in the optimistic case.

Overall, long term investments make the difference for the well-being of present and future generations, independently on the relative weight α. In this respect see the plot for the case $\alpha = 0.1$ in Figure 3.
These results have strong policy implications. In particular, different branches of science, from economics to biology and engineering, focus on the effectiveness of various policy measures – such as emission pricing in the form of carbon taxes and tradable emission allowances (cap-and-trade system), technology mandates, performance standards, and hybrid approaches – in limiting ecological and environmental damages from the use of fossil fuels. This paper shows that a long-term sighted view favoring investments in a broad sense, such as those in human capital, education, and R&D, aimed at improving the efficiency of the use of fossil fuels, plays a key role in increasing the well-being of individuals for both present and future generations. This is not to say that worldwide suggested climate change policies are not useful in improving the well-being of individuals. Yet, all such measures should complement policies oriented at long term investments, and the eventually collected proceeds from environmental taxes should be used for financing and supporting such type of investments.
To summarize, the type of use of fossil fuels matters. Policy makers should not penalize the use of fossil fuels for some types of long term investments, such as those in R&D, human capital accumulation, education, etc.

Looking at Figure 4 it can be observed that the value function always increases w.r.t. n. The trajectories of the value function are computed for $T = 30$, for different levels of initial values, $n \in \{0, 0 + h, 50 + h, ..., 50 + 19h\}$ with $h = 50$, and different levels of weights $\alpha \in \{0.1, 0.5, 0.9\}$, $\beta_c \in \{0.25, 0.5, 0.75\}$ and $\gamma \in \{2.1, 10\}$.

[Insert Figure 4 about here]

Caption: value function when $T = 30$, for different initial values n and weights α, β_c and γ.

Figure 4 shows that the value function is rather flat when $\beta_c = 0.75$, while it grows faster as β_c decreases. At the same time, a small value of α reduces the utility level. This outcome explains that utility is high when much attention is paid to current generations, even if the overconsumption of the present is penalized by the weight function, $\Delta(c)$. This is due to the low level of optimal consumption which neutralizes the deterrence effect carried out by the weight function.

Figure 5 presents the rates of change of the value function when $T = 30$, and for different initial values, n, and weights α, β_c and γ.

[Insert Figure 5 about here]

Caption: the rate of changes of the value function when $T = 30$, for different values of initial values, n, and weights α, γ, and β_c.

Figure 5 builds the increments of the value function. As the initial value of the stock n increases, the value function exhibits decreasing increments, and such behavior is more evident when $\beta_c = 0.25$. This parameter indicates the prominent role of the available stock of resource rather than the optimal consumption for the well-being of individuals. Needless to say that a high relative weight of α reverses this outcome. Moreover, numerical analysis leads to a marginally decreasing optimized objective function, and this is in line with the decreasing satisfaction of consumers in standard utility theory.

5 Conclusions

This paper deals with sustainability and environmental concerns, under the constraint of the standard flow of human activities. In particular, we develop an overlapping-generations model to state policies on consumption and utilization of a given fossil fuel.

We include uncertainty in our setting and build a framework whose nature is of stochastic type. In doing so, we take into account the randomness associated with the evolution of the available stock of fossil fuels and the consequent randomness of the intergenerational expected utility maximization model.

We face the problem by firstly highlighting which world would be under dictatorships of either the current or future generations and then studying the non-dictatorship cases of combination between present and future. The dictatorship cases have been treated from a theoretical perspective while the combination case has been analyzed through numerical simulations. In general, the utilized amount of the resource is the same in each case and meets the evidence that the maximum outcome from resource utilization must be pursued. For what concerns the optimal consumption path, we observe that it is strongly related to the altruism of the policy maker. In the particular case of the dictatorship of the future, we obtain the green golden rule of consume-nothing, which is in line with the classical model of Chichilnisky (1996). In the non-dictatorship case, the forward-looking perspective of the social planner plays a key role in generating ecological and environmental results. In particular, we find that a more forward-looking social planner – i.e. a social planner taking into account the well-being of future generations – should adopt an environmental sustainability policy per se, even if ecological concerns are not in order.

The adopted methodological approach allows us to capture a rather large part of the complexity of the problem we are dealing with. The inclusion of the jumps in the random dynamics of the stock of resource and the development of a dynamic stochastic optimization model are, in fact, key ingredients for a proper formalization of a new model which can be viewed as the stochastic version of the deterministic Chichilnisky set-up. However, even if the developed model is not trivial, we do not pretend to manage here all the sources of complexity related to the issue of sustainable management of fossil fuels. This said, we feel that the level of sophistication achieved in the present research contributes to planning further investigations in this field. In this respect, a more extensive treatment of the control variables, with a particular focus on possible controlled dynamics for consumption and utilized resource, could represent a remarkable extension of the theoretical model.

