Economics Letters 235 (2024) 111573

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Unobservable costly effort in security design

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ARTICLE INFO

JEL classification:
D82
G23
C72

Keywords:
Signalling
Security design
Securitisation
Adverse selection
Costly effort

ABSTRACT

I introduce unobservable costly effort into the canonical signalling model of security design. The choice of effort determines the quality of the security and creates an additional optimisation problem the firm must solve. My main result is that both optimal effort and optimal profits are decreasing in the firm's preference for liquidity. I also show that optimal effort is greater under symmetric information. My approach provides a theoretical explanation for empirical results that cannot be obtained within the standard signalling framework. In addition, I extend existing methods to solve signalling models with endogenous private information.

1. Introduction

In many economic settings that involve adverse selection, the assumption of exogenous private information is not the most realistic. One salient example is a firm that underwrites assets before transforming them into securities to sell on secondary markets. The conventional approach would model the quality of the security as the realisation of a random variable. Intuitively, however, each asset's quality should be determined, to a large extent, by the effort of the firm during the underwriting process.

To more closely approximate reality, I introduce endogenous private information, modelled as an unobservable costly effort choice, into the canonical signalling model of security design proposed by DeMarzo and Duffie (1999). I extend a general result of Mailath and von Thadden (2013) and employ the equilibrium refinement of In and Wright (2017) to fully characterise the firm's optimal choice of signal and effort. By doing so, I obtain a number of novel predictions that lend theoretical support to several stylised facts that cannot be derived without costly effort: I show that the firm's optimal effort is decreasing in its preference for liquidity, as are the optimal profits. The former result implies that optimal effort is greater under symmetric information. Therefore, a key insight is that adverse selection dampens effort incentives, which leads to optimal effort being less than under symmetric information. In addition, I characterise the incentive compatible signal in signalling games with endogenous private information. This result builds on an existing approach in the literature and could be applied to other signalling models that incorporate, for example, costly effort.

In a seminal contribution, DeMarzo and Duffie (1999) showed that a monopolist issuer of asset-backed securities can signal the quality of the security using the quantity that it offers for sale. Their model has been extended to analyse the pooling and tranching of securities (DeMarzo, 2005) and the impact of market power (Biais and Mariotti, 2005). Chemla and Hennessy (2014) introduce a binary effort choice into a signalling model of securitisation to study the welfare implications of optimal retention. In contrast, I allow for a more general choice of effort and focus on analysing the determinants of optimal effort. Daley et al. (2020, 2023) study the effect of credit ratings, and scrutiny more generally, on loan origination and retention. Rather than signalling, Vanasco (2017) focuses on the implications of endogenous effort in a screening model. A related strand of literature examines moral hazard in securitisation (e.g, Hartman-Glaser et al., 2012; Malamud et al., 2013; Malekan and Dionne, 2014) but does not consider signalling.

In Section 2, I lay out the details of the model. In Section 3, I state my general characterisation theorem, derive the optimal signal and set up the firm's effort optimisation problem. In Section 4, I state my main results, which I relate to the empirical literature in Section 5. Proofs are in the Appendix.

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[1] Thanks are due to Alex Dickson, Giuseppe De Feo, Marco Fongoni and Jasmin Droege for feedback on previous drafts of this paper, as well as the participants of the 33rd European Economic Association conference, the 17th SAET conference, the 2018 Royal Economic Society conference and the University of Strathclyde Brown Bag seminar series for useful comments and discussion. I'd also like to thank an anonymous referee for very helpful suggestions that improved the paper. Elements of this paper were taken from the second chapter of my PhD thesis: Signalling with Endogenous Private Information.

https://doi.org/10.1016/j.econlet.2024.111573
Received 30 October 2023; Received in revised form 10 January 2024; Accepted 27 January 2024
Available online 30 January 2024
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2. Model

Consider a monopolist that underwrites and issues asset-backed securities. Unlike in DeMarzo and Duffie (1999), the quality of the security $q(e)$ is determined by the firm's unobservable effort choice $e \in [\underline{e}, \overline{e}] = \mathcal{E}$, which has cost e. One interpretation could be that effort represents the money spent on the underwriting and due-diligence process. In this case, the quality of the security would be a non-decreasing function of the expenditure and the cost would simply be the money spent.[2] Assume $q(\underline{e}) > 0$ as well as $q'(e) > 0$ and $q''(e) < 0$ for all $e > \underline{e}$. The firm's signal is the fraction $x \in [0, 1]$ of the security offered for sale. Given x, the firm discounts the remaining $(1 - x)$ of retained earnings at rate $\delta \in (0, 1)$. Given a price of p for the fraction x offered for sale, the firm's payoff is

$$v(e, p, x) = \delta(1 - x)q(e) + p - e, \tag{1}$$

and the market investors' payoff is $u(e, p, x) = xq(e) - p$. Competition between investors means the best response is $\hat{p} = xq(\hat{e})$ when $\hat{e}$ is the investors' belief about the firm's effort. This implies that (1) can be written in a form analogous to Mailath and von Thadden (2013):

$$V(e, q(\hat{e}), x) := \delta(1 - x)q(e) + xq(\hat{e}) - e = \delta q(e) + [q(\hat{e}) - \delta q(e)]x - e. \tag{2}$$

3. Analysis

Reordering Invariance (In and Wright, 2017) pins down investors' beliefs by considering a reordered version of the game in which the firm first chooses the observable signal before the unobservable effort.[3] Therefore, I first solve for the optimal signal $X : \mathcal{E} \to \mathcal{X}$. For separation, the signal must be one-to-one and satisfy incentive compatibility

$$X(e) \in \underset{x \in X(\mathcal{E})}{\arg\max}\, V(e, q(X^{-1}(x)), x). \tag{IC}$$

To characterise the optimal signal when the security's quality is endogenously generated through an unobservable effort choice, I extend Mailath and von Thadden's Theorem 3 to the framework in Section 2.

