Some Mobile Overconstrained Parallel Mechanisms

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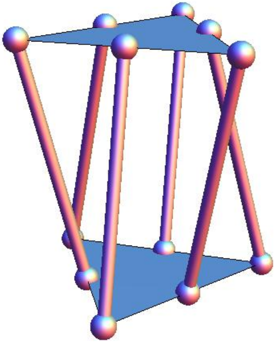
Griffis-Duffy platform



Motivation: Karger and Husty(2000), showed Griffis-Duffy platform mobile.

Want to find other mobile overconstrained parallel linkages.

Two Theorems

Two simple ways to produce mobile over-constraind linkages.

- line symmetry
- translation

Results "well known" but not well understood. State as theorems with formal proofs.

Translation - Theorem

Let $\mathcal{M}$ be an arbitrary mechanism having a coupler with 3 degrees-of-freedom. Duplicate the mechanism $\mathcal{M}$ and subject the new one to a fixed translation. The translation must include all links and joints including the base link. After the translation the translated base-link is again fixed. Rigidly join the coupler bars of the two mechanisms to form a combined coupler. This combined coupler bar will be able to move and will, in general, follow a 1 degree-of-freedom Schönflies motion.

Translation - Proof

Assume $g(\mu_1, \mu_2, \mu_3)$ is dual quaternion representing the three parameter motion of $\mathcal{M}$. After a translation t, shifted mechanism can perform the motion, $tg(\mu_1, \mu_2, \mu_3)t^-$, where t^- is the dual quaternion conjugate of t.

When couplers are joined, motion of joined coupler satisfies,

$$g(\mu_1, \mu_2, \mu_3) = tg(\mu_1, \mu_2, \mu_3)t^-.$$

Solutions are $g(\mu_1, \mu_2, \mu_3)$ that commute with t. Set of all elements in group which commute with a translation consist of the subgroup of all translations and all rotations about axes parallel to t—Schönflies group.

In the Study quadric a Schönflies group is the intersection of the Study quadric with a 5-plane. Intersecting with the 3-dimensional set of displacements $g(\mu_1, \mu_2, \mu_3)$ generally gives a 1-dimensional set, necessarily lying in the Schönflies subgroup.

Line Symmetry - Theorem

Let $\mathcal{M}$ be an arbitrary mechanism that has a coupler with 3 degrees-of-freedom. Again, duplicate the mechanism $\mathcal{M}$ but now subject the new one to a half-turn about a line ℓ_0. This time the base-link of the new mechanism is rigidly fixed to the coupler link of the original and the coupler link of the new machine is fixed to the base. The coupler bar of the new combined mechanism will generally follow a 1 degree-of-freedom line-symmetric motion.

Line Symmetry - Proof

After the half-turn motion of the coupler is $\ell_0 g(\mu_1, \mu_2, \mu_3)\ell_0^-$ but motion of base with respect to the coupler is, $\ell_0 g^-(\mu_1, \mu_2, \mu_3)\ell_0^-$.

After connecting bases and couplers, motion of combined coupler satisfies,

$$g(\mu_1, \mu_2, \mu_3) = \ell_0 g^-(\mu_1, \mu_2, \mu_3)\ell_0^-.$$

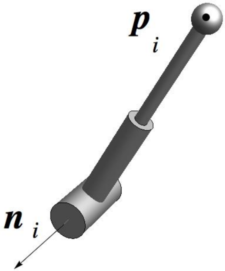
Rearrange,

$$g(\mu_1, \mu_2, \mu_3)\ell_0^- + \ell_0 g^-(\mu_1, \mu_2, \mu_3) = 0,$$

since $\ell_0^- = -\ell_0$.

Can be shown that this equation characterises line-symmetric motions, also, line-symmetric motions lie in the intersection of the Study quadric with a 5-plane.

RPS Legs



An RPS Leg.

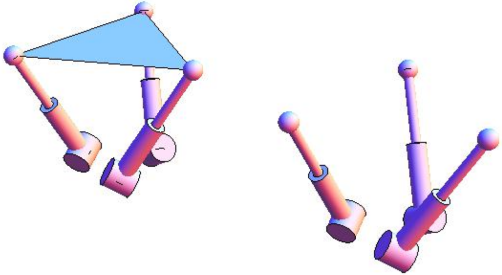
Point $\mathbf{p}_i$ moves on plane perpendicular to $\mathbf{n}_i$. Point-plane constraint. Set of rigid displacements that move $\mathbf{p}_i$ in such a way that it remains in the plane normal to $\mathbf{n}_i$ can be expressed as a quadratic in the dual quaternions.

Leg determines a 5-dimensional subspace in $\mathbb{P}^7$. Intersection of the Study quadric with another hyperquadric.

Parallel mechanism consisting of 3 RPS legs, each leg remove one degree-of-freedom so coupler/platform has 3 degrees-of freedom.

A Mobile 6RPS Mechanism

Translate 3RPS legs and join bases and platforms. Can show that motion of the coupler/platform is a general Darboux motion.



Line Symmetric 6RPS Mechanisms

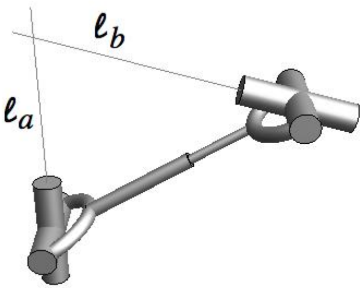
Can use the construction for line-symmetric mechnisms to produce a mobile 6RPS parallel mechanism.

Can show that, in general, line symmetric motion will be generated by reflection in a degree 9 ruled surface.

The curve representing the ruled surface projects to a plane cubic hence is either rational or elliptic (genus 0 or 1). But a degree 9 rational curve in a $\mathbb{P}^5$ can only lie on at most 3 linearly independant quadrics not 4 as ours does. So the ruled surface is genus 1.

Might degenerate into 2 or 3 components.

UPU Legs



A UPU Leg.

Rigid displacements which move a line so that is stays in a linear line complex also lie in the intersection of the Study quadric with another hyperquadric. In general no mechanism to model this constraint.

Special case. UPU linkage on left moves in such a way that the line ℓ_b, axis of the final joint, always meet or is parallel to ℓ_a, the axis of the first joint. Line ℓ_b lies in special linear line complex.

Mobile 6UPU Mechanisms

Duplicating a 3UPU and translating gives a mobile 6UPU. Generic points on the coupler/platform follow trajectories which are rational curves of dgree 6.

Duplicating and reflecting in a line gives a mobile 6UPU where the line symmetric motion is generated by reflecting in the generators of a degree 21 ruled surface.

Conclusions

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THANK YOU