Spatial Fourier transform method to determine reflection and absorption coefficient of porous rigid materials applying Johnson-Champoux-Allard model

Haydar Aygun, School of the Built Environment and Architecture, London South Bank University, 103 Borough Road, London, UK.

1 INTRODUCTION

This paper describes spatial Fourier transform method which is based on calculation of complex pressure distributions on two parallel surfaces and decomposing the complex pressure distributions into plane-wave components by using two-dimensional spatial Fourier transform which is used to separate the incident and reflected plane wave components. Johnson-Champoux-Allard model is utilized to predict effective density and bulk modulus of the air in the material, which are used to calculate wave number and characteristic impedance. Consequently, they are used to determine reflection coefficient of the porous materials at a range of angle of incidence.

2 THEORY OF SPATIAL FOURIER TRNASFORM

2.1 Reflection coefficient

A material with an infinite thickness is considered in this work. The reflection coefficient of surface of the test material at a given angle is determined using following equation [1, 2].

$$C_r(k_r) = \frac{Z_1 \cos\theta - Z_0[1-(k_0/k_1)^2 \sin^2\theta]^{1/2}}{Z_1 \cos\theta + Z_0[1-(k_0/k_1)^2 \sin^2\theta]^{1/2}}, \quad (1)$$

Where $C_r(k_r)$ is the reflection coefficient of the surface at an angle of incidence, Z_1 is the characteristic impedance in the test material, k_1 is the complex wave number in the test materials, $k_r = k_0 \sin\theta$, Z_0 is the characteristic impedance of air, and k_0 is the wave number of air, and θ is the angle of incidence.
The characteristic impedance and the complex wave number in the test materials are calculated using Jonhson-Champoux-Allard (JCA) model [3], see appendixes for more information.

2.2 Complex sound pressure distribution

A monopole sound source is located at a point (0,0, z_0) above the centre of the test material. The complex pressure distribution of this monopole source at a point (r, z) could be calculated using the following equation.

$$P(r,z) = exp(\,ik_0R_1)/4\pi R_1 + \int_0^\infty C_r(k_r) i \frac{exp[ik_z(z_0+z)]}{4\pi k_z} \mathrm{x} J_0(rk_r)k_r dk_r \quad (2)$$

Where $r = (x^2+y^2)^{1/2}$, R_1 is the length of the direct path, $R_1 = [r^2 + (z_0 - z)^2]^{1/2}$, $k_r = (k_x^2 + k_y^2)^{\frac{1}{2}}$, k_x and k_y are the wave vectors in x and y direction respectively, k_z is the wave vector in z direction, J_0 is the Bessel function of the first kind for order 0.

The complex pressure distribution over a surface of the material given in equation 2 is numerically calculated from repeated application of the trapezoidal integration formula [4].

3 THEORETICAL RESULTS

A sound source was assumed to be located at 200mm height above the centre of the material. Two circular planes with a radius of 1000mm and with heights of 10mm and 30mm are considered for calculations in this work. Complex sound pressure that was applied to the surface of the material at 1000 Hz is given in Figure 1. Complex sound pressure reduces with moving towards outer of the circular planes.

The spatial spectrum of the complex sound pressure is shown in Figure 2. Spatial spectrum becomes smoother when k_r/k_0 ratio is higher than 1.2 while it fluctuates when k_r/k_0 ratio is lower than 1.2.

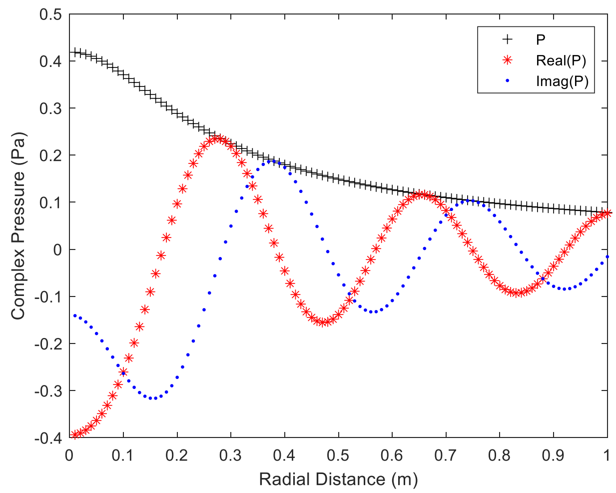

Figure 1: Complex sound pressure distribution on a plane versus radial distance.

