Genetic Modelling and Simulation of Flexible Structures

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Abstract: This paper presents an investigation into the utilization of genetic algorithms (GAs) for dynamic modelling and simulation of flexible beam structures. The global search technique of GA is utilised to identify the parameters of a beam model based on one-step-ahead prediction in fixed-free mode; a simple representation of an aircraft wing or robot arm. A pseudo random binary sequence (PRBS) signal is used as the input excitation, covering the dynamic range of interest of the system. The developed model is validated using several validation tests. A comparative performance of the GA model and conventional recursive least squares (RLS) scheme in characterising the system is carried out in the time and frequency domains. Simulation results highlighting the advantages of GA over RLS in linear parametric modelling of flexible structures are given. The developed genetic-modelling approach will further be utilized in the design and implementation of suitable controllers, for vibration suppression in such systems.

Keywords: Flexible beam system, genetic algorithms, parametric modelling, system identification.

1. Introduction

System identification is extensively used as a fundamental requirement in many engineering and scientific applications. The objective of system identification is to find exact or approximate models of dynamic systems based on observed inputs and outputs. Once a model of the physical system is obtained, it can be used for solving various problems such as, to control the physical system or to predict its behaviour under different operating conditions. In many science and engineering applications it is not always possible to have complete knowledge of all the parameters of a dynamic system. To provide accurate modeling or control of such a system, the unknown parameters must be estimated during real-time operation. In this work, parametric identification refers to the determination of the numerical values of structural parameters which minimize the difference between the system to be identified and its model [1].

Least squares (LS), recursive least squares (RLS) and prediction error method (PEM) are among many conventional methods of identification of auto-regressive with exogenous inputs (ARX) and auto-regressive moving average with exogenous inputs (ARMAX) models. These methods, however, have the disadvantage of getting stuck at local minima, which can result in poor models. Therefore, for a highly non-linear optimisation problem, genetic algorithm (GA) identification technique is sought to avoid the problem of convergence to local minima [1]. Typically the GA starts with little or no knowledge of the correct solution and depends entirely on responses from an interacting environment and its evolution operators to arrive at good solutions. By dealing with several independent points, the GA samples the search space in parallel and hence is less susceptible to converging to a suboptimal solution. In this way, GAs have been shown to be capable of locating high performance areas in complex domains without experiencing the difficulties associated with high dimensionality or false optima, as may occur with gradient descent techniques. Thus, GAs have been recognised as a powerful tool in many control applications such as parameter identification and control structure design [2, 3, 4]. This paper investigates the utilisation of GA optimisation technique based on one-step-ahead (OSA) prediction for modelling a single-input single-output (SISO) flexible beam system; a simple representation of an aircraft wing or robot arm. The resulting model is subjected to several validation methods, and a comparative assessment of the result with RLS modelling is presented and discussed. The developed approach will further be utilized in the design and implementation of suitable controllers, for vibration suppression in such systems.

2.   The Flexible Beam System

A flexible beam in fixed-free mode is shown in Figure 1, where $U(x,t)$ represents an applied force at a distance x from the fixed end at time t and $y(x,t)$ is the resulting beam deflection from its stationary position at the point where the force has been applied. L is the length of the beam and dx is a differential length of the beam.
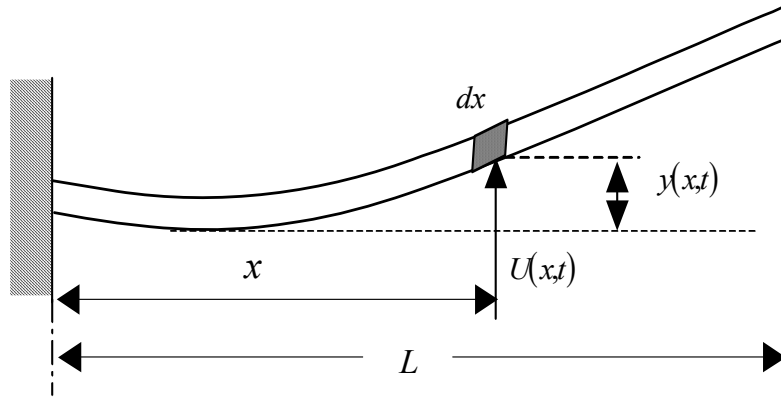
Figure 1: Fixed-free beam

The motion of a beam in transverse vibration in response to an applied force $U(x,t)$ is governed by the well-known fourth-order partial differential equation (PDE) [5]:

$$\frac{\mu^2 \partial^4 y(x,t)}{\partial x^4} + \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{1}{m} U(x,t) \qquad (1)$$

where μ is a beam constant given by $\mu^2 = \frac{EI}{\rho A}$ with E, I, ρ and A representing Young modulus, moment of inertia of the beam, mass density and cross-sectional area respectively, and m is the mass of the beam. The boundary conditions at the fixed end are given as:

$$y(0,t) = 0 \quad \text{and} \quad \frac{\partial y(0,t)}{\partial x} = 0 \qquad (2)$$

and the boundary conditions for free end of the beam are given as:

$$\frac{\partial^2 y(L,t)}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^3 y(L,t)}{\partial x^3} = 0 \qquad (3)$$

