Bribes, Lobbying and Industrial Structure

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Abstract

This paper deals with the relationship between regulatory compliance, bureaucratic corruption, lobbying and the industrial structure of a country. We show that lobbying and bureaucratic corruption can coexist at the macro level when we allow for heterogeneity in firm size. Countries with similar level of development are often characterized by very different industrial structures: we show the implications this has for the level of compliance, corruption and lobbying in that country. Welfare implications of our model point toward encouraging policies that support the small business sector of an economy and toward flexible regulatory policies meant to suppress regulation for small enough firms.

JEL Code: H26, L51, K42.
Keywords: Bureaucratic Corruption, Lobbying, Industrial Organization.

1 Introduction

Lobbying and bureaucratic corruption have been and still are relevant economic and political phenomena of most societies and political systems. In the United States, for example, political lobbying is as old as the Nation:

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indeed, lobbying is protected under the right of petition in the First Amendment of the Constitution. The comprehensive reporting of lobbying expenditure required by the 1995 Lobbying Disclosure Act (LDA) is only a final step of the regulatory process that started in 1945 with the Federal Regulation of Lobbying Act. Also, the US federal lobbying industry has experienced startling growth: between 1998 and 2009, lobbying expenditures approximately doubled, reaching almost USD 4 billion a year.[1]

Corruption is also a widespread and rampant phenomenon, notwithstanding the various prevention and repressive actions taken by governments and civil society. As Pellegrini and Gerlagh (2008) aptly remind us, "corruption is a widespread phenomenon affecting all societies to different degrees, at different times. On the one hand, as corruption scandals have repeatedly shown, bribes are common in all countries notwithstanding differences in income levels and law systems, as they are common in democracies and in dictatorships. Recent scandals over corruption have shown that also supposedly free-from-corruption societies are affected".[2]

In this paper we show that the spread of lobbying and bureaucratic corruption depends on the specific industrial structure considered. Even though they are two different phenomena, often the differences between the two remain blurred. For example, the successful and pioneering Grossman and Helpman's (1994) model on lobbying can be read as one for corruption: in fact, the authors assume that lobbyists influence politicians' policy-making decisions by providing them with resources, which could easily be considered as bribes. In reality, one of the most relevant differences between lobbying and bureaucratic corruption is that lobbying is often legal while bureaucratic corruption is not (see the discussions in Lambsdorff (2002) or Begovic (2005)) and, rather, detectable and punishable.

In this paper we define lobbying as campaign contributions or influence buying meant to change existing rules or policies relevant for the lobbying entity. Similarly, we define corruption as the illegal use of public power and resources for personal gain. In this context, bureaucratic corruption is an activity aimed at bending existing rules or policies (see also Damania et al. (2004) and Campos and Giovannoni (2007) and (2008)). As stressed by Giovannoni (2011) "At the conceptual level, the distinction is important because it raises a natural question: if lobbying and corruption are both rent-seeking activities which operate with different targets, are they complements or substitutes?".

Lobbying and bureaucratic corruption can be complementary as in Dama-

[1] Calculation by the Centre for Responsive Politics based on data from the Senate Office of Public Records. See http:\\wwww.open secrets.org.

[2] Modern research on the economics of corruption began with Rose-Ackerman (1975) and (1978) and has attracted later the interest of a number of scholars; see e.g. Celimene et al. (2016), Cerqueti and Coppier (2009) and (2011), D'Agostino et al. (2016), Enikolopov et al. (2018) and Lim (2019).

nia et al. (2004) where the authors consider that lobbying activity is mostly directed at laws that undermine law enforcement, so as to make corruption easier. In their model, firms lobby high-level government politicians in order to resist legal reform directed at improving judicial efficiency and reduce corruption. In this case, lobbying makes the institutions necessary to enforce compliance weaker, and thus it makes bureaucratic corruption less risky.
Harstad and Svensonn (2011) argue that lobbying and corruption are substitutes and the choice of the firm between bribing the bureaucrat or lobbying the government depends on the level of development of the considered country.
In this paper, we study the role played by the industrial structure of a country in affecting the relationship between the two phenomena. The role of the industrial structure as a possible determinant of lobbying was analyzed by Bombardini (2008). In her model, the size distribution of firms affects lobby participation shares and therefore the level of protection in a sector. In our paper, we consider not only the lobbying activity but also the possibility of bureaucratic corruption and of compliance with regulation. We proxy the industrial structure[3] with its heterogeneity among firms with respect to size and discuss the connection between compliance, lobbying and bureaucratic corruption at two levels: under a microeconomic perspective, by exploring the behavior of the individual firm; under a macroeconomic viewpoint, by aggregating firms and considering the impact of the overall industrial structure.[4] At the micro level we find that, *ceteris paribus*, in any given country, small firms do comply with regulation, middle-size firms are most likely to bribe and large firms engage in lobbying.
The empirical results show that medium and large firms are likely to be engage in non compliant behaviours.[5] In addition, Campos and Giovannoni (2008) find that larger firms are systematically associated with lobbying, while smaller firms are systematically associated with bureaucratic corruption; Bennedsen et al. (2009) show that larger firms pay bribes less frequently but have more political influence.
At the macro level, the introduction of the important element of heterogeneity between firms allows us to analyzes the relationship between compliance, bureaucratic corruption and lobbying in the context of different industrial structures, a key element of heterogeneity across countries. In fact, in our model compliance, bureaucratic corruption and lobbying may coexist, and the nature of such coexistence is strongly affected by the industrial struc-

[3]Following Dasgupta and Stiglitz (1980), we define industrial structure as the degree of concentration in an industry.

[4]We consider that firm size is measured through its capital level.

[5]European Commission (2010, Fig. 4.11) finds that in public procurement tenders micro and small firms have less concern for "tenders evaluated fairly" than medium and large ones, who are likely to experiment "in full" the consequences of non compliance.
See *https* : *//ec.europa.eu/docsroom/documents/*14808/*attachments*/1/*translations/en/renditions/pdf*.

ture of the country. More precisely, countries dominated by small and/or medium firms should see, in the aggregate, relatively less lobbying and more compliance and/or corruption than countries where a few large firms dominate the industry. Therefore, differently from Harstad and Svensonn (2011) who look at the impact of the level of development of the country itself, we check for the relevance on bureaucratic corruption and lobbying of the industrial structure of the country.[6] Our approach – grounded on the evidence that the small size of the firm may be the expression of the specific country's industrial structure rather than the result of the level of development of the country itself – allows us to gather relevant intuition also for rich (poor) countries with a high (low) share of small firms. In fact, considering the MSMEs indicator (Micro, Small, Medium Enterprises per 1,000 people) provided by the World Bank as a proxy of the industrial structure, we can see how equally developed countries (high income) can have profoundly different industrial structures: for example, Japan shows a MSME indicator of 1.6 against a New Zealand indicator of 103.8.