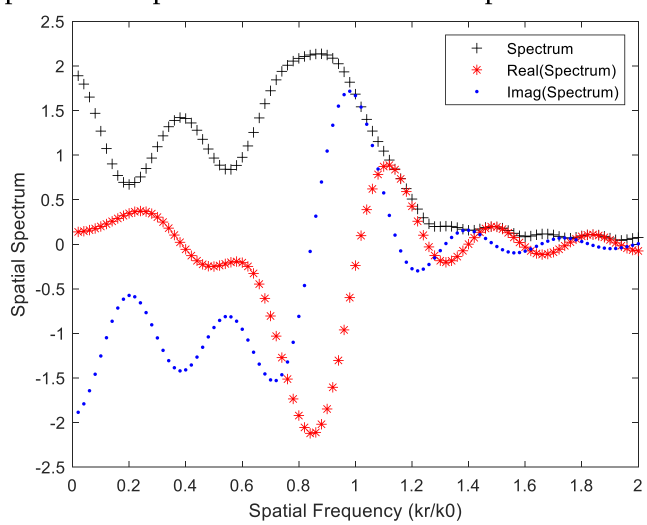

Figure 2: Spatial spectrum of the complex pressure distribution.

The reflection coefficients obtained from equation (1) are shown in Figure 3a and Figure 3b for different flow resistance values of 5460 Pa/m^2.s and 100 Pa/m^2.s, respectively. The discrepancy is attributed to the fact that the reduction of the sound pressure towards outer of surface is slower for material with higher flow resistance value.

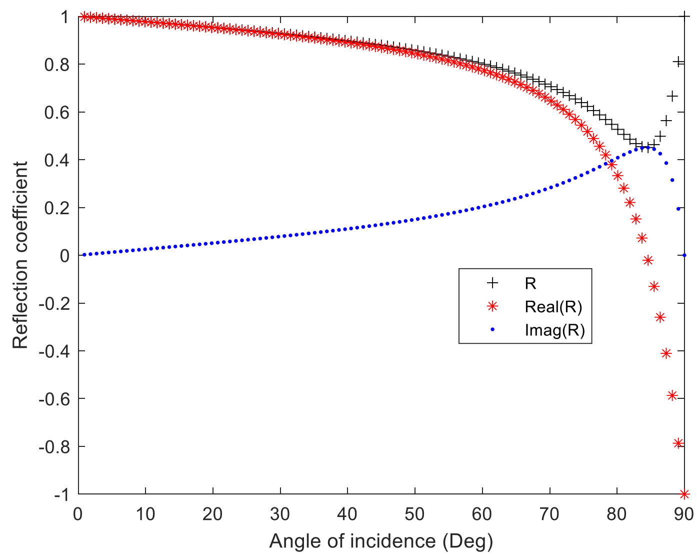

Figure 3a: Reflection coefficient versus angle of incidence for flow resistance value of 5460 Pa/m^2.s at 1000 Hz.