Theorem 1. *Let $X : \mathcal{E} \to \mathcal{X}$ be one-to-one, where $\mathcal{X}$ is compact, and satisfy (IC). Let $q : \mathcal{E} \to \mathbb{R}_+$ be differentiable and one-to-one with $q'(e) \neq 0\ \forall\ e \in \mathcal{E}$, except at either $\underline{e}$ or $\overline{e}$. Then, for any $e \in \mathcal{E}$, if $V_3(e, q(\hat{e}), x) \neq 0\ \forall\ x \in \mathcal{X}$, X is differentiable at e. At all points of differentiability X satisfies*

$$X'(e) = -\frac{V_2(e, q(\hat{e}), x) \cdot q'(\hat{e})}{V_3(e, q(\hat{e}), x)}\bigg|_{\hat{e}=e, x=X(e)} = -\frac{V_2(e, q(e), X(e)) \cdot q'(e)}{V_3(e, q(e), X(e))}. \tag{DE}$$

Applying Theorem 1 to the transformed payoff (2) yields the following optimal signal.

Lemma 1. $X(e) = [q(\underline{e})/q(e)]^{\frac{1}{1-\delta}}$.

The optimal signal $X(e)$ is decreasing in effort: the greater the effort exerted during the underwriting and due-diligence process, the smaller the fraction of the security that is offered for sale by the firm.[4] Substituting $X(e)$ from Lemma 1 into (2) gives

$$\begin{aligned}\mathscr{V}(e, \delta) := V(e, q(e), X(e)) &= \delta q(e) + (1 - \delta)q(e)[q(\underline{e})/q(e)]^{\frac{1}{1-\delta}} - e, \\ &= \delta q(e) + (1 - \delta)q(e)^{\frac{\delta}{\delta-1}} q(\underline{e})^{\frac{1}{1-\delta}} - e. \end{aligned}\tag{3}$$

[2] A convex cost function $c(e)$ could capture non-monetary costs, such as opportunity cost, but is not necessary for the results.

[3] Reordering Invariance eliminates equilibria with unreasonable beliefs through a similar, but weaker, notion of invariance that is used in strategic stability (Kohlberg and Mertens, 1986) and proper equilibrium (Myerson, 1978).

[4] The relationship between this result and the empirical literature will be discussed in Section 5.

Lemma 2. *$\mathscr{V}$ has increasing marginal returns on $(\underline{e}, \overline{e}] \times (0, 1)$.*

The novelty of my approach is that the firm now solves a second optimisation problem to find the optimal choice of effort

$$e(\delta) \in \underset{e \in \mathcal{E}}{\arg\max}\, \mathscr{V}(e, \delta). \tag{OE}$$

Whether (OE) has a solution will depend, in many cases, on the function q. The following example characterises a class of effort production functions for which $\mathscr{V}$ is strictly concave in effort.

Example 1. Suppose $q(e) = 1 + e^{\alpha}$ for $\alpha \in (0, 1)$ and $\underline{e} = 0$. Then $\mathscr{V}$ is strictly concave in e.

4. Results

My main result provides a set of qualitative predictions that cannot be derived in the standard signalling framework, and which arise from the analysis of (OE).

Proposition 1. *Suppose q is such that $\mathscr{V}$ is strictly concave in e. Then,*

(i) optimal effort is greater under symmetric information;
(ii) optimal effort $e^*(\delta) = \arg\max_{e\in\mathcal{E}} \mathscr{V}(e, \delta)$ is increasing in δ for $e \in (\underline{e}, \overline{e}]$; and,
(iii) the value function $\mathscr{V}(e^*(\delta), \delta) = \max_{e\in\mathcal{E}} \mathscr{V}(e, \delta)$ is increasing in δ for $e \in (\underline{e}, \overline{e}]$.

Conversely, suppose $\arg\max_{e\in\mathcal{E}} \mathscr{V}(e, \delta)$ in nonempty. Then,

(iv) any interior optimal effort $e^*(\delta) \in \arg\max_{e\in\mathcal{E}} \mathscr{V}(e, \delta)$ is increasing in δ for $e \in (\underline{e}, \overline{e}]$.

Intuitively, property (i) states that adverse selection depresses effort incentives.[5] When the firm has to use a costly action to signal its effort, optimality requires effort below the first-best. Similar results can arise in moral hazard models, but the novelty of my approach shows that it also holds in an adverse selection framework. Properties (ii) and (iii) highlight that both optimal effort and optimal profits are decreasing in the rate at which the firm discounts retained earnings. When the payoff from retaining the security is relatively low, the firm finds it optimal to exert low effort. The firm then securitises a greater fraction of these poorer quality assets to sell to investors, as the relatively low price will be preferred to retention. Conversely, when the payoff from retaining the security is relatively high, the firm is more willing to expend effort on the due-diligence process knowing that this will feed through to a relatively smaller amount sold to investors, but at a greater price. The key insights of Proposition 1 are illustrated in Fig. 1. Finally, property (iv) highlights that concavity is not necessary for property (ii) to hold. By using tools from monotone comparative statics, property (ii) will be true as long as there is an interior solution to (OE).