Figure 1 compares the levels of perceived corruption of developed OECD

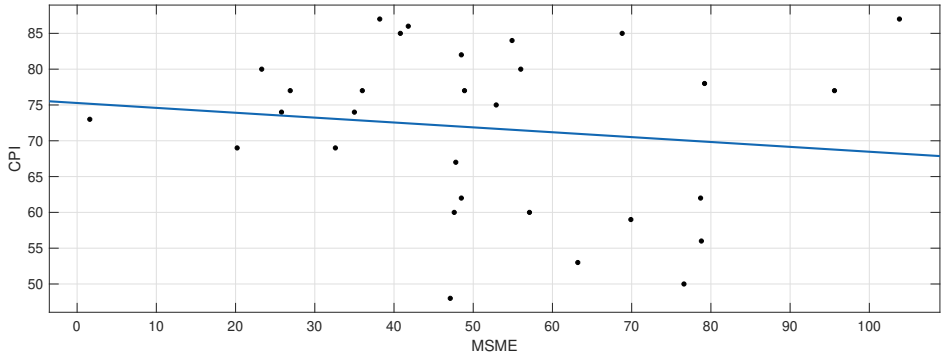

Figure 1: The CPI index versus MSME density for the OECD countries.

countries[7] to their industrial structures: in a developed country within

[6]Harstad and Svensonn (2011) consider in their paper an infinite number of identical firms not allowing for heterogeneity. Differently, in our paper, studying heterogeneity across firms, gives us the opportunity to obtain new insights at the macro level for any given country.

[7]In this figure we consider all OECD countries with high income: Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea Republic, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom, and United States. We do not consider the following countries that despite belonging to the OECD are classified as countries with a lower level of development (upper middle income): Colombia, Mexico, Latvia, Lithuania, Poland, Turkey, and Hungary. The corruption level is measured by the CPI (Corruption Perception Index) of Transparency International. The CPI Index measures the perception of corruption in the public sector and in politics in many countries around the world. It is based on the opinion of experts

OECD, small firms (higher MSME) and more corruption (lower CPI) appear to coexist, a result which is confirmed in our model.
This result, which shows a high degree of heterogeneity even among developed economies, is also coherent with Pellegrini and Gerlagh's (2008) above-mentioned quote, that even in rich countries corruption can be pervasive.
The paper is organized as follows. In Section 2, we present the model. In Section 3, we describe the timing of the game and provide the main results. Section 4 concludes. All proofs of propositions are in the Appendix, along with some remarks on the free-riding case.

2 Theoretical model

The economy is assumed to be populated by three players: a high–level public official (politician), a low level public official (bureaucrat) and firms. Following the mainstream political economy literature, we assume that firms, bureaucrats and politicians are not affected by psycho-biases and do not have bounded rationality.
Firms are assumed to be open to non compliant behavior with respect to the existing legislation, even if compliance is mandatory. The firm can try to change existing legislation in its favor by lobbying the politician. Moreover, it may seek to avoid the application of the law by bribing a bureaucrat (bureaucratic corruption).
The paradigmatic example we have in mind is that of polluting firms, whose emissions are constrained by environmental laws which identify a suitable pollution tax.[8] Although our model is presented in terms of environmental policy, our results may have more general applicability. In order to manage pollution emissions, there are two levels of public official involved: a high-level public official, i.e. a politician, who decides environmental policies, and a low-level public official, i.e. a bureaucrat, who is responsible for the application of the environmental law and for controlling the behavior of firms.[9] To be more precise, we assume that the government cannot directly observe the level of pollution emitted by the firm and therefore uses bureaucrats to monitor pollution levels in order to reduce environmental evasion.

and assigns a rating ranging from 0, for countries that are perceived as very corrupt, to 100, for "clean" ones.

[8] Cerqueti and Coppier (2016) present a game where environmental protection acts as a tool for morally persuading firms to be compliant with regulation. Munoz-Garcia and Akhundjanov (2016) study the effects of environmental regulation in an environment with heterogeneous firms.

[9] For a more detailed analysis on the bureaucrat's behavior see also Cerqueti and Coppier (2013) who discuss the role of incentives for tax evasion for controllers open to bribery, and study the problem through a Bayesian game.

Time is an ingredient of our model. In fact, an important difference between bureaucratic corruption and lobbying is the time-span of the two actions: while the former activity applies temporarily (for one period in our model) – since firms deal with different officials over time (in each period) – the latter one implies a legislative change and, therefore, alters the *status quo* for a longer period. To capture this fact, in our model we consider that the effects of lobbying, i.e. the change of the rule, refer to a period which goes from $t = 0$ to $t = T$, while the effects of bureaucratic corruption apply to a single period.
The production of the j-th firm for a single period is:

$$y_j = f(k_j) = rk_j \tag{1}$$

where r is a productivity parameter and k_j is the specific capital level of the j-th firm. If the j-th firm complies with the regulation, it has to pay a proportional cost c on production, i.e. proportional to k_j, plus a fixed cost C_0.[10] This is the j-th firm profit function for a single period:

$$\pi_j = rk_j - ck_j - C_0. \tag{2}$$

In order to describe the heterogeneity of capital across firms, we consider the cumulative probability function F which defines the distribution of individual capital levels k's. The shape of the relative density function f provides information on the level of firm capitals, and it is used as a proxy for the industrial structure. In particular, the symmetry properties of the function f provide information on the distribution of firms in the economic context in terms of high or low capital levels.[11]
We assume that lobbying and bureaucratic corruption are both directed at eliminating the proportional compliance cost of regulation ck_j. Firms affect the costs of compliance in two possible ways: either through lobbying, a legal activity leading to the removal of the variable cost by a long-term change of the regulation or through bureaucratic corruption, an illegal activity that leads to the removal of the same cost but only for one period.
The effect of the lobbying activity concerns only those firms which participate in it: in other words, we assume that a change of regulation produced by lobbying affects only the firms which have paid the contribution to the politician. We simply assume realistically that the benefits and impact of lobbying are stronger for firms that do lobby compared to the firms that do not engage in lobbying.[12] We demonstrate in Appendix that this simplifies

[10]The presence of fixed and variable costs is in accord with the regulation on the emissions of polluting firms (see e.g. waste management activities).
[11]See Section 4 for a discussion on this.
[12]See for example the cases cited in
https : *//www.americanprogress.org/issues/economy/report*/2014/05/02/88917/.

the analysis but the results also hold in the case in which we consider the free-riding presence.