Note that the model in equation (1) does not incorporate damping. The finite difference (FD) method is used as a numerical solution to the PDE in equation (1) [5]. This involves discretization of the beam into a finite number of equal length sections, each of length Δx, and the beam motion (deflection) for the end of each section is considered at equally spaced time steps of duration Δt. Using a first order central FD method the partial derivatives $\frac{d^2 y}{dt^2}$ and $\frac{d^4 y}{dx^4}$ can be approximated as:

$$\frac{d^2 y(x,t)}{dt^2} = \frac{y_{i,j+1} - 2y_{i,j} + y_{i,j-1}}{(\Delta t)^2} \text{ and}$$

$$\frac{d^4 y(x,t)}{dt^4} = \frac{y_{i+2,j} - 4y_{i+1,j} + 6y_{i,j} - 4y_{i-1,j} + y_{i-2,j}}{(\Delta x)^4} \tag{4}$$

Substituting for $\dfrac{d^2 y}{dt^2}$ and $\dfrac{d^4 y}{dx^4}$ from equation (4) into equation (1) and simplifying yields

$$y_{i,j+1} = 2y_{i,j} - y_{i,j-1} - \lambda^2 \left\{ y_{i+2,j} - 4y_{i+1,j} + 6y_{i,j} - 4y_{i-1,j} + y_{i-2,j} \right\} + \frac{(\Delta t)^2}{m} U(x,t) \tag{5}$$

where $\lambda^2 = \dfrac{(\Delta t)^2}{(\Delta x)^4} \mu^2$.

The boundary conditions in equations (2) and (3) can be approximated using the FD method as:

$$y_{o,j=0}, \qquad \frac{y_{i,j} - y_{-1,j}}{2\Delta x} = 0$$

$$\frac{y_{n+1,j} - 2y_{n,j} + y_{n-1,j}}{(\Delta x)^2} = 0, \quad \frac{y_{n+2,j} - 2y_{n+1,j} + 2y_{n-1,j} - y_{n-2,j}}{2(\Delta x)^3} \tag{6}$$

Substituting the discretised boundary conditions for the fixed and free ends from equation (6) into equation (5) yields the deflection at the grid points along the beam as:

$$y_{1,j+1} = -y_{1,j-1} - \lambda^2 \left\{ \left(7 - \frac{2}{\lambda^2}\right) y_{1,j} - 4y_{2,j} + y_{3,j} \right\} + \varphi$$

$$y_{2,j+1} = -y_{2,j-1} - \lambda^2 \left\{ -4y_{1,j} + \left(6 - \frac{2}{\lambda^2}\right) y_{2,j} - 4y_{3,j} + y_{4,j} \right\} + \varphi$$

$$\vdots$$

$$y_{n,j+1} = -y_{n,j-1} - \lambda^2 \left\{ 2y_{n-2,j} + -4y_{n-1,j} + \left(2 - \frac{2}{\lambda^2}\right) y_{n,j} - 4y_{2,j} \right\} + \varphi$$

where, $\varphi = \dfrac{(\Delta t)^2}{m} U(x,t)$. The above equations can be written in a matrix form as:

$$\mathbf{Y}_{j+1} = -\mathbf{Y}_{j-1} - \lambda^2 \mathbf{S} \mathbf{Y}_j + \frac{(\Delta t)^2}{m} U(x,t) \tag{7}$$

where,

$$\mathbf{Y}_{j+1} = \begin{bmatrix} y_{1,j+1} \\ y_{2,j+1} \\ \vdots \\ \vdots \\ y_{n,j+1} \end{bmatrix}, \quad \mathbf{Y}_j = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ \vdots \\ y_n \end{bmatrix}, \quad \mathbf{Y}_{j\text{-}1} = \begin{bmatrix} y_{1,j-1} \\ y_{2,j-1} \\ \vdots \\ \vdots \\ y_{n,j-1} \end{bmatrix},$$

and **S** is a matrix, known as stiffness matrix, given (for $n = 20$, say) as:

$$\mathbf{S} = \begin{bmatrix} a & -4 & 1 & 0 & 0 & 0 & \cdots & \cdots & 0 \\ -4 & b & -4 & 1 & 0 & 0 & \cdots & \cdots & 0 \\ 1 & -4 & b & -4 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & -4 & b & -4 & 1 & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & 1 & -4 & b & -4 & 1 \\ \cdots & \cdots & \cdots & \cdots & 0 & 1 & -4 & c & -2 \\ \cdots & \cdots & \cdots & \cdots & 0 & 0 & 2 & -4 & d \end{bmatrix}$$

where $a = 7 - \dfrac{2}{\lambda^2}$, $b = 6 - \dfrac{2}{\lambda^2}$, $c = 5 - \dfrac{2}{\lambda^2}$ and $d = 2 - \dfrac{2}{\lambda^2}$. The stability of the algorithm in equation (7) is satisfied by $0 < \lambda^2 \le 0.25$ [6]. The first five resonance modes of the fixed-free beam, as obtained through theoretical analysis, are located at 1.875 Hz, 11.751 Hz, 32.902 Hz, 64.476 Hz and 106.583 Hz respectively with the first two modes being dominant [7]. The resonance modes obtained from the simulation exercise were found to be at 1.98 Hz, 11.74 Hz, 32.19 Hz, 62.10 Hz, and 100.55 Hz respectively. Hence, the simulation algorithm thus developed gives a reasonably accurate representation of the dynamic behaviour of the beam.