5. Discussion

A common interpretation of δ is that it represents the firm's preference for liquidity, which can arise due to more profitable investment opportunities, or a need to satisfy capital adequacy requirements. Under this interpretation, the findings of Proposition 1 provide a theoretical explanation for a number of empirical results. Specifically, $\delta_H > \delta_L$ implies that $e^*(\delta_H) > e^*(\delta_L)$, which in turn implies that $q(e^*(\delta_H)) > q(e^*(\delta_L))$ and $X(e^*(\delta_H)) < X(e^*(\delta_L))$. Therefore, my model predicts that firms in need of liquidity will underwrite assets of lower quality than

[5] Symmetric information implies that investors have correct beliefs about the firm's effort and that the firm always sells the entire security: the choice of effort is observable. Formally, this implies $\hat{e} = e$ and $X(e) = 1$ for all e. Conversely, under asymmetric information, $\hat{e}$ may not equal e and $X(e)$ may be less than one.

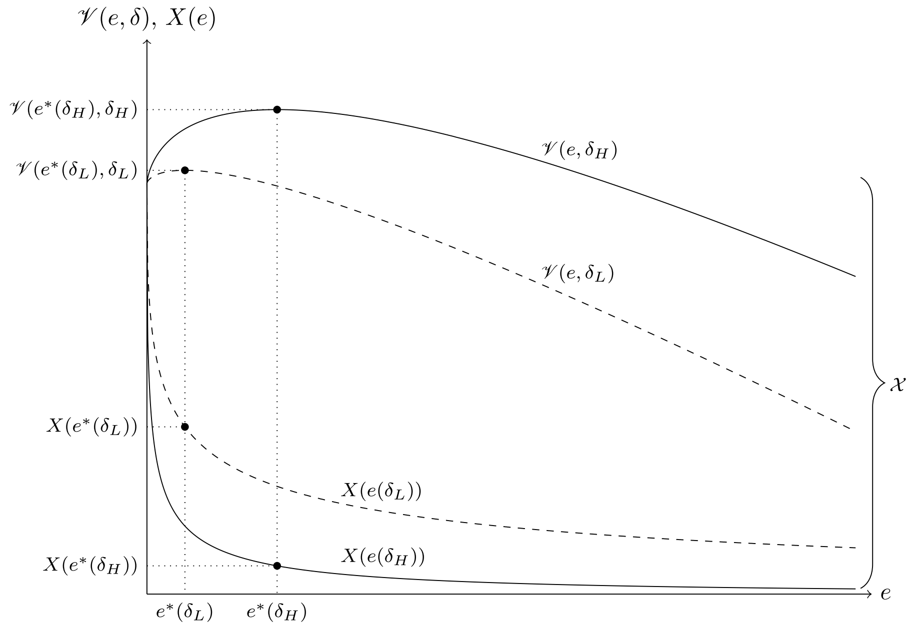

Fig. 1. The optimal signal and effort for $\delta_H > \delta_L$.

those that are less in need of liquidity. The firms with lower quality assets will then securitise a larger proportion of these assets to sell.

These theoretical predictions align with Cardone-Riportella et al. (2010), who show that firms with low liquidity will have lower performance and subsequently securitise more. The driving force behind my results is δ, which is supported by Martin-Oliver and Saurina (2007), Agostino and Mazzuca (2009) and Cerrato et al. (2012) who find that the key motivating factor behind securitisation in Spanish, Italian and UK banks, respectively, over the period 1999–2006 was a need for liquidity. My results can also accommodate Bannier and Hansel's (2008) finding that banks with lower liquidity are more likely to securitise and sell a larger proportion of their assets, and Affinito and Tagliaferri's (2010) finding that less liquid banks with more troubled loans are more likely to securitise assets and at a larger quantity than otherwise.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix. Proofs

Proof of Theorem 1. First, I derive versions of Mailath and von Thadden's preliminary results using my modified framework. In particular, the following condition is analogous to Mailath and von Thadden's (A5):

$$0 \geq g(e_0, q(e), X(e)) \geq -(e-e_0)\Bigg\{\frac{1}{2}g_{11}([e;\theta])_1(e-e_0) + g_{12}([e;\mu]_{23})(q(e)-q(e_0)) + g_{13}([e;\mu]_{23})(X(e)-X(e_0))\Bigg\}, \quad (4)$$

where

$$[e;\theta]_1 := (\theta e_0 + (1-\theta)e, q(e), X(e)),$$

$$[e;\mu]_{23} := (e_0, \mu q(e_0) + (1-\mu)q(e), \mu X(e_0) + (1-\mu)X(e))$$

and

$$g(e, q(\hat{e}), x) := V(e, q(\hat{e}), x) - V(e, q(e_0), X(e_0))$$

for fixed $e_0 \in \mathcal{E}$, arbitrary $e, \hat{e} \in \mathcal{E}$, $\theta \in [0,1]$, $\mu \in [0,1]$ and $X \in \mathcal{X}$. The key change in (4) relative to Mailath and von Thadden's (A5) is the presence of the informed agent's effort technology, which alters the g_{12} term.