At time $t = 0$, the politician wants a contribution (for electoral purpose) p_j from each j-th firm for changing the current regulation, i.e. for removing the proportional cost of compliance. All firms which, in equilibrium, find it worthwhile to engage in lobbying, pay individually the cost of their lobbying activity. The politician asks for a contribution which is a percentage $\alpha \in [0, 1]$ of the benefits that the firm obtains thanks to the lobbying activity: since the effects of the lobbying activity relate to $(T + 1)$ periods, the contribution paid to the politician will be a percentage of the benefits (absence of the variable costs of compliance) relating to $(T + 1)$ periods. Since, as we said, firms are heterogeneous with respect to their capital level k_j, each firm will get a specific benefit from lobbying; hence, the contribution p_j to be paid by firm j for the lobbying activity depends on the j-th firm's proportional costs of compliance.[13]

Critically, we assume that capital markets are imperfect and, therefore the firms must have at time $t = 0$ all the necessary resources to pay the contribution for the politician p_j, which as we said refers to $(T + 1)$ periods. This implies that the only reason in our model for a firm not to lobby is that by being liquidity constrained (a less likely outcome for large firms) it opts for either compliance or (risky) corruption.[14] Other firms, which cannot take part in the lobbying activity, must decide whether to comply with the existing regulation or to bend the rule in order to avoid paying the proportional part of compliance cost ck_j. In the latter case, facing a bureaucrat that may decide to ask for a bribe, the j-th firm must decide whether to engage or not in a negotiation on the bribe. In the case in which the agreement between the bureaucrat and the j-th firm is not achieved, the bureaucrat denounces the non–compliant firm. We also assume that the j-th firm must pay a fine mk proportional to their capital level in the case in which the agreement between the bureaucrat and the j-th firm is not achieved. In addition, with an exogenous probability q, the corrupt transaction between the bureaucrat and the firm is controlled and detected. In this case, not only the j-th firm

[13] In our model, following Grossman and Helpman (1994), Bombardini (2008), Catola and D'Alessandro (2020), we consider that each lobbying firm operates through a political contribution to the government, which is structured as a contribution schedule. In other words, a situation arises in which the decision of whether to lobby and how much to contribute is made by individual firms which offer different contributions to the politician depending on the possible favorable regulatory change. Therefore, this literature highlights a close link between the size of the contribution paid to the politician and the extent of the expected benefit. In so doing, we consider that there is only one possible change in regulation (i.e., total elimination of the variable cost of compliance in our case) and therefore there is a single contribution offered by the firm to the politician of the take-it-or-leave-it type (see for example Harstad and Svensson (2011)).

[14] In fact, since lobbying is a legal activity, when the lobbying costs and those associated with bribing the bureaucrats are identical, it will be always preferred to bureaucratic corruption.

must pay a fine m on the capital level, but also the corrupt bureaucrat is fined with a penalty λ^B. The value of the bribe is the result of a bargaining process between the firm and the bureaucrat performed in each period. Since lobbying is a legal activity that can give only advantages to the politician, then the latter will always be willing to ask for a contribution p_j to firm j for changing the existing legislation. We now proceed to describe the model through a sequential game.
Given the heterogeneity of firms, their behavior will vary according to their capital level k_j. The payoff vectors will be indicated with a couple

$$\underline{\pi}_{j,t} = (\pi_{j,t}^{(F)}, \pi_t^{(B)}), \qquad t = 0, 1, \ldots, T, \tag{3}$$

where $\pi_{j,t}^{(F)}$ and $\pi_t^{(B)}$ represent the payoffs of the j-th firm and of the bureaucrat for one period, respectively.
The bureaucrat earns a salary equal to w^B, irrespective of whether he accepts the bribe or not .

3 The game: description and solution

As mentioned above, we assume that there is a lobby in this industry which pays a politician in order to change regulations. Firms belong to this lobby only if they have enough capital to pay the contribution to the politician. If the j-th firm engages in lobbying activity, (Lobbying -L-), then the one period payoff vector is given by:

$$\underline{\pi}_L = (rk_j - p_j - C_0; w^B) \tag{4}$$

Thus, the necessary condition for the j-th firm to engage in the lobbying activity is that the payoff of engaging in lobbying is positive. This leads to:

$$\underline{\pi}_L = rk_j - p_j - C_0 \geq 0 \tag{5}$$

Inequality (5) is equivalent to

$$k_j \geq \frac{C_0 + p_j}{r} = k_j^{(0)}. \tag{6}$$

We take condition (6) as the requirement to be satisfied by the capital of the j-th firm to engage in lobbying activity. Therefore, the j-th firm with capital k_j greater or equal to $k_j^{(0)}$ belongs to the lobby and engages in lobbying.[15]
As we said, the j-th firm which does not have a sufficient capital level to

[15] As we will see below in detail and as mentioned in the previous section, the value of p_j will depend also on the level of capital k_j. By substituting such a value into (6), we will obtain a "universal"threshold for the capital which does not depend on j.

engage in lobbying, i.e. $k_j < k_j^{(0)}$, must decide whether to comply with environmental regulation or try to bend the rules. In the latter case, the j-th firm must decide whether to bribe the bureaucrat in order to avoid being reported for having violated the rules.
To describe bureaucratic corruption for the firms which have a capital level $k_j < k_j^{(0)}$, we use a three-stage game with imperfect information. The steps are the following:

First stage

In this stage, the j-th firm must decide whether to comply with the environmental regulation (honest -H-) or to bend the rule in order not to pay the proportional cost ck_j. In the latter case, the game continues to the second stage. If the j-th firm decides to comply with the rule (honest -H-), then the game ends with the following one period payoff vector:

$$\underline{\pi}_{1,H} = (rk_j - C_0 - ck_j; w^B). \tag{7}$$

If the j-th firm decides to bend the rule, the game continues to the second stage.

Second stage

The bureaucrat, who inspects the j-th firm, must decide whether to report the violation of the environmental law or to ask for a bribe ($b > 0$), comparing the payoffs relating to the two different situations.
In the case in which the bureaucrat decides not to ask for a bribe, i.e. to report the bending of the rule (report corruption- RC-), the j-th firm must pay a fine mk_j and the game ends with the following one period payoff vector:

$$\underline{\pi}_{2,RC} = (rk_j - ck_j - C_0 - mk_j; w^B). \tag{8}$$

Otherwise, the game continues to stage three.

Third stage

If the bureaucrat asks the j-th firm for a bribe, then the firm must decide whether to negotiate the bribe or refuse the negotiation. When the agreement between the j-th firm and the bureaucrat is not achieved, the bureaucrat denounces the non–compliant firm which must pay a fine m on the capital level. The game ends with the following one period payoff vector:

$$\underline{\pi}_{3,RC} = (rk_j - ck_j - C_0 - mk_j; w^B). \tag{9}$$

If the agreement between the j-th firm and the bureaucrat is achieved, then the two parties will find an agreement on the bribe b_j^{NB}, which corresponds to the Nash solution to a bargaining game. The corrupt transaction may

be inspected by a controller with probability q. If corruption is discovered, then the j-th firm pays a fine m on the capital and the bureaucrat a fine λ^B. Otherwise, we have undetected corruption.
The game ends with the following random one period payoff vector:

$$\underline{\pi}_C = \begin{cases} (rk_j - C_0 - b_j^{NB}; w^B + b_j^{NB}), & \text{with probability } 1-q; \\ (rk_j - C_0 - ck_j - mk_j; w^B - \lambda^B), & \text{with probability } q. \end{cases} \tag{10}$$

3.1 Solution of the game

In order to proceed to the solution of the game for the j-th firm, we first provide an explicit expression for the bribe b_j^{NB} for the bureaucrat and for the contribution p_j for the politician. We assume that the bargaining strength of the firm versus the bureaucrat is equal to β and that the j-th firm pays to the politician a percentage α of the benefit deriving from the lobbying activity, i.e. the proportional cost of compliance that the lobbying firm does not pay for $(T+1)$ periods.
As for the corruption game, reserved to those firms that are not able to lobby, Proposition 3.1 illustrates the outcome of our Nash–bargaining game between the firm and the bureaucrat.