3. Identification techniques

The characteristics of flexible structure systems are generally of distributed nature, amounting to resonance modes of vibration [8]. The primary interest in this work lies in locating frequencies of these resonance modes, which represent the dynamic behaviour of the system. An important stage in a system identification process is the selection of the type and characteristics of plant excitation signal [9]. In order

to allow the dynamics of interest of the system be incorporated within the model, a PRBS signal covering the dynamic range of interest of the system was used in estimating the model parameters. A comparative assessment of RLS and GA is carried out with the view to find accurate model of the flexible beam system.

3.1 Genetic algorithm

Genetic algorithms, introduced and studied by John Holland and his students at the University of Michigan [10], are search algorithms that are based on the mechanics of natural selection and natural genetics [11]. They perform a global, random, parallel search for an optimal solution using simple computations. Starting with an initial population of genetic structures, genetic inheritance operations based on selection, mating, and mutation are performed to generate "offspring" that compete for survival ("survival of the fittest") to make up the next generation of population structures.

From an operational perspective, a GA comprises two basic elements-a set of individuals, i.e., potential solutions (the population) and a set of biologically inspired operators active over the population. A new set of approximations/ solutions is created at each generation, by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using the operators. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adaptation [12, 13]. The GA search process is carried out according to the algorithm depicted in Figure 2 [11].

To begin the search cycle, a fitness or performance measure must be defined as a function of members of the population. Secondly, several parameters must be set. These include, as a minimum: the population size, and frequency of mutation and crossover. An initial population is then generated (randomly, possibly with constraint), and the fitness of each individual is computed. If the fitness of any individual satisfies the stopping criteria, the search is terminated and the solution returned. Otherwise, a new generation is created through reproduction, crossover, mutation and possibly other operations.
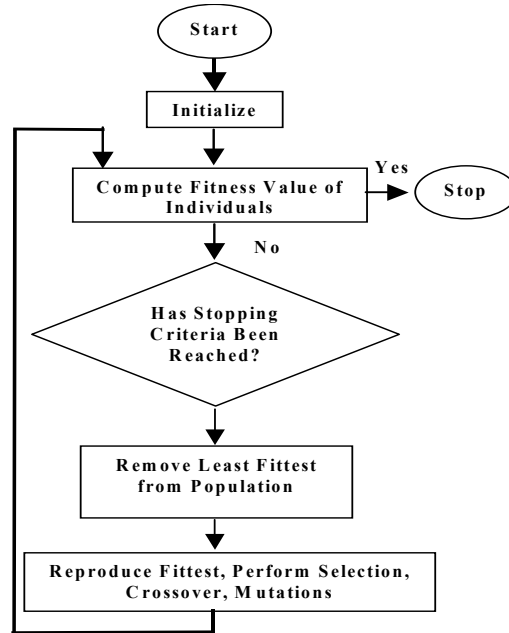
Figure 2: Flowchart of basic genetic algorithm

In this investigation, randomly selected parameters are optimised for different, arbitrarily chosen order to fit to the system by applying the working mechanism of GA as explained above. In identifying the parameters of the model the mean-squared error between the actual output $y(n)$ and the predicted output $\hat{y}(n)$ is adopted as the fitness function:

$$f(e) = \frac{1}{m} \sum_{i=1}^{m} \left(\left| y(m) - \hat{y}(m) \right| \right)^2 \tag{8}$$

where m is the number of input/output samples. With the fitness function given above, the global search technique of the GA is utilised to obtain the best parameters among all attempted orders for the system.

3.2 Recursive least squares algorithm

Adaptive algorithms such as RLS are able to provide a complete model of a system based on known parameters and previous history of inputs and outputs. The RLS algorithm, based on the well known least squares (LS) method, uses an iterative refinement technique to continuously tune estimated parameters using knowledge of some existing parameters as well as information obtained from the continuous

operation of the system [5]. While LS provides a best-fit estimate for a set of recorded data, the RLS algorithm creates a continuous estimate for a set of unknown system parameters.

The RLS algorithm is described by the following set of equations:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\,E(t) \tag{9}$$

$$K(t) = \frac{\lambda^{-1}\,P(t-1)\,u(t)}{1 + \lambda^{-1}\,u(t)^T\,P(t-1)\,u(t)} \tag{10}$$