I perform a Taylor series expansion on $g(e_0, q(e), X(e))$ around $(e_0, q(e_0), X(e_0))$ and simplify the resulting expression to yield

$$\begin{aligned} g(e_0, q(e), X(e)) &= g_2(e_0, q(e_0), X(e_0))(q(e)-q(e_0)) \\ &+ g_3(e_0, q(e_0), X(e_0))(X(e)-X(e_0)) \\ &+ \frac{1}{2}g_{22}([e;\gamma]_{23})(q(e)-q(e_0))^2 + \frac{1}{2}g_{33}([e;\gamma]_{23})(X(e)-X(e_0))^2 \\ &+ g_{23}([e;\gamma]_{23})(q(e)-q(e_0))(X(e)-X(e_0)), \end{aligned} \quad (5)$$

for some $\gamma \in [0,1]$. Substituting (5) into (4),

$$\begin{aligned} 0 \geq\; & g_2(e_0, q(e_0), X(e_0))(q(e)-q(e_0)) + g_3(e_0, q(e_0), X(e_0))(X(e)-X(e_0)) \\ &+ \frac{1}{2}g_{22}([e;\gamma]_{23})(q(e)-q(e_0))^2 + \frac{1}{2}g_{33}([e;\gamma]_{23})(X(e)-X(e_0))^2 \\ &+ g_{23}([e;\gamma]_{23})(q(e)-q(e_0))(X(e)-X(e_0)) \\ \geq\; & -(e-e_0)\Bigg\{\frac{1}{2}g_{11}([e;\theta]_1)(e-e_0) + g_{12}([e;\mu]_{23})(q(e)-q(e_0)) \\ &+ g_{13}([e;\mu]_{23})(X(e)-X(e_0))\Bigg\}, \end{aligned}$$

and dividing through by $(e-e_0)$,

$$\begin{aligned} 0 \geq\; & g_2(e_0, q(e_0), X(e_0))\frac{q(e)-q(e_0)}{e-e_0} + g_3(e_0, q(e_0), X(e_0))\frac{X(e)-X(e_0)}{e-e_0} \\ &+ \frac{1}{2}g_{22}([e;\gamma]_{23})\frac{(q(e)-q(e_0))^2}{e-e_0} + \frac{1}{2}g_{33}([e;\gamma]_{23})\frac{(X(e)-X(e_0))^2}{e-e_0} \\ &+ g_{23}([e;\gamma]_{23})\frac{(q(e)-q(e_0))(X(e)-X(e_0))}{e-e_0} \\ \geq \end{aligned}$$

$$- \frac{1}{2} g_{11}([e;\theta]_1)(e-e_0) - g_{12}([e;\mu]_{23})(q(e)-q(e_0))$$
$$- g_{13}([e;\mu]_{23})(X(e)-X(e_0)). \quad (6)$$

Finally, taking the limit as e approaches e_0 from above in (6) gives

$$0 \geq g_2(e_0, q(e_0), X(e_0)) \lim_{e \searrow e_0} \left[\frac{q(e)-q(e_0)}{e-e_0} \right] + g_3(e_0, q(e_0), X(e_0)) \lim_{e \searrow e_0} \left[\frac{X(e)-X(e_0)}{e-e_0} \right] \geq 0.$$

This condition implies that X is differentiable at e_0 and that

$$g_2(e_0, q(e_0), X(e_0)) \lim_{e \searrow e_0} \left[\frac{q(e)-q(e_0)}{e-e_0} \right] + g_3(e_0, q(e_0), X(e_0)) \lim_{e \searrow e_0} \left[\frac{X(e)-X(e_0)}{e-e_0} \right] = 0.$$

Therefore, I have shown that when $V_3(e_0, q(e_0), X(e_0)) \neq 0$ and when X is continuous at e_0 that X is differentiable at this point and the derivative satisfies

$$X'(e_0) = -\frac{V_2(e_0, q(e_0), X(e_0)) \cdot q'(e_0)}{V_3(e_0, q(e_0), X(e_0))}.$$

To then show that if $e \to e_0$ then $V(e_0, q(e_0), X(e)) \to V(e_0, q(e_0), X(e_0))$ the proof follows Mailath and von Thadden's Lemma D and so will not be reproduced. The key point to note is that the proof continues to go through as I have assumed that q is continuous and therefore, for each $\epsilon > 0$ and $e \in \mathcal{E}$, there is a $\theta > 0$ such that

$$e_0 \in \mathcal{E} \text{ and } |e-e_0| < \theta \Rightarrow |q(e)-q(e_0)| < \epsilon,$$

and hence I still have, as required,

$$|e-e_0| < \theta \;\to\; |V(e_0, q(e), x) - V(e_0, q(e_0), x)| < \epsilon.$$

Having shown convergence, the final step is to show the continuity of X at e_0. For this, Mailath and von Thadden's proof of Theorem 3 continues to apply. They employ the compactness of $\mathcal{X}$, the assumption on V_3, and the Bolzano–Weierstrass Theorem to find a convergent subsequence within $\mathcal{X}$ that demonstrates continuity of X at e_0. □