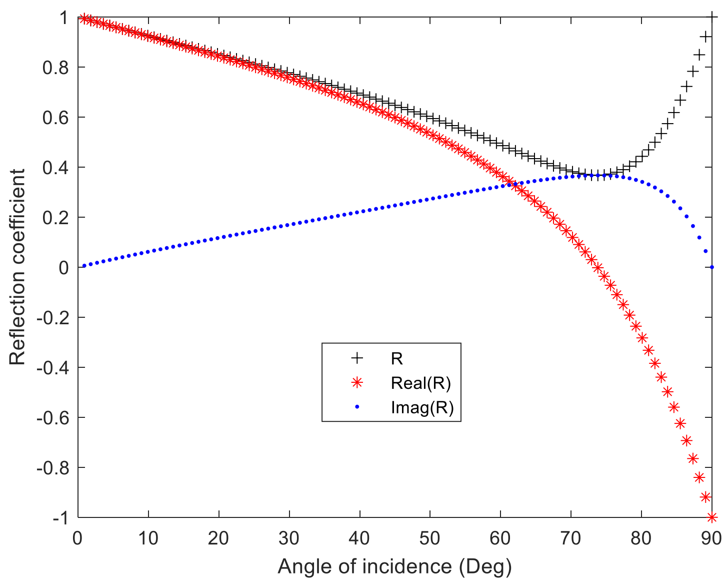

Figure 3b: Reflection coefficient versus angle of incidence for flow resistance value of 100 Pa/m^2.s at 1000 Hz.

4 CONCLUSION

A spatial Fourier transform method is used to determine the reflection coefficient at oblique angles of incidence. The complex parameters such as wave number and characteristic impedance are predicted using Johnson-Champoux-Allard model.

5 REFERENCES

1. M Tamura, Spatial Fourier transform method of measuring reflection coefficients at oblique incidence; I: Theory and numerical examples. *J. Acoust. Soc. Am*. Vol. 88(5), 2259-2264, (November 1990).
2. M Tamura, J F Allard, and D Lafarge, Spatial Fourier transform method of measuring reflection coefficients at oblique incidence; II: Experimental results. *J. Acoust. Soc. Am.* 97, 2255–2262, (April 1995)
3. J F Allard, and N Atalla, Propagation of Sound in Porous Media: modelling sound absorbing materials. 2nd Ed Wiley. (2009)
4. B Carnahan, H A Luther, and J O Wilkes, Applied Numerical Methods (Wiley, New York, 1969).

Appendix: **Summary of the equivalent formulation for the effective density and the bulk modulus calculations.**

Complex Wave number

$$k = \omega\sqrt{\frac{\rho}{K}}$$

Where ρ is the effective density, K is the bulk modulus of the air.
Surface impedance

$$Z = -j\frac{Z_c}{\emptyset} cotg\ kd$$

Where d is the thickness of the sample, Z_c is the Characteristic Impedance and given by $Z_c = \sqrt{\rho K}$, and $\emptyset$ is the porosity.

Effective density

$$\rho = \alpha_\infty \rho_0 \left[1 + \frac{\sigma\emptyset}{j\omega\alpha_\infty\rho_0} G_j(\omega)\right]$$

Bulk Modulus

$$K = \gamma P_0 \Big/ \left[\gamma - (\gamma - 1)\left[1 + \frac{\sigma'\emptyset}{jB^2\omega\alpha_\infty\rho_0} G_j'(B^2\omega)\right]^{-1}\right]$$

$$G_j(\omega) = \sqrt{1 + \frac{4j\alpha_\infty^2 \eta\rho_0\omega}{(\sigma\Lambda\emptyset)^2}}$$

$$G_j'(B^2\omega) = \sqrt{1 + \frac{4j\alpha_\infty^2 \eta\rho_0\omega B^2}{(\sigma'\Lambda'\emptyset)^2}}$$

Characteristic dimension for viscous forces

$$\Lambda = \frac{1}{c}\sqrt{\frac{8\alpha_\infty \eta}{\sigma \emptyset}}$$

Characteristic dimension for

$$\Lambda' = \frac{1}{c'}\sqrt{\frac{8\alpha_\infty \eta}{\sigma \emptyset}}$$

$\alpha_\infty$is the tortuosity given by $\alpha_\infty = 1.025 + 0.864\cos 2(\theta)$
σ is the flow resistivity of the material,
$\emptyset$is the porosity of the material,
η is the viscosity of the air,
$\rho_0$is the density of the air.
$\omega$is the angular frequency,
γ is the ratio of the specific heats (1.4 for air),
P_0 is the atmospheric pressure of air, 1.0132 x 10^5 Pa
B^2 is the Prandtl number and given by $\eta\gamma c_v/K$. It is around 0.71 at 18^{o}C.
$c_v$is the specific heats per unit mass at constant volume, $K$is the thermal conductivity.
c and c' are the coefficients that are used to fit measured and predicted values the effective density and the Bulk Modulus.