$$P(t) = \lambda^{-1}\,P(t-1) \;-\; \lambda^{-1}K(t)\,u(t)^T P(t-1) \tag{11}$$

$$E(t) = y(t) - u(t)^T\,\hat{\theta}(t-1) \tag{12}$$

A diagrammatic representation of the RLS algorithm is given in Figure 3, which describes the relationship between the regression vector $u(t)$, the unknown parameter vector θ, and the system output $y(t)$. At each iteration of the algorithm, a new estimate $\theta(t)$ is calculated based on recent values of $u(t)$ and $y(t)$. The system output $y(t)$ is one time step behind the input $u(t)$ and the parameter vector $\theta(t)$. λ is defined as the forgetting factor used to help the algorithm converge to the global minimum. However, the use of a forgetting factor could cause the predicted values of parameters to fluctuate rather than converge to a certain value. The level of fluctuation depends on the value of λ, the smaller the value the bigger the fluctuation in the parameter values. The computational cost of the RLS algorithm may be reduced by defining:
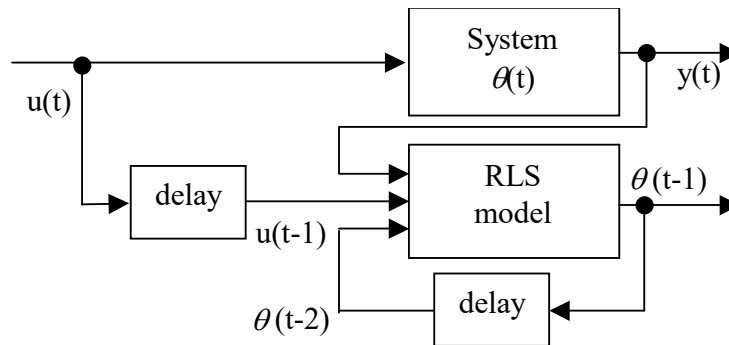

Figure 3: Diagrammatic representation of the RLS algorithm

$$J(t) = \lambda^{-1}\,P(t-1)\,u(t) \tag{13}$$

Hence, equations (10) and (11) can be simplified as:

$$K(t) = \left[1 + u(t)^{T} J(t)\right]^{-1} J(t) \tag{14}$$

$$P(t) = \lambda^{-1} P(t-1) - K(t) J(t)^{T} \tag{15}$$

4. Model Validation

Model validity tests are procedures designed to detect the adequacy of a fitted model. In practice, the model of the system will be unknown and the detection of an inadequate fit is more challenging. A common measure of predictive accuracy used in control and system identification is to compute the one step-ahead prediction of the system output. This is expressed as:

$$\hat{y}(t) = f\left(u(t), u(t-1), \ldots, u(t-n_u), y(t-1), \ldots, y(t-n_y)\right)$$

where $f(\cdot)$ is a non-linear function, u and y are the inputs and outputs respectively. The residual or prediction is given as:

$$\varepsilon(t-1) = y(t) - \hat{y}(t)$$

Often $\hat{y}(t)$ will be a relatively good prediction of $y(t)$ over the estimation set even if the model is biased because the model was estimated by minimizing the prediction error. Correlation tests are also used to validate the model. If a model is adequate then the residuals or prediction errors $\varepsilon(t)$ should be unpredictable from all linear and nonlinear combinations of past inputs and outputs. The derivation of simple tests that can detect these conditions is complex, but it can be shown that the following conditions should hold [9]:

$$\phi_{\varepsilon\varepsilon}(\tau) = E[\varepsilon(t-\tau)\varepsilon(t)] = \delta(\tau) \tag{16}$$

$$\phi_{u\varepsilon}(\tau) = E[u(t-\tau)\varepsilon(t)] = 0 \quad \forall \tau \tag{17}$$

$$\phi_{u^2\varepsilon}(\tau) = E[(u^2(t-\tau) - \bar{u}^2(t))\varepsilon(t)] = 0 \quad \forall \tau \tag{18}$$

$$\phi_{u^2\varepsilon^2}(\tau) = E[(u^2(t-\tau) - \bar{u}^2(t))\varepsilon^2(t)] = 0 \quad \forall \tau \tag{19}$$

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$$\phi_{\varepsilon(\varepsilon u)}(\tau)=E[\varepsilon(t)\varepsilon(t-1-\tau)u(t-1-\tau)]=0 \quad \tau\geq0 \tag{20}$$

where, $\phi_{u\varepsilon}(\tau)$ indicates the cross-correlation function between $u(t)$ and $\varepsilon(t)$, $\varepsilon u(t)=\varepsilon(t+1)u(t+1)$, $\delta(\tau)$ is an impulse function. Ideally the model validity tests should detect all the deficiencies in model performance including bias due to internal noise. The cause of the bias will however be different for different model orders and assignments of network input nodes. Consequently the full five tests defined by equations (16) to (20) should be satisfied if $u(.)$'s and $y(.)$'s are used in the model structure. Correlation function between two sequences $\psi_1(t)$ and $\psi_2(t)$ is given by:

$$\hat{\phi}_{\psi_1\psi_2}(\tau)=\frac{\sum\limits_{t=1}^{N-\tau}\psi_1(t)\psi_2(t+\tau)}{\sqrt{\sum\limits_{t=1}^{N}\psi_1^2(t)\sum\limits_{t=1}^{N}\psi_2^2(t)}}$$

In practice normalised correlations are computed. Normalisation ensures that all the correlation functions lie in the range $-1\leq\hat{\phi}_{\psi_1\psi_2}(\tau)\leq1$ irrespective of the signal strengths. The correlations will never be exactly zero for all lags and the 95% confidence bands defined as $1.96/\sqrt{N}$ are used to indicate if the estimated correlations are significant or not, where N is the data length. Therefore, if the correlation functions are within the confidence intervals the model is regarded as adequate [9].