Proof of Lemma 1. Given (IC), an application of Theorem 1 yields the following differential equation

$$X'(e) = -\frac{x \cdot q'(\hat{e})}{q(\hat{e}) - \delta q(e)}\Big|_{\hat{e}=e, x=X(e)} = \frac{1}{\delta-1} \frac{X(e) \cdot q'(e)}{q(e)}. \quad (7)$$

Condition (7) is a first-order differential equation

$$X'(e)(1-\delta)q(e) + X(e)q'(e) = 0$$

and can, therefore, be solved by separating the variables

$$\frac{X'(e)}{X(e)} = \frac{1}{\delta-1} \frac{q'(e)}{q(e)}.$$

Using integration by parts, this equals

$$\int \frac{X'(e)}{X(e)} de = \frac{1}{\delta-1} \int \frac{q'(e)}{q(e)} de$$
$$\Rightarrow \ln[X(e)] = \frac{1}{\delta-1} \ln[q(e)] + c_1 = \ln\left[q(e)^{\frac{1}{\delta-1}}\right] + c_1.$$

The right-hand side can be simplified as

$$\ln\left[q(e)^{\frac{1}{\delta-1}}\right] + c_1 = \ln\left[e^{\ln\left[q(e)^{\frac{1}{\delta-1}}\right]+c_1}\right] = \ln\left[e^{\ln\left[q(e)^{\frac{1}{\delta-1}}\right]} e^{c_1}\right] = \ln\left[q(e)^{\frac{1}{\delta-1}} e^{c_1}\right],$$

which implies $X(e) = q(e)^{\frac{1}{\delta-1}} c_2$ where $c_2 = e^{c_1}$. For $e = \underline{e}$ the firm will choose $X(\underline{e}) = 1$. Inserting this boundary condition into $X(e)$ yields $c_2 = q(\underline{e})^{\frac{1}{1-\delta}}$, which is then substituted to give $X(e) = q(e)^{\frac{1}{\delta-1}} q(\underline{e})^{\frac{1}{1-\delta}} = [q(\underline{e})/q(e)]^{\frac{1}{1-\delta}}$. □

Proof of Lemma 2. Partially differentiating (3) with respect to the firm's choice of effort yields

$$\mathscr{V}_1(e,\delta) = \delta q'(e) + \frac{\delta}{\delta-1}(1-\delta) q(e)^{\frac{\delta}{\delta-1}-1} q(\underline{e})^{\frac{1}{1-\delta}} q'(e) - 1,$$
$$= \delta q'(e) + \delta\left(\frac{1-\delta}{\delta-1}\right) q(e)^{\frac{1}{\delta-1}} q(\underline{e})^{\frac{1}{1-\delta}} q'(e) - 1,$$
$$= \delta q'(e) - \delta\left(\frac{\delta-1}{\delta-1}\right)\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}} q'(e) - 1,$$
$$= \delta q'(e) - \delta q'(e)\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}} - 1,$$
$$= \delta q'(e)\left[1 - \left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right] - 1.$$

The cross partial derivative with respect to δ is

$$\mathscr{V}_{12}(e,\delta) = q'(e) - q'(e)\left[\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}} + \delta \frac{d}{d\delta}\left(\frac{1}{1-\delta}\right)\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}} \ln\left(\frac{q(\underline{e})}{q(e)}\right)\right],$$
$$= q'(e)\left[1 - \left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right]$$
$$- q'(e)\left[\delta(-1)(1-\delta)^{-2} \frac{d}{d\delta}\left(1-\delta\right)\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}} \ln\left(\frac{q(\underline{e})}{q(e)}\right)\right],$$
$$= q'(e)\left[1 - \left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right] - q'(e)\left[\frac{-\delta}{(1-\delta)^2}(-1)\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}} \ln\left(\frac{q(\underline{e})}{q(e)}\right)\right],$$
$$= q'(e)\left[1 - \left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right] - q'(e)\left[\frac{\delta}{(1-\delta)^2}\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}} \ln\left(\frac{q(\underline{e})}{q(e)}\right)\right],$$
$$= q'(e)\left[1 - \left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right] - q'(e)\frac{\delta}{(1-\delta)^2}\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}} \ln\left(\frac{q(\underline{e})}{q(e)}\right).$$

Note that $\mathscr{V}_{12}(\underline{e},\delta) = 0$ since $[q(\underline{e})/q(\underline{e})]^{\frac{1}{1-\delta}} = 1$ and $\ln[q(\underline{e})/q(\underline{e})] = 0$. This implies that $[q(\underline{e})/q(e)]^{\frac{1}{1-\delta}} < 1$ for all $e > \underline{e}$. Therefore, $\ln[q(\underline{e})/q(e)] < 0$ for any $e > \underline{e}$ as $q'(e) > 0$ and, subsequently, $\mathscr{V}_{12}(e,\delta) > 0$ for all $(\underline{e}, \overline{e}] \times (0,1)$. □

Proof of Example 1. The firm's payoff function is

$$\mathscr{V}(e,\delta) = \delta(1+e^\alpha) + (1-\delta)(1+e^\alpha)^{\frac{\delta}{\delta-1}},$$
$$= \delta + \delta e^\alpha + (1+e^\alpha)^{\frac{\delta}{\delta-1}} - \delta(1+e^\alpha)^{\frac{\delta}{\delta-1}}.$$