Proposition 3.1. *For each period $t = 0, 1, \ldots, T$, there is a unique bribe b_j^{NB}, as the Nash solution to the bargaining game, given by:*

$$b_j^{NB} = (1-\beta)(c+m)k_j + \frac{\beta q \lambda^B}{(1-q)}. \tag{11}$$

where β and $1-\beta$ are the parameters in $[0,1]$ that can be interpreted as measures of bargaining strength, of the firm and the bureaucrat respectively.

By computing this derivative we observe that:

$$\frac{\partial b_j^{NB}}{\partial q} > 0$$

Increasing monitoring increases the risk of a corrupt transaction for the bureaucrat who asks, to support this increased risk, for a greater bribe. Also computing:

$$\frac{\partial b_j^{NB}}{\partial k_j} > 0$$

A greater capital level of the firm means more savings in compliance costs for the firm –as the compliance costs are proportional to capital level– and therefore a greater surplus to be shared between the firm and the bureaucrat. We need to determine the amount of the contribution p_j by each firm. As

we saw, if $k_j \geq k_j^{(0)}$, then the j-th firm engages in lobbying and offers a contribution p_j which is a percentage $\alpha \in [0,1]$ of the proportional part of the compliance costs that the firm participating in the lobbying will not have to pay for the $(T+1)$ periods during which the lobbying produces its effects. Indeed, as we said above, the lobbying activity implies a legislative change and, therefore, alters the *status quo* for a period which goes from $t=0$ to $t=T$.
If the j-th firm does not engage in lobbying, (No Lobbying -NL-), then the aggregate (over time) payoff vector is:

$$\underline{\Pi}_{NL} = \sum_{t=0}^{T} [(rk_j - ck_j - C_0)] = rk_j(T+1) - ck_j(T+1) - C_0(T+1). \tag{12}$$

If the j-th firm engages in the lobbying activity, (Lobbying -L-), it does not pay the proportional cost of compliance ck_j for $(T+1)$ periods, then the payoff vector given by:

$$\underline{\Pi}_{L} = \sum_{t=0}^{T} [(rk_j - C_0)] - p_j = (T+1)rk_j - (T+1)C_0 - p_j. \tag{13}$$

The j-th firm offers to pay a contribution to the politician equal to a share α of the surplus, given by the difference between (13) and (12):

$$p_j = (T+1)\alpha ck_j. \tag{14}$$

where α is the percentage of the benefit deriving from the engaging in the lobbying activity that the j-th firm offers to the politician. In other words, the contribution p_j represents a percentage of the saving coming from not paying the proportional compliance costs for $(T+1)$ periods.[16] Formula (14) clearly states that greater capital level of the firm means more savings in compliance costs for the firm, which implies a greater contribution for the politician.

[16]Notice that in our model we assume a unitary discount rate $e^{-\delta t}$, with $\delta = 0$ for each t. This assumption slightly simplifies the treatment of the model and allows to gain more intuitive outcomes. However, it can be removed. The presence of a discount rate with $\delta > 0$ reduces the role played by future amounts. Formula (12) becomes

$$\underline{\Pi}_{NL} = (rk_j - ck_j - C_0)\sum_{t=0}^{T} e^{-\delta t} = (rk_j - ck_j - C_0)\frac{1-e^{-\delta(T+1)}}{1-e^{-\delta}} < (rk_j - ck_j - C_0)(T+1).$$

Also in the subsequent analysis, as in the expressions of p_j in (14) and $k^{(0)}$ in (15), the term $(T+1)$ should be substituted with $\frac{1-e^{-\delta(T+1)}}{1-e^{-\delta}}$. Thus, the outcome of the introduction of a discount rate smaller than one is that politicians obtain less from the lobbying activity, because the value of the contribution p_j and of the threshold $k^{(0)}$ are reduced. Lobbying becomes cheaper as the term $e^{-\delta}$ becomes smaller, and more firms have a capital large enough to engage in the lobbying activity.

Using (14), we can rewrite condition (6) so as to find the lobbying activity sustainable for the firm:

$$k_j \geq \frac{C_0}{r - c\alpha(T+1)} = k^{(0)}. \tag{15}$$

As we said above, credit markets are assumed imperfect and therefore for firms to be able to engage in lobbying they must have resources sufficient to cover the saving deriving from not paying the proportional compliance cost for $(T+1)$ periods.
We now present the solution of the game (see the Appendix for the proof).

Proposition 3.2. *Consider the capital threshold*

$$k^{(2)} = \frac{\beta q \lambda^B}{\beta(1-q)c - m[1-\beta(1-q)]}. \tag{16}$$

(I) Assume that $k^{(0)} \geq k^{(2)}$.

(I.A) If $k_j < k^{(2)}$*, then the* j*-th firm does not engage in lobbying and it will find it worthwhile not to bend the rule and the expected payoff is* $\pi_{1,H}$

(I.B) If $k^{(2)} \leq k_j < k^{(0)}$*, then the* j*-th firm does not engage in lobbying but engages in bureaucratic corruption and the expected payoff is* π_C.

(I.C) If $k_j \geq k^{(0)}$*, then the* j*-th firm engages in lobbying activity and the expected payoff is* $\pi_{1,L}$.

(II) Assume that $k^{(0)} < k^{(2)}$.

(II.A) If $k_j < k^{(0)}$*, then the* j*-th firm does not engage in lobbying and it will find it worthwhile not to bend the rule and the expected payoff is* $\pi_{1,H}$.

(II.B) If $k_j \geq k^{(0)}$*, then the* j*-th firm engages in lobbying activity and the expected payoff is* $\pi_{1,L}$.

The proposition shows the existence of some capital thresholds beyond which the perfect Nash equilibria in the sub-games are obtained. In order to better clarify the results of the previous proposition, let us rename the thresholds found for the capital critical level:

- $k^{(0)}$. We call this threshold the *Lobbying Threshold (LT)* because this is the necessary and sufficient capital level for lobbying: in fact, if the firm capital level k_j is less than $k^{(0)}$, the j-th firm does not have the necessary capital to lobby; if the firm capital level k_j is greater than $k^{(0)}$, the j-th firm engages in lobbying;

- $k^{(2)}$. We call this threshold the *Compliance Threshold (CT)* because if the firm capital level k_j is lower than $k^{(2)}$, the j-th firm will find it worthwhile to comply with the rule; if the firm capital level k_j is more than $k^{(2)}$, the j-th firm will find it worthwhile to bend the rule.