5. Implementation and Results

An aluminium type fixed-free beam with specifications of length as 0.635, μ as 1.351108, mass as 0.0478 kg and lambda as 0.2948 was considered for simulation over 4 seconds. The beam was divided into 20 sections and a sampling time of 0.2ms that satisfies the stability requirements of the FD simulation algorithm and is sufficient to cover all resonance modes of vibration of interest, was utilised. A PRBS signal covering the dynamic range of interest of the system was used to estimate the system model parameters. The primary force was applied at grid point 13. The input and output sensors were placed at grid points 12 and 19 respectively. Based on the OSA prediction, modelling is carried out using the following input vector format:

$$X(t)=[y(t-1),\dots y(t-n); \ u(t),u(t-1),\dots u(t-m)]^T$$

10

wheren $m = n = 10$ and $m = n = 12$ for GA and RLS estimation techniques, respectively.

5.1 GA modelling

Investigations were carried out using the GA based on OSA prediction with different initial values and operator rates. Satisfactory results were achieved with the following set of parameters; generation gap as 0.7, crossover rate as 0.67 and mutation rate as 0.01. The fitness function of equation (8) was adopted. The parameters of the model were represented by real strings with 70 individuals. The GA showed convergence of the function over 90 generations. The algorithm achieved the best mean-square error level of 0.0003028 in the 90th generation. Figures 4 – 7 show the algorithm convergence and the error in time and the simulated output in time and frequency domains with the best parameter set. The vibration modes, as found from the GA estimated outputs are very close to the simulated vibration modes of the flexible beam. The corresponding correlation test functions for identification using GA were found to be within the 95% confidence intervals, indicating an adequate model fit.

5.2 RLS modelling

For comparison purposes, the system was also modelled with RLS algorithm using the same input-output data. Results of the simulation are shown in Figures 8, 9 and 10. The mean-square error for RLS-based model was slightly bigger, 0.0311. The vibration modes observed from the RLS estimated outputs were close to actual values for the first mode but slightly shifted for other modes. The corresponding results of GA modeling indicate that identification using GA performed better than the conventional RLS based modeling in this application. The correlation tests for the RLS-based model were also found to be within the 95% confidence interval.

6. Conclusion

The development of a GA modelling strategy based on OSA prediction has been presented and verified in the identification of a flexible beam system in comparison to an RLS modelling approach. OSA predictions have been used as estimating method and model validity tests using correlation tests have been carried out. It has been shown that with suitable choice of the input data structure and correct model

order, the system data can be predicted with a minimal prediction error. The significance of the GA modelling strategy has been clearly demonstrated through the level of performance achieved in the identification of dynamics of the flexible beam system. The developed GA modelling approach will further be utilized in the design and implementation of suitable controllers, for vibration suppression in flexible structures.