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Figure 2

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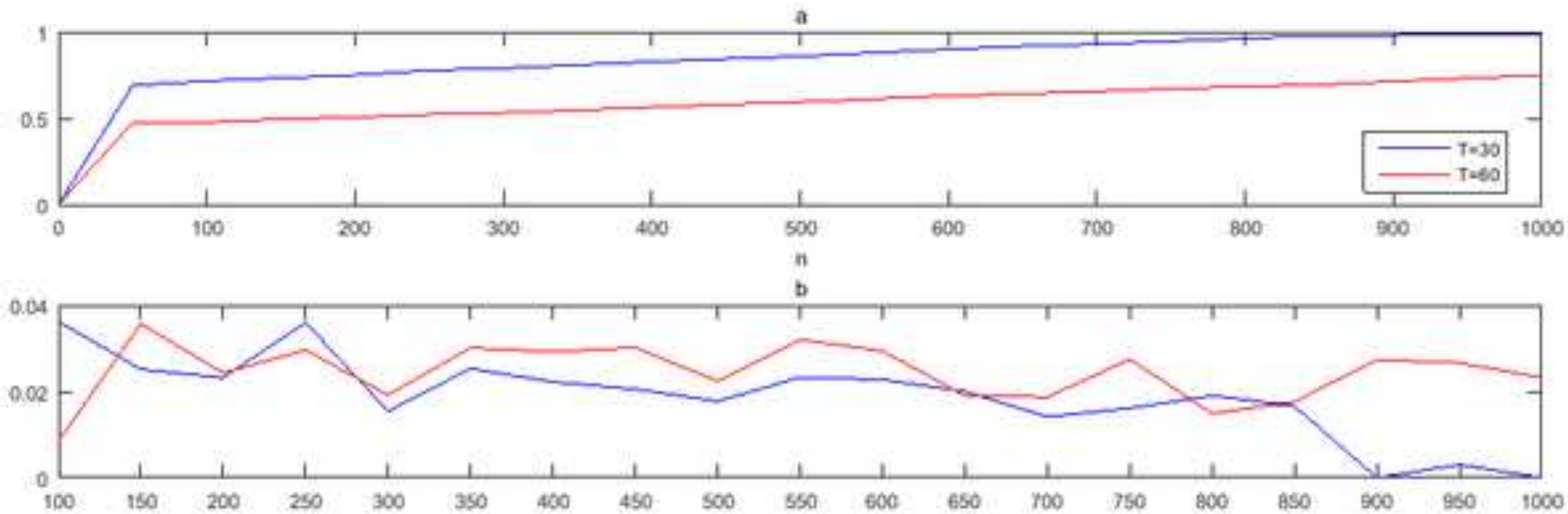

