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NUMERICAL ANALYSIS OF DESICCATION, SHRINKAGE AND CRACKING IN LOW PLASTICITY CLAYEY SOILS

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Abstract. This paper presents a numerical study of the desiccation processes of low-plasticity clayey soils that usually result in shrinkage and often in cracking. For the theoretical development of the numerical model, concepts of Unsaturated Soils Mechanics and of classical Strength of Materials are used as a framework for formulating phenomena such as water flow in deformable porous media and cracking. The mathematical formulation of the problem and its implementation in a hydro-mechanical coupled model is presented, in order to simulate fluid flow and cracking in soils, for which the FEM and the node release technique is combined. The code developed has been used to perform several numerical analyses on radial sections of cylindrical soil specimens subjected to a drying process for which experimental laboratory data was available. The objective of these simulations is to determine the mechanisms by which the soil shrinks and cracks during desiccation. The results show the capabilities of the approach to reproduce the main features of the problem, with desiccation, shrinkage, and cracking being reproduced consistently during a desiccation cycle. The model also highlights the key role of the displacement and suction boundary conditions in the development of cracks as a consequence of tensile stress fields. Finally, the model has revealed the necessity of further research in the study of the soil-container and soil-atmosphere interaction in order to reproduce with more accuracy the changes in the main variables.

Key words: cracking, desiccation, shrinkage, flow in deformable porous media, fracture, numerical simulation.

1 INTRODUCTION

The topic of drying cracks in soils has been the object of considerable experimental research, and many significant contributions have been made in recent decades. However, until the development of Unsaturated Soil Mechanics, the problem has not received proper theoretical research taking into account the parameters that govern the behavior of soil in the unsaturated state, mainly suction. Tensile strength, which is suction dependent, and fracture toughness are shown to be also relevant parameters if the initiation and propagation of the cracks have to be studied.

The main variables involved in this problem are the temperature and relative humidity of the environment, but several other factors are involved in the process. In laboratory tests, specimen size, soil-container type of contact, drying rate and specimen's characteristics (such as heterogeneity, anisotropy, imperfections, water content, particle size, tensile strength or fracture toughness) condition how cracking develops. In the field, the soil's fabric, position of the water table, wind velocity, solar radiation, etc. need also to be considered.

When the soil is dried under laboratory conditions or in an environmental chamber, the first cracks that can be seen on the top surface of the specimen are usually boundary cracks that start at the interface between the soil mass and the container wall. These cracks propagate until the entire soil mass is separated from the wall. During the propagation of this boundary cracks, other cracks may appear in the middle of the specimen. These cracks may initiate at the top or bottom surfaces, or at the middle of the specimen, and propagate simultaneously with the first ones.

Crack formation and propagation in drying soils is a coupled thermo-hydro-mechanical process (not considering the chemical processes that may also take place). However, in the present work and in order to simplify the analysis the thermal component is left out, assuming that the process is isothermal.

The main objective of the numerical analysis is to reproduce the time evolution of the recorded variables (suction, water content, and deformation) during laboratory tests performed in recent years [1, 2] and to estimate the stress evolution before and after the initiation of the cracks. The numerical analysis is carried to simulate the formation and propagation of the first crack, which usually appears at the soil-container interface and initiates from the upper external surface of the specimen and propagates toward the bottom along the interface.

2 NUMERICAL MODEL

The model was formulated assuming that the process of desiccation and cracking take place mainly in unsaturated conditions. In the unsaturated porous medium, the equilibrium equation in terms of the total stresses is:

$$\nabla \cdot (\boldsymbol{\sigma} - u_a \mathbf{1}) + \nabla u_a + \rho \mathbf{g} = \mathbf{0} \tag{1}$$

which is an elliptic partial differential equation where $\boldsymbol{\sigma}(\mathbf{x}, t)$ is the total stress tensor, $(\boldsymbol{\sigma} - u_a \mathbf{1})$ is the net stress tensor, u_a is the air pore pressure, ρ is the average density of the multiphase medium (soil, water and air) and $\mathbf{g}$ is the gravity vector.