The *Lobbying Threshold (LT)* is the minimum level of capital needed to engage in the lobbying activity which derives from the liquidity constraint condition of firms. The lobbying activity is always preferred to bureaucratic corruption from an economic point of view. In fact, the net surplus deriving from lobbying compared with the bureaucratic corruption is given by:

$$\Delta(\pi^F_{1,L} - \pi^F_C) = (T+1)\{[m[1-\beta(1-q)+c\beta q]]k_j + q\beta\lambda^B\} > 0$$

This net surplus is positive for all levels of capital.
Regarding the *Compliance Threshold (CT)*, this represents the minimum capital level that makes it worthwhile for the firm to bend the regulation. The threshold depends on the dimension of the firm (i.e. capital level) because the net surplus for bureaucratic corruption compared with the compliance case is:

$$\Delta(\pi^F_C - \pi^F_H) = k_j\{[c\beta(1-q) - m[1-\beta(1-q)]]\} - q\beta\lambda^B$$

This net surplus has a part proportional to the size of the firm (k_j) and a "fixed cost" component $q\beta\lambda^B$ related to the bureaucrat's expected fine. Thus, intuitively, this "fixed cost" part of the surplus related to the bureaucrat's expected fine drives the fact that only sufficiently large firms (medium) find it worthwhile to engage in bureaucratic corruption. As already illustrated above, the payoffs describe three different situations:

- $\pi_{1,H}$ is the payoff describing the case in which there is neither lobbying nor corruption activity.
- $\pi_{1,L}$ is the payoff describing the case in which the j-th firm engages in lobbying activity.
- π_C is the payoff describing the case in which the j-th firm finds it worthwhile to engage in bureaucratic corruption.

The results show that, in line with the theoretical and empirical literature, a sufficiently large size of the firm, i.e. its capital level, is a necessary condition for engaging in the lobbying activity. More precisely, our model predicts that firms with a sufficiently low capital level comply with the existing regulation; the firms with an intermediate capital level may prefer to engage in bureaucratic corruption, while firms with a high capital level prefer to engage in lobbying.

4 Macroeconomic implications of the model

This section aims at providing the macroeconomic insights that can be obtained from the solution of the game. In particular, we here focus on the aggregation of levels of capital to describe a country. Specifically, the assessment of the distribution of capital gives information on the industrial structure of a "country". We then intend here to present the analysis of the relationship between compliance, lobbying, bureaucratic corruption and the industrial structure of the country in which firms are located.
First of all, the identification of a distribution of the firms according to their capital is needed. We select a Gamma law, which is particularly versatile in this case, since it can describe different situations. In fact, Gamma depends on two nonnegative parameters h and θ, which represent shape and scale, respectively. The variation of h and θ drives the shape of the density function of a Gamma distribution, which serves to describe countries with different industrial structures.
For $h, \theta > 0$, the related Gamma random variables is denoted as $X \sim \Gamma(h, \theta)$, and its probability density function is

$$f(x) = \frac{x^{h-1} \exp\{-x/\theta\}}{\Gamma(\theta, h)}, \qquad x > 0, \tag{17}$$

being $\Gamma(\theta, h)$ the normalizing constant.
The numerical experiments here performed have the aim of assessing the relationship between compliance, bureaucratic corruption and lobbying, as in Proposition 3.2. The industrial structure is then described through two different situations: $h = 1.5, \theta = 0.5$ and $h = 1, \theta = 5$. The graphs of related functions f in (17) are given in Figures 2 and 3, respectively. The first case represents a country with mostly small firms, while the second one is the case with medium and large firms. We set $r = 280, c = 50, m = 1, \lambda^B = 120, T = 4$. As for C_0, we consider four values which capture the entities of the cost of compliance: $C_0 = 100, 150, 200, 300$.
We also let β, α, and q vary: $q = 0.3; 0.7$ (low and high monitoring activity, respectively), $\beta = 0.3; 0.7$ (asymmetric bargaining strength between bureaucrat and firm, in favor of bureaucrat and firm, respectively) and $\alpha = 0.3; 0.7$ (low and high percentage for political lobbying activity). The truncation of the capitals, when needed in the numerical computation of the integrals (see below), is reasonably performed at $H = 200$.
Thus, the aggregation of the capital levels divides the space in three regions, whose sizes are L, BC and C. Such sizes denote lobbying, bureaucratic corruption and compliance, respectively.
Cases (I) and (II) of Proposition 3.2 are considered. Cases (I) and (II) are mutually exclusive, and this depends on the relationship between thresholds $k^{(0)}$ and $k^{(2)}$. We can therefore distinguish two cases in the simulations.