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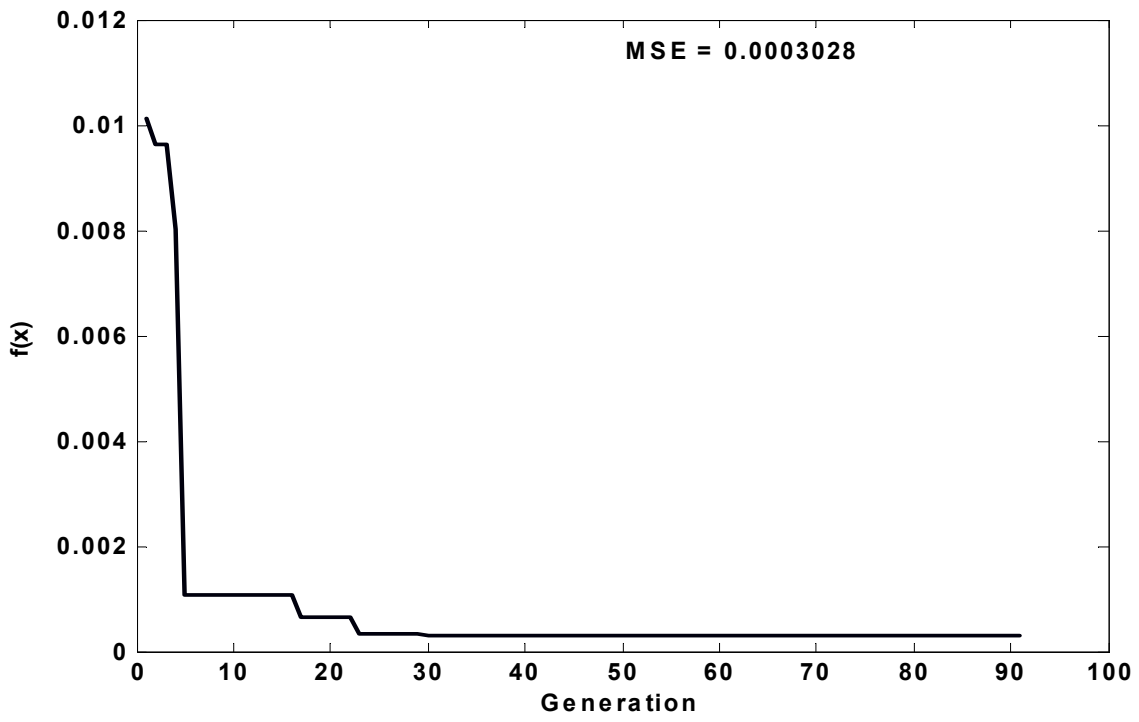
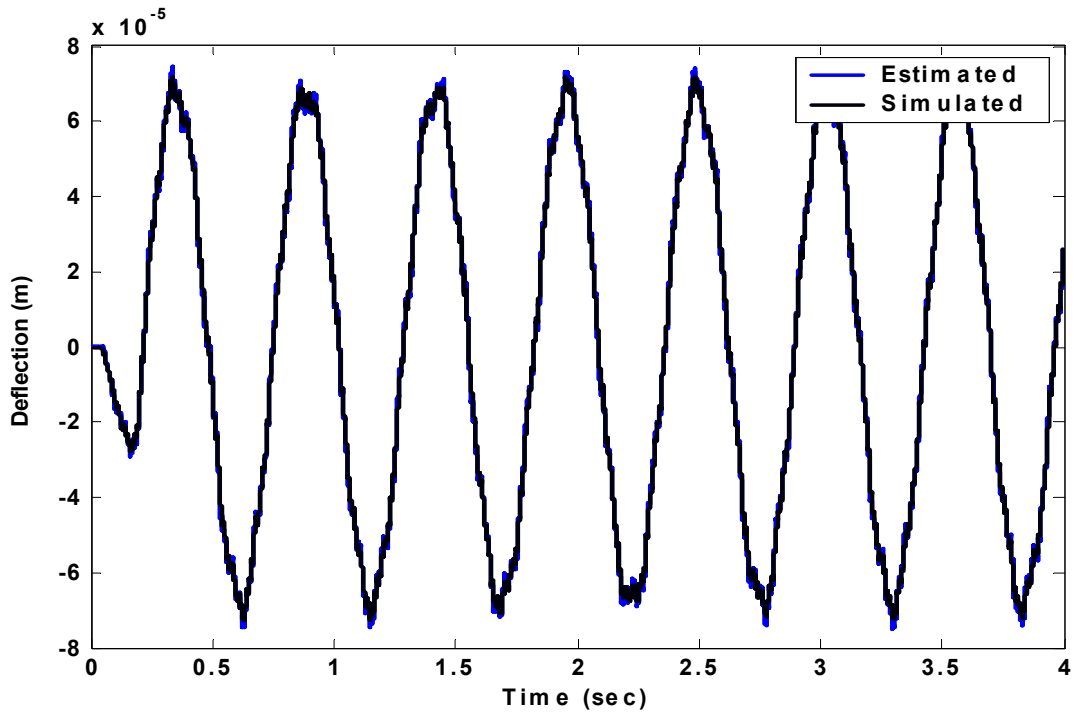Figure 4: Objective value as a function of number of generations

Figure 5: Estimated and target output in time domain with GA modelling
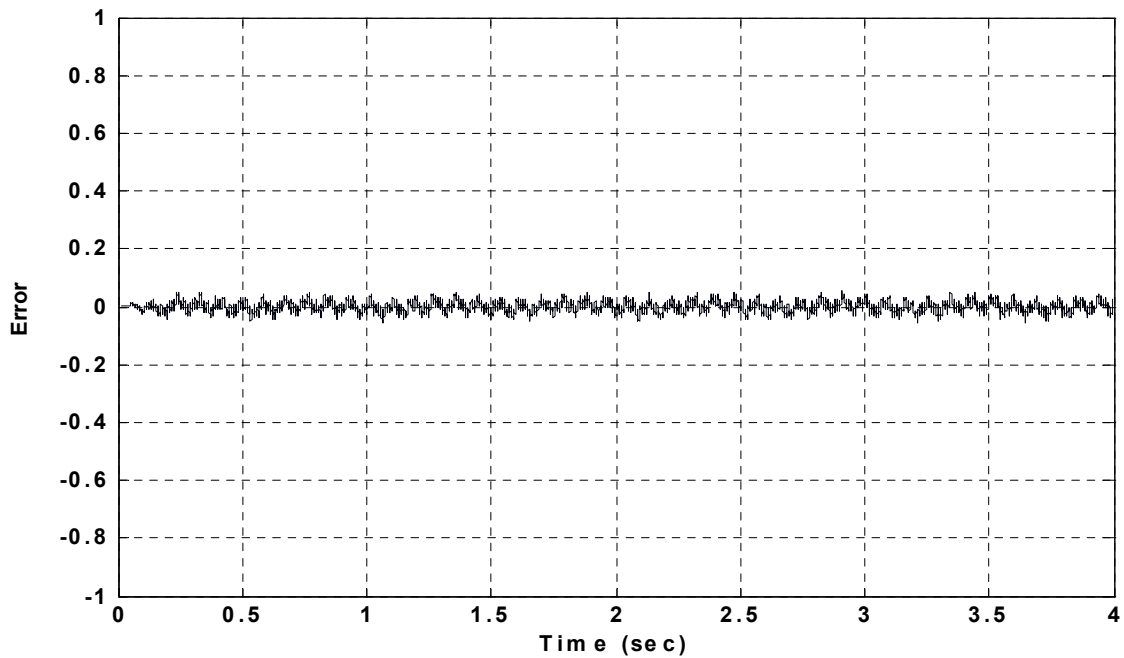