Note that the cost e has been suppressed as it will play no role in the proof: the result holds for any weakly convex function. The first-order condition with respect to effort is

$$\mathscr{V}_1(e,\delta) = \delta\alpha e^{\alpha-1} + (1-\delta)\left(\frac{\delta}{\delta-1}\right)\left(1+e^\alpha\right)^{\frac{\delta}{\delta-1}-1} \alpha e^{\alpha-1},$$
$$= \delta\alpha e^{\alpha-1} - \delta\alpha e^{\alpha-1}(1+e^\alpha)^{\frac{1}{\delta-1}},$$
$$= \delta\alpha e^{\alpha-1}\left[1 - \left(1+e^\alpha\right)^{\frac{1}{\delta-1}}\right].$$

The second-order condition is

$$\mathscr{V}_{11}(e,\delta) = \delta(1-\alpha)\alpha e^{\alpha-2} - \delta\alpha\left[(\alpha-1)e^{\alpha-2}(1+e^\alpha)^{\frac{1}{\delta-1}} + e^{\alpha-1}\left(\frac{1}{\delta-1}\right)\left(1+e^\alpha\right)^{\frac{1}{\delta-1}-1} \alpha e^{\alpha-1}\right],$$
$$= -\delta(1-\alpha)\alpha e^{\alpha-2} + \delta(1-\alpha)\alpha e^{\alpha-2}(1+e^\alpha)^{\frac{1}{\delta-1}} + \left(\frac{\delta}{1-\delta}\right)\alpha^2 e^\alpha e^{\alpha-2}(1+e^\alpha)^{\frac{1}{\delta-1}-1},$$
$$= -\delta(1-\alpha)\alpha e^{\alpha-2}\left[1 - \left(1+e^\alpha\right)^{\frac{1}{\delta-1}}\right] + \left(\frac{\delta}{1-\delta}\right)\alpha^2 e^\alpha e^{\alpha-2}(1+e^\alpha)^{\frac{1}{\delta-1}-1}.$$

Note that the second-order condition has the same sign as the function $g(e)$, where

$$g(e) := \left(1+e^\alpha\right)^{\frac{1}{\delta-1}}\left[1 + \frac{\alpha}{(1-\delta)(1-\alpha)} \frac{e^\alpha}{(1+e^\alpha)}\right] - 1,$$

and so the problem is equivalent to showing that $g(e) < 0$. Evaluating $g(e)$ at $\underline{e}$ yields $g(\underline{e}) = g(0) = 0$. Moreover, $g(e)$ is strictly decreasing. The first-order condition is

$$g'(e) = \frac{1}{\delta-1}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}-1}\alpha e^{\alpha-1} + \frac{\alpha}{(1-\delta)(1-\alpha)}\left[\frac{\alpha e^{\alpha-1}(1+e^{\alpha})-e^{\alpha}\alpha e^{\alpha-1}}{(1+e^{\alpha})^{2}}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}} + \frac{1}{\delta-1}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}-1}\alpha e^{\alpha-1}\left(\frac{e^{\alpha}}{1+e^{\alpha}}\right)\right],$$

$$= \frac{\alpha e^{\alpha-1}}{(\delta-1)}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}-1} + \frac{\alpha}{(1-\delta)(1-\alpha)}\left[\frac{\alpha e^{\alpha-1}}{(1+e^{\alpha})^{2}}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}} + \frac{\alpha e^{\alpha-1}}{(\delta-1)}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}-1}\left(\frac{e^{\alpha}}{1+e^{\alpha}}\right)\right].$$

Simplifying further yields

$$g'(e) = -\frac{\alpha e^{\alpha-1}}{(1-\delta)}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}-1} + \alpha e^{\alpha-1}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}-2}\left[\frac{\alpha}{(1-\delta)(1-\alpha)} - \frac{\alpha e^{\alpha}}{(1-\delta)^{2}(1-\alpha)}\right].$$

Then, using

$$\frac{1}{(1-\delta)} = \frac{(1-\delta)(1+e^{\alpha})}{(1-\delta)^{2}(1+e^{\alpha})} \quad \text{and} \quad \frac{\alpha}{(1-\delta)(1-\alpha)} = \frac{\alpha(1-\delta)}{(1-\delta)^{2}(1-\alpha)},$$

implies that

$$g'(e) = -\alpha e^{\alpha-1}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}-2}\left[\frac{(1-\delta)(1+e^{\alpha})-\alpha(1-\delta)+\alpha e^{\alpha}}{(1-\delta)^{2}(1-\alpha)}\right],$$

$$= -\alpha e^{\alpha-1}\left(1+e^{\alpha}\right)^{\frac{1}{\delta-1}-2}\left[\frac{(1-\delta)[1-\alpha+e^{\alpha}]+\alpha e^{\alpha}}{(1-\delta)^{2}(1-\alpha)}\right] < 0.$$

Therefore, $g(e)$ is a strictly decreasing function over the domain $e > 0$. Since $\mathscr{V}_{11}(e,\delta)$ has the same sign as $g(e)$, this implies that $\mathscr{V}(e,\delta)$ is strictly concave in e. □

Proof of Proposition 1.