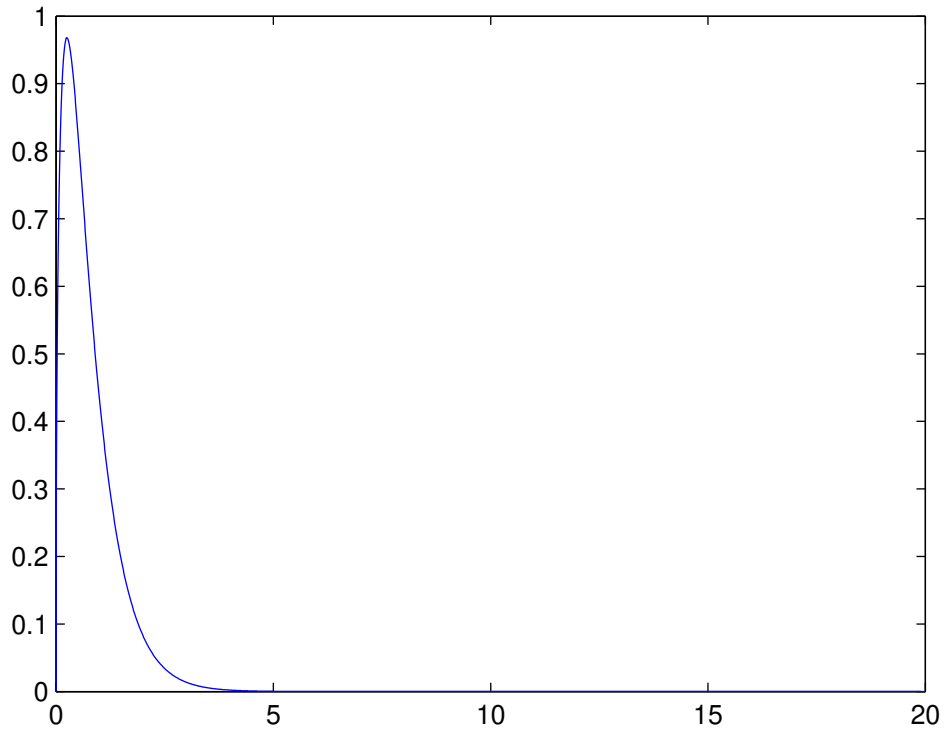

Figure 2: Density function of a Gamma distribution: $h = 1.5$, $\theta = 0.5$

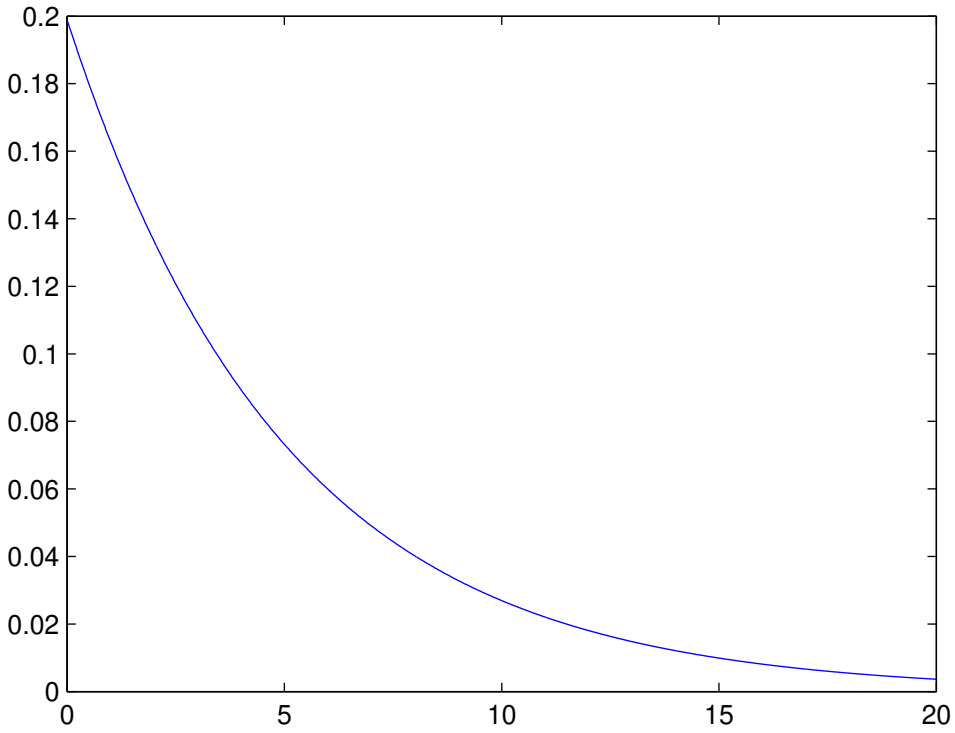

Figure 3: Density function of a Gamma distribution: $h = 1$, $\theta = 5$

Table 1: Labels of the case (I), with the values of the parameters and of $k^{(0)}$ and $k^{(2)}$.

Label of the case	q	β	α	C_0	$k^{(0)}$	$k^{(2)}$
I-1	0.3	0.7	0.3	300	1.4634	0.078
I-2	0.3	0.3	0.7	150	1.4285	1.1122
I-3	0.3	0.3	0.7	200	1.9047	1.1122
I-4	0.3	0.3	0.7	300	2.8571	1.1122

Table 2: Labels of the case (II), with the values of the parameter $k^{(0)}$.

Label of the case	q	β	α	C_0	$k^{(0)}$
II-1	0.3	0.7	0.3	100	0.4878
II-2	0.3	0.7	0.3	200	0.9756
II-3	0.7	0.7	0.3	100	0.4878
II-4	0.7	0.7	0.3	200	0.9756
II-5	0.7	0.7	0.3	300	1.4634
II-6	0.3	0.3	0.7	100	0.9523
II-7	0.7	0.3	0.7	100	0.9523
II-8	0.7	0.3	0.7	200	1.9047
II-9	0.7	0.3	0.7	300	2.8571

- Case (I)

$$\begin{cases} C = \frac{1}{\Gamma(\theta,h)} \int_0^{k^{(2)}} x^{h-1} \exp\{-x/\theta\} dx, \\ L = \frac{1}{\Gamma(\theta,h)} \int_{k^{(0)}}^{+\infty} x^{h-1} \exp\{-x/\theta\} dx, \\ BC = \frac{1}{\Gamma(\theta,h)} \int_{k^{(2)}}^{k^{(0)}} x^{h-1} \exp\{-x/\theta\} dx. \end{cases} \tag{18}$$

- Case (II)

$$\begin{cases} C = \frac{1}{\Gamma(\theta,h)} \int_0^{k^{(0)}} x^{h-1} \exp\{-x/\theta\} dx, \\ L = \frac{1}{\Gamma(\theta,h)} \int_{k^{(0)}}^{+\infty} x^{h-1} \exp\{-x/\theta\} dx. \end{cases} \tag{19}$$

The different cases are labeled, for the convenience of the reader, according to Tables 1 and 2 in which we present the related values of the parameters and of the thresholds k's.
Tables 3 and 4 collect the simulation results. Table 3 describes the situation in which the *Lobbying Threshold* is greater than the *Compliance Threshold*, i.e. $k^{(0)} > k^{(2)}$. In this circumstance, all the cases of bureaucratic corruption, lobbying and compliance appear in a given economy.
Table 4 describes the situation in which the *Lobbying Threshold* is lower than the *Compliance Threshold*, i.e. $k^{(0)} < k^{(2)}$. In this case, there is no

Table 3: Values of L, BC, C in case I

Label	$h = 1.5$, $\theta = 0.5$			$h = 1$, $\theta = 5$		
	BC	C	L	BC	C	L
I-1	0,84	0,04	0,12	0,24	0,02	0,74
I-2	0,09	0,78	0,13	0,05	0,20	0,75
I-3	0,16	0,78	0,06	0,12	0,20	0,68
I-4	0,21	0,78	0,01	0,23	0,20	0,57

Table 4: Values of L, BC, C in case II

Label	$h = 1.5$, $\theta = 0.5$			$h = 1$, $\theta = 5$		
	BC	C	L	BC	C	L
II-1	0	0.42	0.58	0	0.09	0.91
II-2	0	0.73	0.27	0	0.18	0.82
II-3	0	0.42	0.58	0	0.09	0.91
II-4	0	0.73	0.27	0	0.18	0.82
II-5	0	0.88	0.12	0	0.25	0.75
II-6	0	0.72	0.28	0	0.17	0.83
II-7	0	0.72	0.28	0	0.17	0.83
II-8	0	0.94	0.06	0	0.32	0.68
II-9	0	0.99	0.01	0	0.43	0.57

bureaucratic corruption and firms which do not have sufficient capital to engage in lobbying comply with the rule. Therefore, Tables 3 and 4 show the relationship between the industrial structure and the behavior of firms in regards to compliance.
In what follows, we remind the reader of the different implications that arise when parameters acquire different values.

- β, i.e. the bargaining power of the firm. Regarding the role of β, we consider two different values: $\beta = 0.7$ and $\beta = 0.3$. The first value means that the bargaining power of the firm is higher than that of the bureaucrat. In such cases, *ceteris paribus*, the firm will have a higher incentive to engage in bureaucratic corruption. Conversely, when $\beta = 0.3$, the bargaining strength of the firm is low and therefore, the firm has a lower economic incentive, *ceteris paribus*, to engage corruption.

- α, i.e. the percentage due to the politician for lobbying. For α, we consider two different values: $\alpha = 0.3$ and $\alpha = 0.7$. In the first case, i.e. low percentage due to the politician for his lobbying activity, the threshold capital level $k^{(0)}$ necessary to be able to engage in lobbying is, *ceteris paribus*, lower and, consequently it is easier for any given

firm to do lobbying. The reverse occurs in the case in which $\alpha = 0.7$.