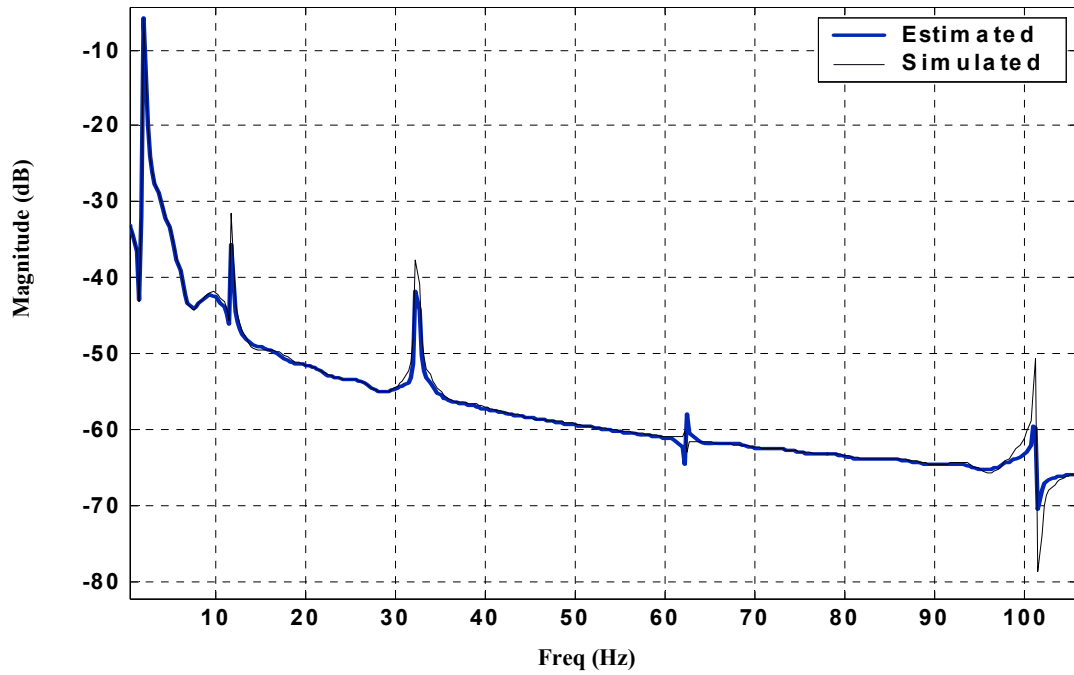Figure 6: Error between estimated and target outputs with GA modelling

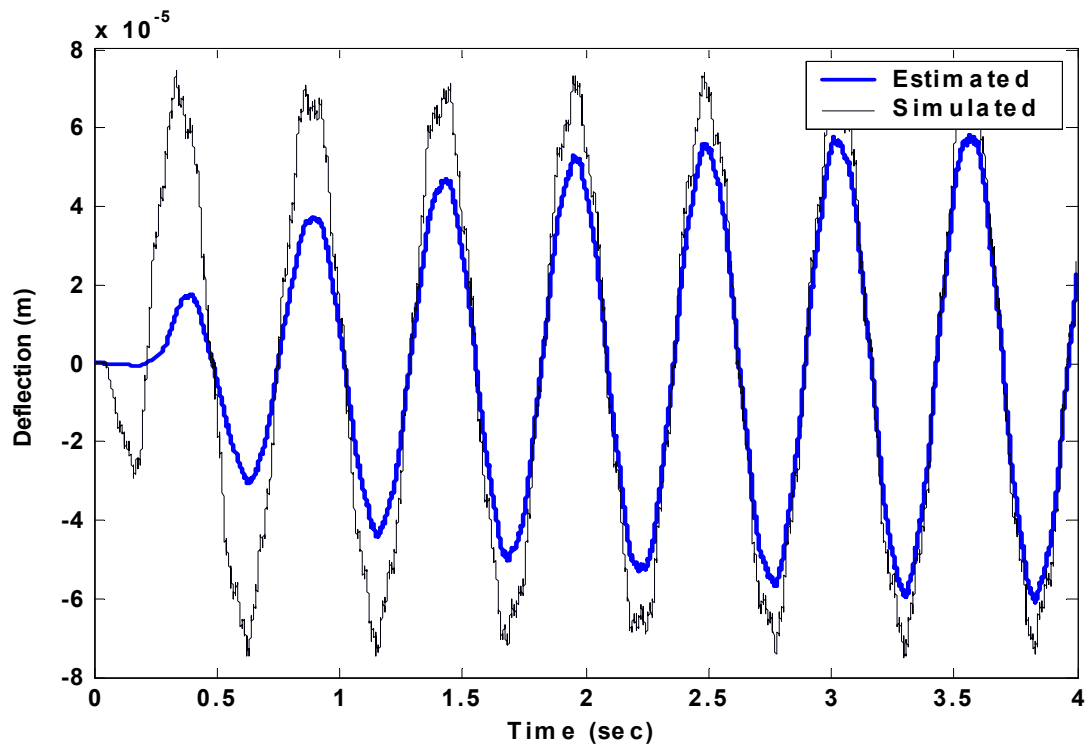Figure 7: Estimated and target output in frequency domain with GA modelling