The stress-strain relation used in the present model can be written as

$$d\boldsymbol{\sigma} = \mathbf{D}(d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}^s) = \mathbf{D}\left(d\boldsymbol{\varepsilon} + \mathbf{m}\frac{du_w}{3K_t^s}\right) \tag{2}$$

where $d\boldsymbol{\sigma}$ is the increment of the total stress tensor, $\mathbf{D}$ in the elastic tangent stiffness matrix, $d\boldsymbol{\varepsilon}$ and $d\boldsymbol{\varepsilon}^s$ are the total and suction related infinitesimal deformations, u_w is the suction or negative pore water pressure, K_t^s is the suction modulus and $\mathbf{m} = (1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0)^T$ is the identity tensor in vector form.

The generalized Darcy's law for unsaturated soils, which is the constitutive equation for the flow problem, is written as

$$\mathbf{q} = -\mathbf{K}(S_r) \cdot (\nabla u_w - \rho^w \mathbf{g}) \tag{3}$$

where $\mathbf{q}$ is Darcy's velocity vector; ∇u_w is the porewater pressure gradient; $\mathbf{K}(S_r)$ is the permeability tensor which depends on the saturation degree (S_r); and ρ^w is the water density.

The fluid mass balance equation is

$$\nabla \cdot (\rho^w \mathbf{q}) + \frac{\partial}{\partial t}(\rho^w n S_r) = 0 \tag{4}$$

For the present formulation, it is assumed that the air flow is produced without friction and without phase changes. After the application of the finite element method, the system of partial differential equations (1) that emerges can be written in matrix notation as:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_\mathrm{T} & \mathbf{Q}_\mathrm{T} \\ \mathbf{P} & \mathbf{S} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \bar{\mathbf{u}}}{\partial t} \\ \dfrac{\partial \bar{\mathbf{p}}}{\partial t} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \mathbf{f}^u}{\partial t} \\ \mathbf{f}^p \end{bmatrix} \tag{5}$$

where $\bar{\mathbf{u}}$ is the nodal displacement vector; $\bar{\mathbf{p}}$ is the nodal porewater pressure vector; $\mathbf{H}$ is the diffusion matrix; $\mathbf{K}_T$ is the mechanical stiffness matrix; $\mathbf{S}$ is the storage matrix; $\mathbf{Q}_T$ and $\mathbf{P}$ are coupling matrices between the mechanical and hydraulic problems; and $\mathbf{f}^u$ and $\mathbf{f}^p$ are the external nodal displacement and flow vectors respectively. The Dirichlet boundary conditions of this problem are written in terms of suction and displacements.

3 SIMULATION OF A TEST ON A 80×20 CYLINDRICAL SAMPLE

Several cylindrical specimens of clayey soils were tested at the laboratory to study the problem of desiccation. The sizes of the specimens were 40/80 cm in diameter and 10/20 cm height. The tests were performed in laboratory conditions or in an environmental chamber [3].

The simulation of the desiccation of a 2D radial section (40×20 cm) of an 80×20 cylindrical specimen is presented (Figure 1). This simulation reproduces the initiation and propagation of a crack between the soil and the container on the right lateral edge of the specimen which is typically the first crack to appear in the experiments. The suction boundary condition is applied to the top of the section and the displacement boundary conditions are applied to the right and bottom edges. The left edge of the radial section is the axis of symmetry of the specimen.

The formation of cracks at the soil-container interface is justified by the presence of tensile stresses larger than the tensile strength of the soil, which depends on moisture content. The crack propagates from the top to the bottom of the container. Although the direction of propagation in terms of the maximum principal stress would have to be roughly calculated, vertical propagation was simulated. To simulate crack propagation, the node separation technique has been applied, which modifies the boundary conditions as the tensile strength of the soil is reached. In the numerical simulation, the condition for the first crack was reached during the first day. Although in the laboratory experiments the start of cracking in the 80×20 specimen occurred on day 8, with the 80/40×10 cm specimens the first crack appeared between days 1 and 10. This shows that the variability in the initial cracking time is very large and the repeatability of the tests and cracking start times cannot be guaranteed neither in the laboratory nor in the numerical simulations.

It has been possible to simulate quite accurately the evolution of the moisture content and of shrinkage of the soil due to desiccation. Also, the simulations of the specimen's shrinkage agree quite well with the experimental observations.