Figure 1
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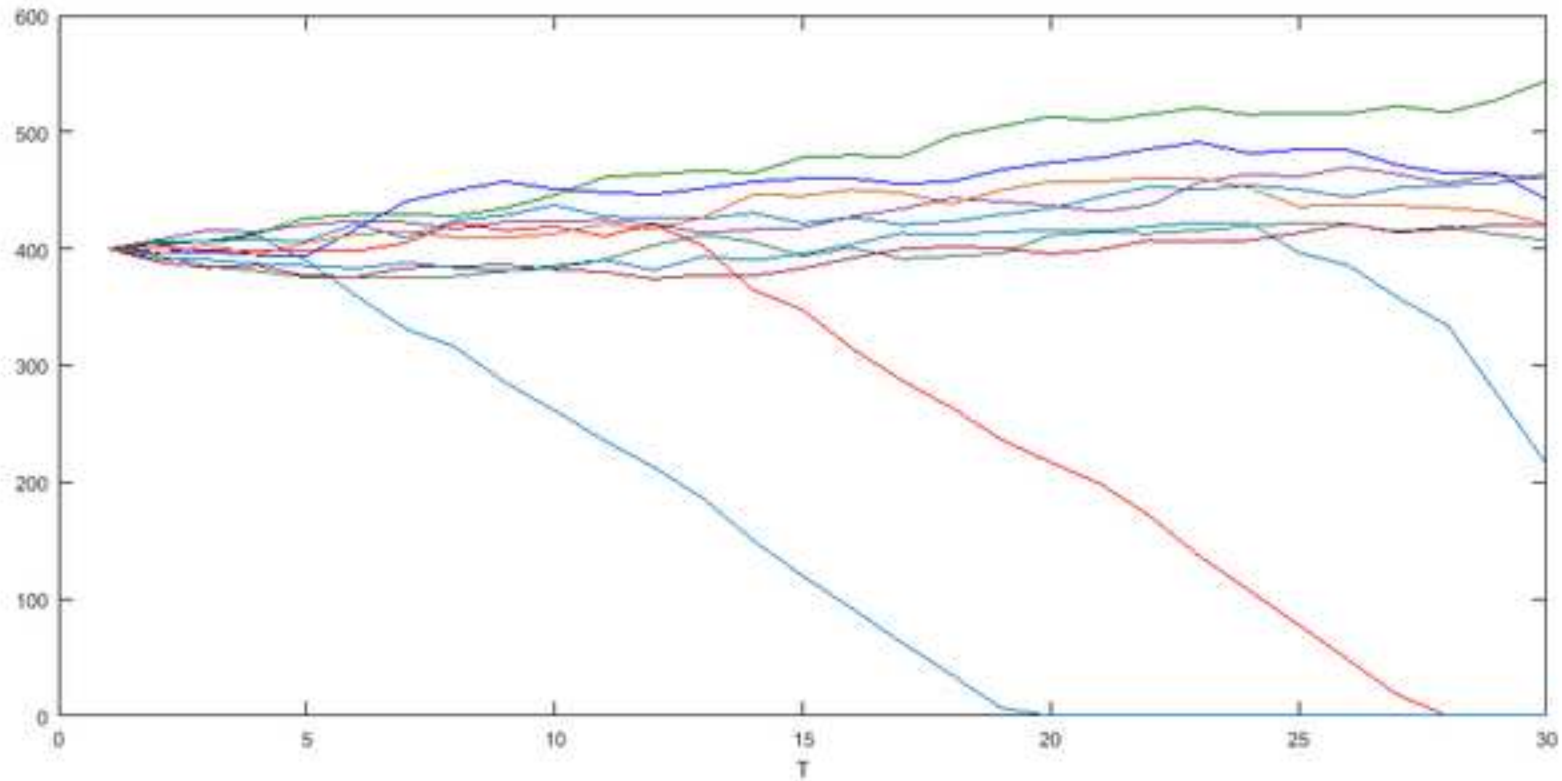

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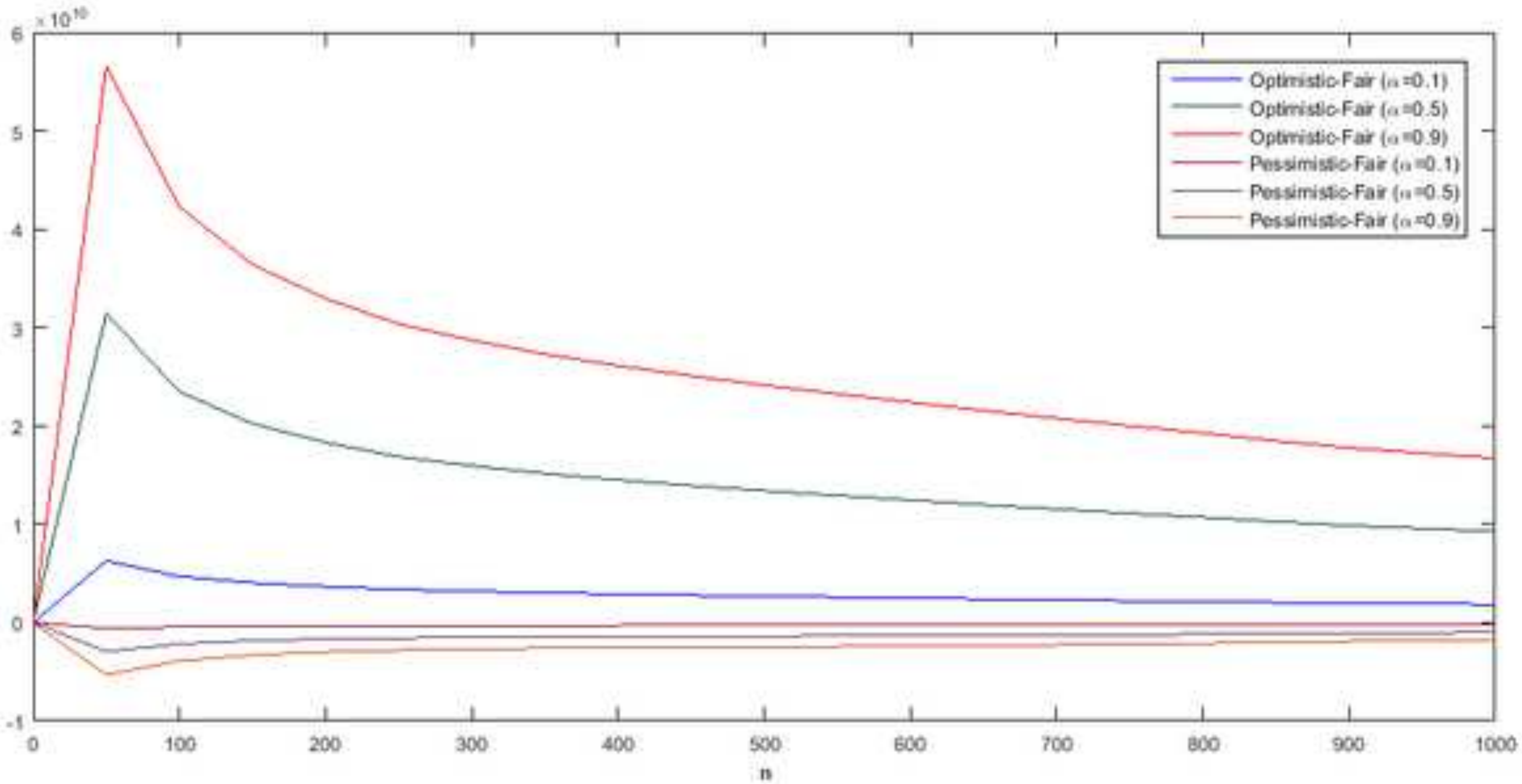