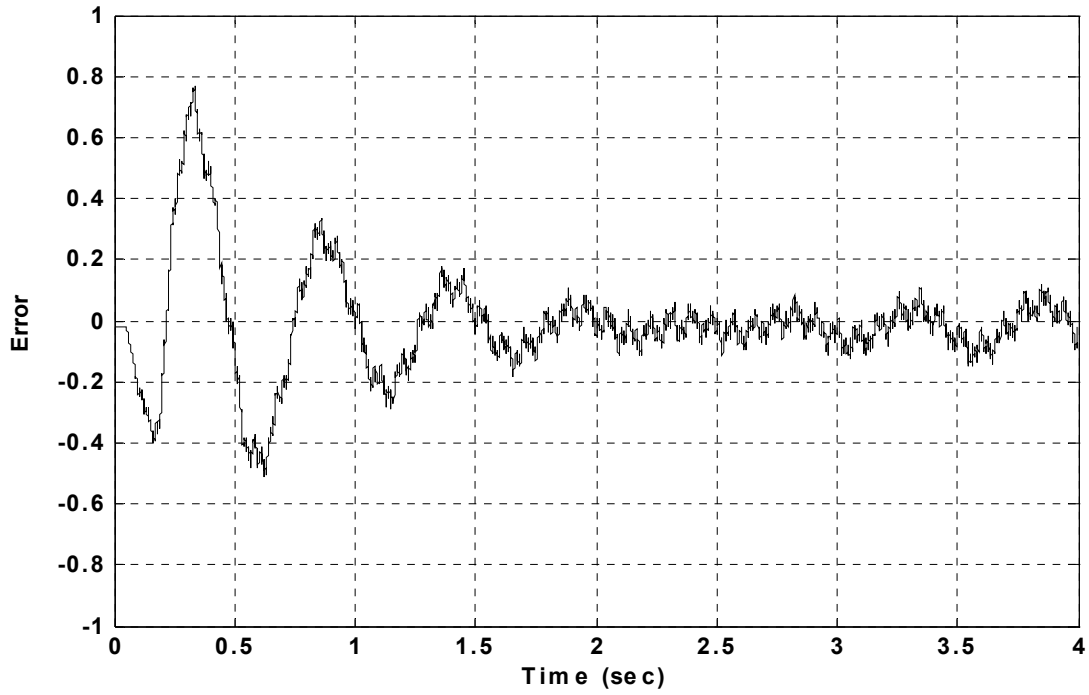Figure 8: Estimated and target output in time domain with RLS modelling

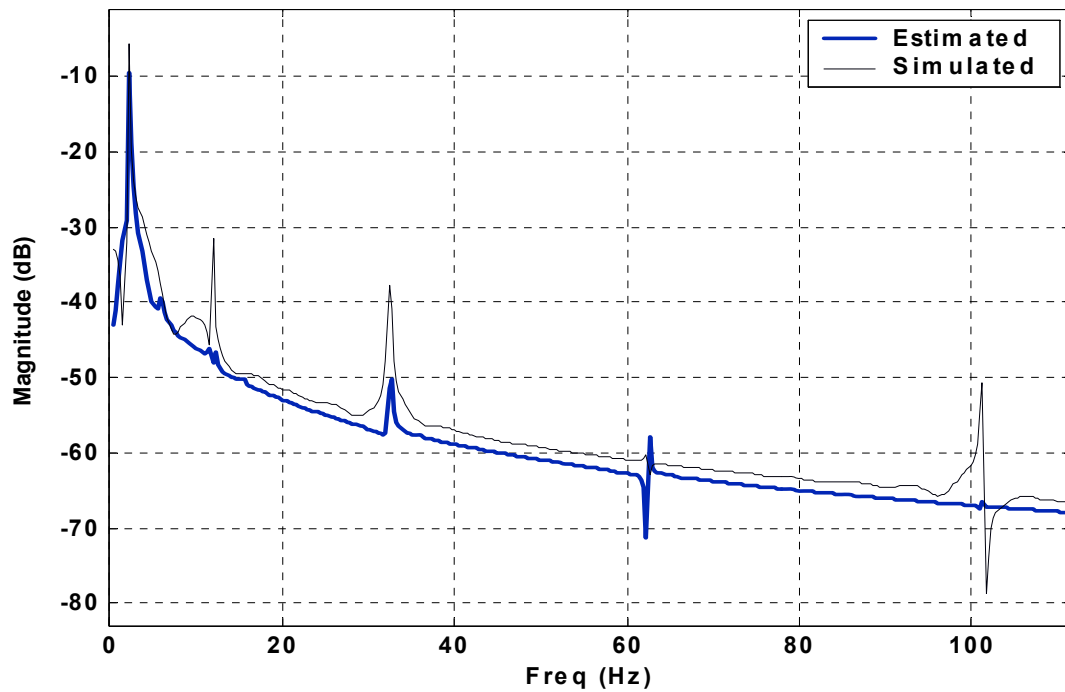Figure 9: Error between estimated and target outputs with RLS modelling

Figure 10: Estimated and target output in frequency domain with RLS modelling