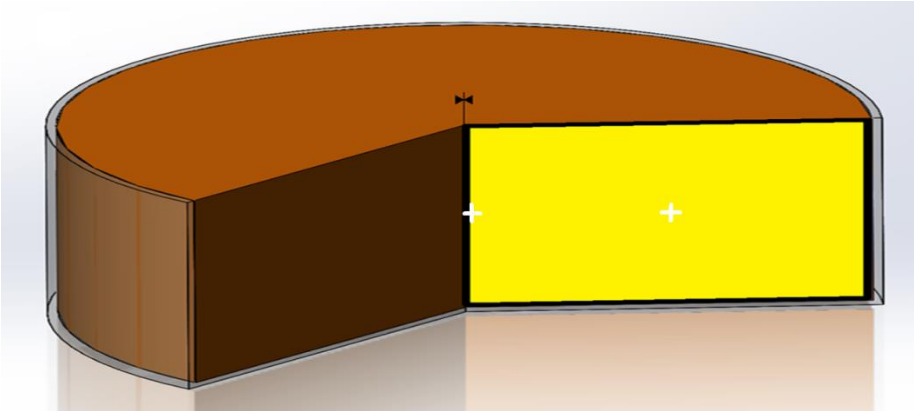

Figure 1 – 80×20 cylindrical specimen with the radial section (40×20 cm, yellow) used in the numerical simulation of the desiccation process. The two reference points are shown with the white crosses.

Although there were some small temperature fluctuations in the laboratory during the experiments, the process can be assumed to be isothermal. The relative humidity during the laboratory tests was maintained around 40% for most of the test. In the simulation, a constant suction equal to 60 MPa has been imposed on the boundary exposed to the atmosphere. This value has allowed the best possible adjustment although it can be considered a somewhat low value compared to what was measured in the laboratory which reached 100 MPa.

Figure 2 shows the evolution of the suction field during the 120 days of the simulation. It is assumed in this case that there is no adherence between the soil and the bottom of the container, although the separation from the container is prevented. Thus, once the crack reaches the bottom of the container there is no possibility that other cracks will form. The propagation of the crack has been fast because the tensile stress conditions rapidly exceeded the tensile strength, which is consistent with the tests.

The tensile strength depends largely on the moisture content of the soil. The following equation, that governs the initiation of the crack [4], has been adopted:

$$\sigma_t = -0.0191w^2 + 0.6874w - 2.88 \tag{6}$$

where σ_t is the tensile strength and w is the moisture content of the soil.

The dimensions of the cracks in the simulations correspond also well with the dimensions obtained in the laboratory. Crack formation and propagation modifies the normal stress field and how it changes with time.

Figure 3 shows the evolution of the moisture content and of suction during the test at the reference points shown in Figure 1.

Table 1 summarizes the parameters used in the simulation. A fairly small permeability is needed, but it does not deviate too far from the usual values for clays.

Table 2 lists the parameters of the water retention curve for the used soil [4] obtained using the van Genuchten function [5]:

$$S_r = \left[1 + \left(\frac{s}{P_0 \cdot f_n}\right)^{\frac{1}{1-\lambda}}\right]^{-\lambda} \qquad f_n = \exp[-\eta(n - n_0)] \tag{7}$$