Figure 4

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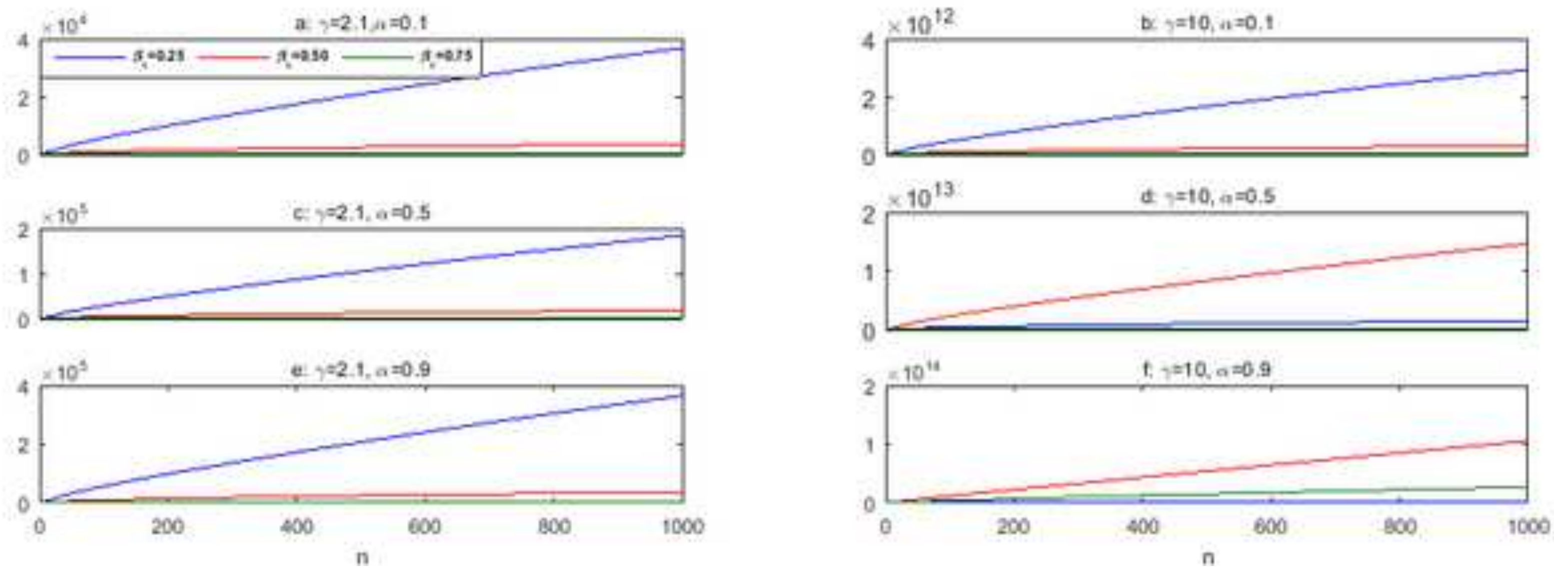

Figure 5
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