(i) I first solve for optimal effort in the symmetric information case. In this first-best case, the firm sets $X(e) = 1$ for all e. This yields the problem $\max_{e\in\mathcal{E}} V(e,q(e),1) = \max_{e\in\mathcal{E}} q(e) - e$, which provides the unique optimal effort e^{*} implicitly defined by the solution of $q'(e) = 1$ since $q''(e) < 0$. The problem under asymmetric information is $\max_{e\in\mathcal{E}} V(e,q(e),X(e)) = \max_{e\in\mathcal{E}} \delta q(e) + (1-\delta)q(e)^{\frac{\delta}{\delta-1}}q(\underline{e})^{\frac{1}{1-\delta}} - e$ and, by Lemma 2, optimal effort $e^{*}(\delta)$ is implicitly defined by the solution to

$$\delta q'(e)\left[1-\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right] = 1.$$

Given that $\mathscr{V}$ is strictly concave, this first-order condition is both necessary and sufficient. Since $\delta < 1$ and $[q(\underline{e})/q(e)]^{\frac{1}{1-\delta}} < 1$ for all $e > \underline{e}$, optimal effort is greater under symmetric information $e^{*} > e^{*}(\delta)$.

(ii) Under the supposition that $\mathscr{V}$ is strictly concave, Berge's maximum theorem implies that optimal effort $e^{*}(\delta)$ is single-valued and continuous. I can, therefore, use the implicit function theorem to derive this comparative static because $\max_{e\in\mathcal{E}} \mathscr{V}(e,\delta)$ has a unique maximiser. The second-order condition is

$$\mathscr{V}_{11}(e,\delta) = \delta q''(e) - \delta q(\underline{e})^{\frac{1}{1-\delta}}\left[q''(e)q(e)^{\frac{1}{\delta-1}} + q'(e)\left(\frac{1}{\delta-1}\right)q(e)^{\frac{1}{\delta-1}-1}q'(e)\right],$$

$$= \delta q''(e)\left[1-\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right] + q'(e)\left(\frac{\delta}{1-\delta}\right)\left(\frac{q'(e)}{q(e)}\right)\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}.$$

Consequently, an application of the implicit function theorem yields

$$\frac{d}{d\delta}e^{*}(\delta) = -\left[\frac{q'(e)\left[1-\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right] - q'(e)\frac{\delta}{(1-\delta)^{2}}\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\ln\left(\frac{q(\underline{e})}{q(e)}\right)}{\delta q''(e)\left[1-\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}\right] + q'(e)\left(\frac{\delta}{1-\delta}\right)\left(\frac{q'(e)}{q(e)}\right)\left(\frac{q(\underline{e})}{q(e)}\right)^{\frac{1}{1-\delta}}}\right].$$

The numerator is positive by Lemma 2 and the denominator is negative by the assumption of strict concavity. Therefore, $\frac{d}{d\delta}e^{*}(\delta) > 0$ for $e > \underline{e}$.

(iii) Since $\mathscr{V}$ is continuous, strict concavity implies that I can use the envelope theorem to derive

$$\frac{\partial}{\partial\delta}\mathscr{V}(e,\delta)\Big|_{e=e^{*}(\delta)} = q(e^{*}(\delta)) + q(e^{*}(\delta))\frac{1}{(1-\delta)^{2}}\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)^{\frac{1}{1-\delta}}\ln\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right) - q(e^{*}(\delta))\left[\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)^{\frac{1}{1-\delta}} + \frac{\delta}{(1-\delta)^{2}}\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)^{\frac{1}{1-\delta}}\ln\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)\right].$$

Taking common factors and simplifying yields

$$q(e^{*}(\delta))\left[1-\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)^{\frac{1}{1-\delta}}\right] + q(e^{*}(\delta))\frac{(1-\delta)}{(1-\delta)^{2}}\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)^{\frac{1}{1-\delta}}\ln\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)$$

$$= q(e^{*}(\delta))\left[1-\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)^{\frac{1}{1-\delta}}\right] + q(e^{*}(\delta))\frac{1}{1-\delta}\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right)^{\frac{1}{1-\delta}}\ln\left(\frac{q(\underline{e})}{q(e^{*}(\delta))}\right).$$

Now, define $y := q(\underline{e})/q(e^{*}(\delta))$ and note that, since $q(e^{*}(\delta)) > q(\underline{e})$ for all $e^{*}(\delta) > \underline{e}$, $y \in (0,1)$. Moreover, as $q(\cdot)$ is just a positive constant, showing that the value function is increasing in δ is equivalent to showing that $f(y) > 0$, where, by taking common factors,

$$f(y) := 1 - y^{\frac{1}{1-\delta}} + \frac{1}{1-\delta}y^{\frac{1}{1-\delta}}\ln(y).$$

I first find the limit values of $f(y)$ and then show that this function is strictly decreasing from one limit value to another. Using L'Hopital's rule, the upper limit is

$$\lim_{y\to 0} f(y) = 1 - \lim_{y\to 0} y^{\frac{1}{1-\delta}} + \frac{1}{1-\delta}\lim_{y\to 0} y^{\frac{1}{1-\delta}}\ln(y) = 1 + \frac{1}{1-\delta}\lim_{y\to 0} y^{\frac{1}{1-\delta}}\ln(y),$$

$$= 1 + \frac{1}{1-\delta}\lim_{y\to 0}\frac{\ln(y)}{y^{\frac{1}{\delta-1}}} = 1 + \frac{1}{1-\delta}\lim_{y\to 0}\frac{\frac{d}{dy}\ln(y)}{\frac{d}{dy}y^{\frac{1}{\delta-1}}},$$