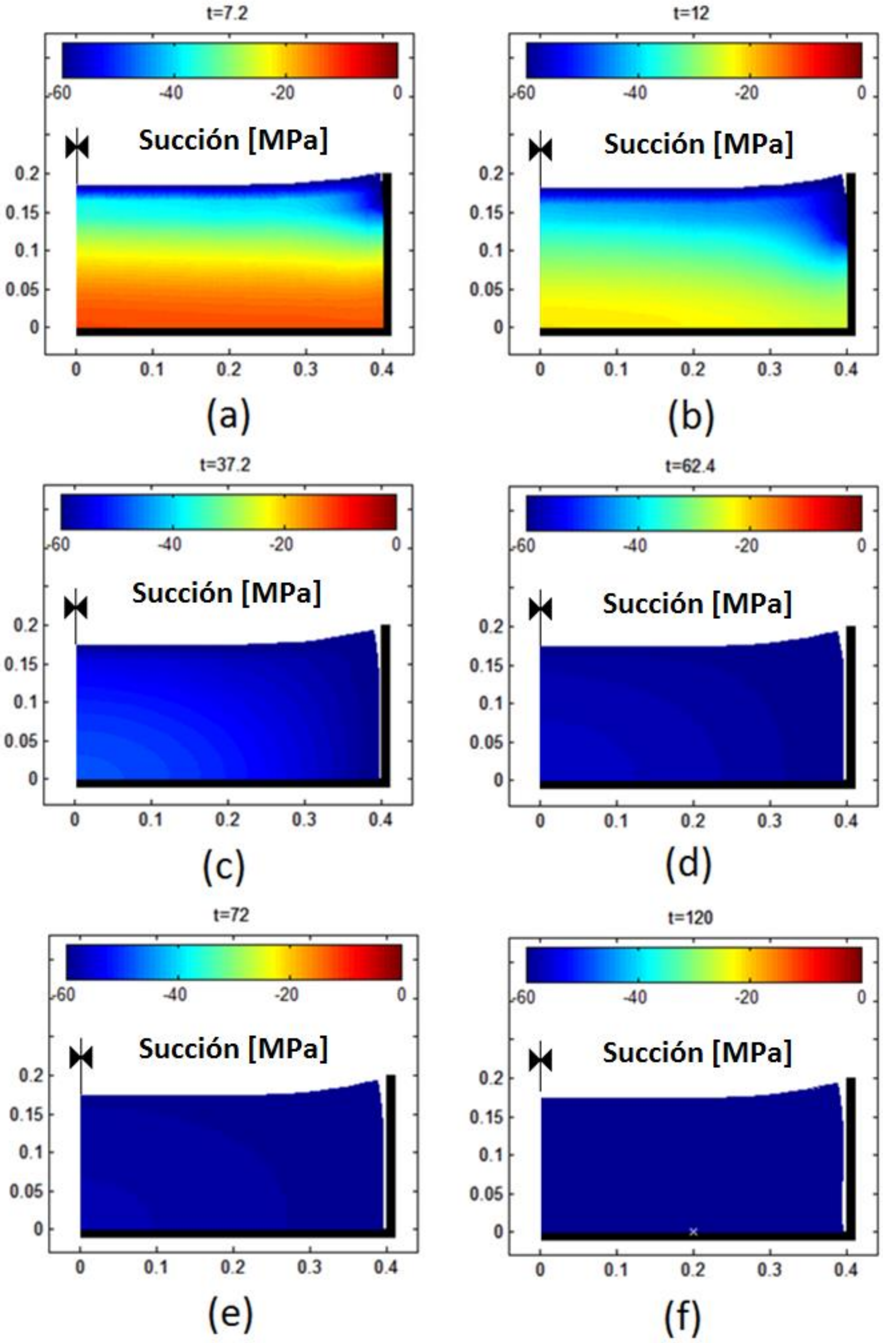

Figure 2 – Evolution of the suction field during drying and propagation of a crack between the soil and the container. Suction an interface crack after a) 7 days of drying; b) 12 days of drying; c) 37 days of drying; d) 62 days of drying; e) 72 days of drying; and f) 120 days of drying.