- q, i.e. monitoring level of non compliant behaviors. For what concerns the role of the monitoring level q, a greater level of control ($q = 0.7$ in our study) implies that it is less convenient for the firm, *ceteris paribus*, to engage in bureaucratic corruption; the opposite applies for lower levels of monitoring ($q = 0.3$).

- C_0, i.e. fixed cost of compliance. In our analysis we consider different values of C_0: 100, 150, 200 and 300. As we mentioned above, the firms must always pay the fixed cost C_0. When this cost increases, the *Lobbying Threshold (LT)* $k^{(0)}$ increases too and then, *ceteris paribus*, less firms have sufficient capital to engage in lobbying activity.

Summing up, the bureaucratic corruption, *ceteris paribus*, is higher when the bargaining power of the firm is higher, the monitoring level is lower, and the proportional cost of compliance is higher. Viceversa, the lobbying activity, *ceteris paribus*, is more likely occurring when the contribution to the politician is lower and the fixed cost C_0 of compliance is lower.
The introduction within the model of the heterogeneity of the firm capital allows us to analyze the implications at the macroeconomic level of different industrial structures. The equilibria are computed in the entire set of cases, and in the two cases of Gamma distributions mentioned above: $h = 1.5, \theta = 0.5$; $h = 1, \theta = 5$. Considering two different industrial structures allows us to highlight how the industrial structure influences the choice of firms whether to be compliant with regulation, to engage in bureaucratic corruption or in lobbying. As for the Tables 3 and 4, note that, in all the examined cases, at least one of the regions seems to have a prominent role with respect to the others.
As already argued above, *ceteris paribus*, the industrial structures with a significant number of small firms (i.e. $h = 1.5, \theta = 0.5$) show a higher rate of compliant behavior for low capital level of firms. In fact, by reading the Tables 3 and 4 from left to right, i.e. by going from countries with many small firms to countries with a high number of large firms, we observe progressively a decrease in compliant behavior in favor of bureaucratic corruption and an increase in lobbying.
More in detail, we can analyse different cases which we present in the following subtable. Observe Table 5 in which the monitoring level is low ($q = 0.3$), the bargaining strength of the firm is high ($\beta = 0.7$), the percentage for the politician is low ($\alpha = 0.3$). If the prevalence of firms is small (i.e. $h = 1.5, \theta = 0.5$) and the fixed cost C_0 is low (i.e. $C_0 = 100$, case II-1, Table 5), then the capital necessary in order to engage in the lobbying activity is not high, and therefore the majority of firms, despite the relatively small size, can engage in lobbying. Reading Table 5 from left to right, i.e. considering an industrial structure characterized by larger firms, this trend is

Table 5: L, BC, C when $q = 0.3$, $\beta = 0.7$, and $\alpha = 0.3$

Label	$h = 1.5$, $\theta = 0.5$			$h = 1$, $\theta = 5$			C_0
	BC	C	L	BC	C	L	
Table 4							
II-1	0	0.42	0.58	0	0.09	0.91	100
II-2	0	0.73	0.27	0	0.18	0.82	200
Table 3							
I-1	0.84	0.04	0.12	0.24	0.09	0.74	300

Table 6: L, BC, C when $q = 0.3$, $\beta = 0.3$, and $\alpha = 0.7$

Label	$h = 1.5$, $\theta = 0.5$			$h = 1$, $\theta = 5$			C_0
	BC	C	L	BC	C	L	
Table 4							
II-6	0	0.72	0.28	0	0.17	0.83	100
Table 3							
I-2	0,09	0,78	0,13	0,05	0,20	0,75	150
I-3	0,16	0,78	0,06	0,12	0,20	0,68	200
I-4	0,21	0,78	0,01	0,23	0,20	0,5	3007

strengthened.

Reading the Table 5 vertically if, *ceteris paribus*, the fixed cost increases, then the fraction of the firms which can engage in lobbying activity decreases first in favor of compliance (case II-2, Table 5) and then, when the fixed cost is $C_0 = 300$, in favor of bureaucratic corruption (case I-1, Table 5).

Consider the Table 6 in which the monitoring level is low ($q = 0.3$) but also the bargaining strength of the firm is low ($\beta = 0.3$), and the percentage for the politician is high ($\alpha = 0.7$). If the prevalence of firms is small (i.e. $h = 1.5, \theta = 0.5$), when the fixed cost C_0 is low (i.e. $C_0 = 100$, case $II - 6$, Table 6), then the majority of firms find it worthwhile to comply with the regulation, because the relatively small size of the firm prevents them from engaging in lobbying. Reading vertically Table 6 and considering an industrial structure characterized by small firms, when the fixed cost increased (cases $I - 2$, $I - 3$ and $I - 4$, Table 6) then the majority of firms finds it worthwhile to comply with the regulation and the fraction of the firms which can engage in lobbying activity decreases in favor of bureaucratic corruption. Reading Table 6 from left to right, i.e. considering an industrial structure characterized by larger firms, this trend is reversed because the majority of the firms can engage in the lobbying activity.

Considering Table 7 and compare the cases $II - 7$, $II - 8$ and $II - 9$ in which firms have a low bargaining power, the percentage for the politician is high and the monitoring level is high ($q = 0.7$) with the similar cases $I - 2$,

Table 7: L, BC, C when $\beta = 0.3$, and $\alpha = 0.7$

Label	$h = 1.5$, $\theta = 0.5$			$h = 1$, $\theta = 5$			C_0	q
	BC	C	L	BC	C	L		
Table 4								
II-7	0	0.72	0.28	0	0.17	0.83	100	0.7
II-8	0	0.94	0.06	0	0.32	0.68	200	0.7
II-9	0	0.99	0.01	0	0.43	0.57	300	0.7
Table 3								
I-2	0,09	0,78	0,13	0,05	0,20	0,75	150	0.3
I-3	0,16	0,78	0,06	0,12	0,20	0,68	200	0.3
I-4	0,21	0,78	0,01	0,23	0,20	0,5	300	0.3

$I-3$ and $I-4$ in which the monitoring level is low ($q = 0.3$). If we focus on the same industrial structure, i.e. fixing h and θ, we can observe that bureaucratic corruption collapses in favor of compliance when the probability of being detected grows. In fact, when the probability of being detected is high, the number of firms which find it worthwhile to comply grows (up to 99%) at the expense of bureaucratic corruption.
Therefore, our model predicts a widespread corruption when, *ceteris paribus*, the industrial structure is concentrated on medium firms and when the probability of being detected in a corrupt transaction is low, the bargaining power of firms is high and also the fixed cost is large (which impairs to engage in lobbying activity). When the industrial structure considered has a prevalence of large firms, *ceteris paribus*, the compliance of firms and bureaucratic corruption decreases while lobbying increases. We stress that this result may occur both in a developed or non developed country.