$$= 1 + \frac{1}{1-\delta}\lim_{y\to 0}\frac{\frac{1}{y}}{(\frac{1}{\delta-1})y^{\frac{1}{\delta-1}-1}},$$

$$= 1 + \frac{1}{1-\delta}\lim_{y\to 0}\frac{1}{(\frac{1}{\delta-1})y^{\frac{1}{\delta-1}}},$$

$$= 1 + \frac{\delta-1}{1-\delta}\lim_{y\to 0} y^{\frac{1}{1-\delta}},$$

$$= 1 - \lim_{y\to 0} y^{\frac{1}{1-\delta}},$$

$$= 1.$$

The lower limit is

$$\lim_{y\to 1} f(y) = 1 - 1 + \frac{1}{1-\delta}\ln(1) = 0.$$

Finally, note that $f(y)$ is strictly decreasing as

$$f'(y) = -\left(\frac{1}{1-\delta}\right)y^{\frac{1}{1-\delta}-1} + \left(\frac{1}{1-\delta}\right)\left[\left(\frac{1}{1-\delta}\right)y^{\frac{1}{1-\delta}-1}\ln(y) + y^{\frac{1}{1-\delta}}\frac{1}{y}\right],$$

$$= -\left(\frac{1}{1-\delta}\right)y^{\frac{\delta}{1-\delta}} + \frac{1}{(1-\delta)^{2}}y^{\frac{\delta}{1-\delta}}\ln(y) + \left(\frac{1}{1-\delta}\right)y^{\frac{\delta}{1-\delta}},$$

$$= \frac{1}{(1-\delta)^{2}}y^{\frac{\delta}{1-\delta}}\ln(y) < 0.$$

Taken together, these results imply that $f(y)$ is a strictly decreasing function that goes from a limit value of $f(0) = 1$ to a limit value of $f(1) = 0$. Therefore, $f(y) > 0$ for all $y \in (0,1)$ and $\delta \in (0,1)$ and I can conclude that $\frac{\partial}{\partial\delta}\mathscr{V}(e,\delta)\Big|_{e=e^{*}(\delta)} > 0$.

Suppose $\arg\max_{e\in\mathcal{E}} \mathscr{V}(e,\delta)$ in nonempty.

(iv) By Lemma 2, $\mathscr{V}$ has increasing marginal returns. Therefore, Theorem 1 and Corollary 1 of Edlin and Shannon (1998) implies that any interior selection of the argmax will be strictly increasing. □

References

Affinito, M., Tagliaferri, E., 2010. Why did (or do) banks securtize their loans? Evidence from Italy. J. Financial Stabil. 6 (1), 189–202.
Agostino, M., Mazzuca, M., 2009. Why did banks securtize? Evidence from Italy. Bancaria 9, 18–38.
Bannier, C.E., Hansel, D.N., 2008. Determinents of European banks' engagement in loan securitization. Technical Report 10, Deutsche Bundesbank.
Biais, B., Mariotti, T., 2005. Strategic liquidity supply and security design. Rev. Econ. Stud. 72, 615–649.
Cardone-Riportella, C., Samaniego-Medinab, R., Trujillo-Ponceb, A., 2010. What drives bank securitisation? The Spanish experience. J. Bank. Finance 34, 2639–2651.
Cerrato, M., Choudhry, M., Crosby, J., Olukuru, J., 2012. Why do UK banks securitize?.
Chemla, G., Hennessy, C.A., 2014. Skin in the game and moral hazard. J. Finance 69 (4), 1597–1641.
Daley, B., Green, B., Vanasco, V., 2020. Securitization, ratings, and credit supply. J. Finance 75 (2), 1037–1082.
Daley, B., Green, B., Vanasco, V., 2023. Designing securities for scrutiny. Rev. Financ. Stud. 36 (9), 3693–3737.
DeMarzo, P.M., 2005. The pooling and tranching of securities: A model of informed intermediation. Rev. Financial Stud. 18, 1–35.
DeMarzo, P., Duffie, D., 1999. A liquidity-based model of security design. Econometrica 67, 65–99.
Edlin, A.S., Shannon, C., 1998. Strict monotonicity in comparative statics. J. Econ. Theory 81, 201–219.
Hartman-Glaser, B., Piskorski, T., Tchistyi, A., 2012. Optimal securitization with moral hazard. J. Financ. Econ. 104 (1), 186–202.
In, Y., Wright, J., 2017. Signaling private choices. Rev. Econ. Stud. 1, 1–24.
Kohlberg, E., Mertens, J.-F., 1986. On the strategic stability of equilibria. Econometrica 54 (5), 1003–1037.
Mailath, G.J., von Thadden, E.-L., 2013. Incentive compatibility and differentiability: New results and classic applications. J. Econ. Theory 143, 1841–1861.
Malamud, S., Rui, H., Whinston, A., 2013. Optimal incentives and securitization of defaultable assets. J. Financ. Econ. 107 (1), 111–135.
Malekan, S., Dionne, G., 2014. Securitization and optimal retention under moral hazard. J. Math. Econ. 55, 74–85.
Martin-Oliver, A., Saurina, J., 2007. Why do banks securitize assets.
Myerson, R.B., 1978. Refinements of the Nash equilibrium concept. Internat. J. Game Theory 7 (2), 73–80.
Vanasco, V., 2017. The downside of asset screening for market liquidity. J. Finance 72 (5), 1937–1982.