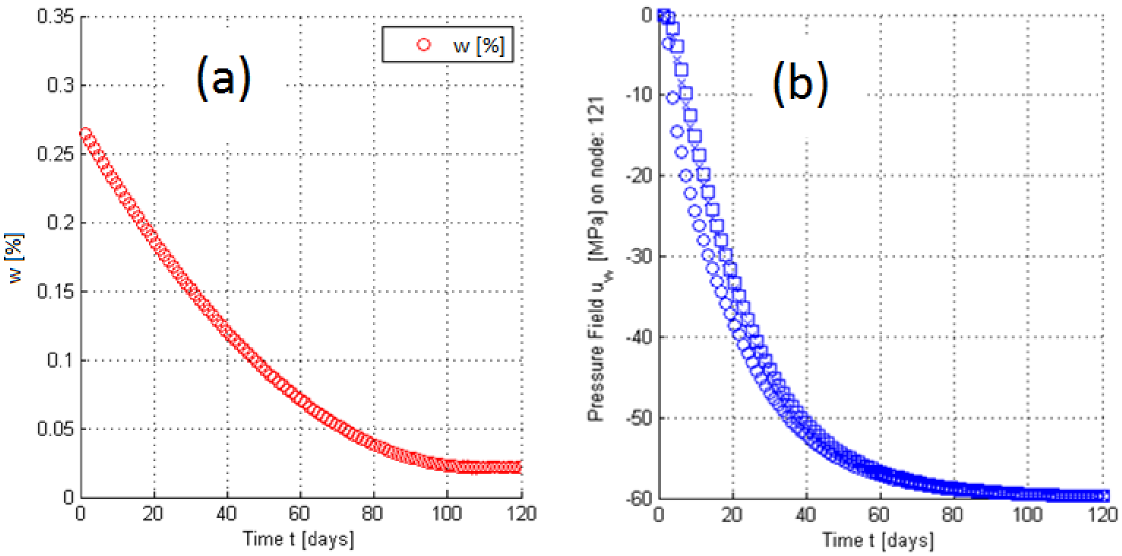

Figure 3 – Simulation of a cracking test: a) change with time of the specimen's moisture content; and b) change of suction at the three reference points in the radial section.

Table 1 – Parameters used in the numerical simulation

<table>
<tr><td colspan="9">Mechanical parameters</td></tr>
<tr><td>a_1</td><td>a_2</td><td>a_3</td><td>a_4 (MPa)</td><td>P_{ref} (MPa)</td><td>ν</td><td colspan="2">G (MPa)</td><td>σ_c (MPa)</td></tr>
<tr><td>-0.02</td><td>-0.0025</td><td>-0.000039</td><td>0.023</td><td>0.1</td><td>0.4</td><td colspan="2">$\frac{3K(1-2\nu)}{2(1+\nu)}$</td><td>0.0035</td></tr>
<tr><td colspan="3">Retention curve parameters</td><td rowspan="3"></td><td colspan="5">Hydraulic parameters</td></tr>
<tr><td>P_0 (MPa)</td><td>λ</td><td>n_0</td><td colspan="2">k_0 (m/s)</td><td>b</td><td>n_0</td><td>r</td></tr>
<tr><td>1.05</td><td>1.55</td><td>0.6</td><td colspan="2">9.27×10^{-10}</td><td>25</td><td>0.6</td><td>3</td></tr>
</table>

Table 2 – Parameters of the water retention curve [4]

e	γ_d (kN/m^3)	f_n	λ	n (porosity)
0.87	14.5	0.828	1.524	0.545
0.75	15.5	0.756	1.447	0.527
0.64	16.5	0.485	1.554	0.479
0.55	17.5	0.033	3.052	0.373
0.53	17.7	0.066	2.420	0.380

4 CONCLUSIONS

The behaviour of the numerical model is adequate to simulate the process of desiccation and soil cracking. It has been possible to implement an algorithm capable of triggering crack initiation and of simulating crack propagation using a tensile strength criterion and a node release technique. The hydraulic problem needs special care because of its nonlinearity and because the suction boundary condition generates some instability to the system. The propagation of the cracks introduces still greater instability and requires care to maintain the balance, since imbalances are introduced whenever a node is released. The change in boundary conditions affects the stress field in the vicinity of the crack and these stresses must be redistributed in the soil matrix. In spite of the complications derived from the implementation and the development of the model, it has been demonstrated that it is possible to simulate the drying and cracking process with a relatively simple technique.

In the laboratory experiments, the first crack usually appears at the specimen's boundary (soil/container interface) and propagates from the top surface towards the bottom of the container. This process can be studied numerically in two dimensions, using a radial section.

Laboratory experiments show a great variability in the time of beginning of the first visible crack on the surface, between 1 and 10 days in the tests. Previous research also shows that there is no guarantee that the first crack will always begin at the upper boundary, although this is likely to be the case when adequate boundary conditions are imposed and a minimum of homogeneity is ensured in the specimen. The numerical model has demonstrated the reasons for cracking to start at the edges, because of a state of tensile stresses that the specimen cannot sustain.

The rate of crack propagation is relatively large during the tests. The model allows to propagate the crack in lengths proportional to the length of the edge of the finite element so that in principle it can be adjusted to the measurements made in the laboratory.

The model parameters have been calibrated using all available test information. However, in order to properly calibrate the hydromechanical parameters, simpler tests should be designed on which a smaller number of variables are controlled. On the other hand, due to the natural variability of the phenomenon, it is necessary to perform a large number of tests and conduct a statistical study on the behaviour of some variables, such as the time at which the first crack appears.

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