5 Conclusions

This paper deals with the relationship between compliance, bureaucratic corruption and lobbying in the context of different industrial structures, a key element of heterogeneity across countries. We demonstrate that compliance, lobbying and bureaucratic corruption can coexist at the macro level. In our model, firms with a sufficiently low capital level comply with the existing regulation; firms with an intermediate capital level may prefer to engage in bureaucratic corruption, while firms with a high capital level prefer to engage in lobbying. Our results show that, in line with the theoretical and empirical literature, a certain size of the firm, i.e. its capital level, is a necessary condition for the existence of a lobbying activity and firms find it worthwhile to move from bureaucratic corruption to lobbying as the size of the firm grows.
The welfare and policy implications of our paper are relevant. Insofar as

corruption and lobbying are not merely redistributive activities but impact on welfare through, for example, the under-provision of a public good (e.g. environmental well-being), policies that manage to reduce them are welfare improving. While indeed improving the monitoring technology against corruption may prove to be useful in this sense, we should acknowledge that a sector with, *ceteris paribus*, more small firms might be relevant in sustaining welfare (e.g. in supporting environmental improvements). If instead regulation is not welfare-improving but merely a red-tape exercise (see Lambsdorff, (2007)) that introduces costs, removing it for small firms might prove to be relevant because it lowers costs not only due to red tape but also due to corruption meant to avoid it. Inserting legislation like, in the US, the Regulatory Flexibility Act, which ensures that regulation is imposed only if the size of the firm is large enough might therefore have an additional advantage for society.

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Appendix

Proof of Proposition 3.1

Let us fix $t = 0, 1, \ldots, T$ and let $\underline{\phi}_{\Delta}(t) = \left(\phi_{\Delta}^{(F)}(t), \phi_{\Delta}^{(B)}(t)\right)$ be the vector of the differences in the expected payoffs between the case of agreement and disagreement regarding the bribe between the j-th firm and the bureaucrat, i.e.

$$\begin{cases} \phi_{\Delta}^{(F)}(t) = \mathbf{E}[\pi_C^{(F)}(t)] - \pi_2^{(F)}(t), \\ \\ \phi_{\Delta}^{(B)} = \mathbf{E}[\pi_C^{(B)}(t)] - \pi_2^{(B)}(t), \end{cases}$$

where $\mathbf{E}$ indicates the expected value operator.
Follow the generalized Nash bargaining theory, the bribe of agreement comes out from:

$$\max_{b_t \in (0,+\infty)} \left\{\phi_{\Delta}^{(F)}(t)\right\}^{\beta} \cdot \left\{\phi_{\Delta}^{(B)}(t)\right\}^{1-\beta}, \tag{20}$$

i.e.:

$$\max_{b \in (0,+\infty)} \left[(ck_j + mk_j)(1-q) - (1-q)b\right]^{\beta} \cdot \left[-q\lambda^B + (1-q)b\right]^{(1-\beta)}. \tag{21}$$

The objective function in (21) is a reversed U-shaped function in b. Therefore, the first order condition leads to the bribe of agreement:

$$b_j^{NB} = (1-\beta)(c+m)k_j + \frac{\beta q \lambda^B}{1-q},$$

which is the unique bureaucratic equilibrium bribe in the last subgame.

Proof of Proposition 3.2

The game is solved by using backward induction, which enables the equilibria to be obtained. Fix a level of time $t = 0, 1, \ldots, T$.

(3) At stage three, the j-th firm negotiates the bribe if and only if:

$$\mathbf{E}[\pi_C^{(F)}] - \pi_{3,RC}^{(F)} > 0. \tag{22}$$

Condition (22) is verified when:

$$k_j > \frac{\lambda^B q}{(c+m)(1-q)} =: k^{(1)}. \tag{23}$$

(2) Ascending the decision-making tree, at stage two the bureaucrat decides whether to ask for a bribe or not. The bureaucrat knows that if she/he asks for a bribe, then the bribe will be negotiated when $k_j > k^{(1)}$, and refused otherwise.

(I) If $k_j > k^{(1)}$, then the bureaucrat asks for a bribe if and only if

$$\mathbf{E}[\pi^{(B)}_{(C)}] - \pi^{(B)}_{(2,RC)} > 0, \tag{24}$$

which is always verified.

(II) If $k_j \leq k^{(1)}$, then the bureaucrat asks for a bribe if and only if

$$\pi^{(B)}_{(3,RC)} - \pi^{(B)}_{(2,RC)} > 0, \tag{25}$$

which is never verified.

(1) At stage one, the j-th firm must decide whether to comply with regulation or to bend the rule. To proceed, we need to observe the cases occurring in the previous stage.

(I) If $k_j > k^{(1)}$, then the j-th firm bends the rule if and only if

$$\mathbf{E}[\pi^{(F)}_{(C)}] - \pi^{(F)}_{(1,H)} > 0, \tag{26}$$

This condition is verified when:

$$k_j > \frac{\beta\lambda^B q}{\beta(1-q)c - m[1-\beta(1-q)]} = k^{(2)}. \tag{27}$$

(II) If $k_j \leq k^{(1)}$, then the j-th firm bends the rule if and only if

$$\pi^{(F)}_{(3,RC)} - \pi^{(F)}_{(1,H)} > 0, \tag{28}$$

which is never verified.

It is easy to check that $k^{(1)} < k^{(2)}$. This completes the proof.

Some remarks on the presence of free-riding opportunities

This section is devoted to the discussion of the case in which the lobbying activity is not only in favor of the firms implementing it, but it offers also free-riding opportunities to the other firms.
Proposition 3.2 states that each firm j satisfying $k_j \geq k^{(0)}$ engages in lobbying. Now, assume that there exists at least one firm engaging in lobbying – i.e., satisfying the related condition on the capital. Moreover, let us hypothesize that the gains from the lobbying activity are enjoyed also by a generic firm j having $k_j < k^{(0)}$ by adding a free-riding parameter $\gamma \in (0,1)$, so that the variable cost of compliance paid by j is $\gamma c k_j$, instead of ck_j.
The presence of the free-riding parameter γ does not imply any additional

complexity in the mathematical solution of the model. However, the economic content of the obtained outcomes is particularly relevant.
First of all, in the presence of free riding, the threshold $k^{(2)}$ in (16) becomes

$$k^{(2)}(\gamma) = \frac{\beta q \lambda^B}{\beta(1-q)\gamma c - m[1-\beta(1-q)]}. \tag{29}$$

According to (29), the threshold $k^{(2)}(\gamma)$ decreases with respect to γ. Therefore, cases $(I.A)$ and $(I.B)$ in Proposition 3.2 assure that a high value of γ – which is associated to a small effect of free-riding, with a large part of variable costs paid by the non-lobbying firms – implies that firms are more likely involved in bureaucratic corruption. This outcome is in line with the economic decision to engage in bureaucratic corruption or, alternatively, of not bending the rule.
Interestingly, we notice that the presence of the free-riding parameter leads also to a new formulation of the bribe b_j^{NB} in (11), as follows:

$$b_j^{NB}(\gamma) = (1-\beta)(\gamma c + m)k_j + \frac{\beta q \lambda^B}{(1-q)}. \tag{30}$$

The bribe $b_j^{NB}(\gamma)$ now increases with respect to γ; thus, we find that has the free-riding become stronger– i.e., as γ declines – the equilibrium bribe declines too. Firms are, therefore, less ready to pay bribes for corrupting bureaucrats when there is the interesting alternative of a meaningful free-riding from the lobbying activity.