4 Model

4.1 Hydro-Mechanical model

Two state stress variables are adopted in this model: net stress $\boldsymbol{\sigma}^*$ and suction s .

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma} - p^a \mathbf{1} \tag{5}$$

$$s = p^a - p \tag{6}$$

With the assumption that atmospheric pressure p^a is constant and equal to zero, the state variables become the total stress $\boldsymbol{\sigma}$ and the negative pore water pressure p.

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma} \qquad \text{and} \qquad s = -p \tag{7}$$

Under the assumptions of small-strain theory, isothermal equilibrium and negligible inertial forces, we obtain the following balance equations. First, the linear momentum balance equation for a two phase medium, where ρ is the density and $\mathbf{g}$ is the gravity vector.

$$\operatorname{div} \mathbf{g} + \rho \mathbf{g} = 0 \tag{8}$$

Second, the mass balance equation for water, where ρ^w is the water density, $\mathbf{q}$ is Darcy's velocity, n the porosity and S_r the degree of saturation.

$$\operatorname{div}(\rho^w \mathbf{q}) + \frac{\partial}{\partial t}(\rho^w n S_r) = 0 \tag{9}$$

We can summarize the coupled problem through the next system of differential equations:

$$\begin{cases} \mathbf{K}\dfrac{\partial \overline{\mathbf{u}}}{\partial t} + \mathbf{Q}\dfrac{\partial \overline{\mathbf{p}}}{\partial t} - \mathbf{f}^u = \mathbf{0} \\ \mathbf{P}\dfrac{\partial \overline{\mathbf{u}}}{\partial t} + \mathbf{S}\dfrac{\partial \overline{\mathbf{p}}}{\partial t} + \mathbf{H}\overline{\mathbf{p}} - \mathbf{f}^p = \mathbf{0} \end{cases} \tag{10}$$

Where $\overline{\mathbf{u}}$ and $\overline{\mathbf{p}}$ are respectively nodal displacements and nodal pore pressure. $\mathbf{K}$, $\mathbf{Q}$, $\mathbf{P}$, $\mathbf{S}$ and $\mathbf{H}$ are matrices, and $\mathbf{f}^u$ and $\mathbf{f}^p$ vectors, that result from the FEM approach.

The algebraic system of equations that results from this formulation is highly nonlinear and non-symmetric, in general. For this reason we need to use iterative strategies to solve it. The first equation in (10) is the mechanical part and the second one is the hydraulic part.

4.2 Unsaturated soil mechanics concepts

For the hydro-mechanical formulation we need to employ two constitutive models. First we apply a mechanical constitutive model based on the concept of state surfaces. Next a hydraulic constitutive model, including unsaturated flow (Darcy's law) is used. We write both constitutive models as follow:

$$\varepsilon_v = -\frac{e}{1+e_0} = a_1 \ln(\boldsymbol{\sigma} + a_4) + a_2 \ln\left(\frac{p + p_{ref}}{p_{ref}}\right) + a_3\left[\ln(\boldsymbol{\sigma} + a_4)\ln\left(\frac{p + p_{ref}}{p_{ref}}\right)\right] \tag{11}$$

$$\mathbf{q} = -\mathbf{K}(S_r)\cdot(\nabla p - \rho^w \mathbf{g}) \tag{12}$$

Where ε_v is the volumetric strain, e and e_0 are the current and initial void ratios, a_1, a_2, a_3, a_4 are state surface constants and p_{ref} is a reference pressure.

Furthermore, we need the relation between suction and degree of saturation. In our case we chose the Van Genuchten equation (Van-Genuchten 1980) [13]:

$$S_r = \left[1 + \left(\frac{p}{P_0 f_n}\right)^{\frac{1}{1-\lambda}}\right]^{-\lambda} \tag{13}$$

In (13) S_r is the degree of saturation, λ is a material parameter, P_0 is the air entry value at the reference porosity n_0 and f_n is a function of the porosity and of the material parameter η.

4.3 Fracture parameters

Tensile strength of the soil is used for the crack initiation (Lee et al. 1988), whereas for crack propagation concepts of LEFM (Atkinson 1991) are used. This section explains how the conditions of stress state in a drying soil obtained from the H-M model are used for the crack initiation and propagation.

4.3.1 Crack initiation

Drying soil can be considered as a continuum media before the crack initiation, therefore applying a tensile strength criterion whereby computed principal stresses (from H-M model) are compared with an experimentally determined tensile strength (σ_T) to predict the crack initiation.

4.3.2 Crack propagation and direction

Considering only Mode-I failure and applying the $\sigma(\theta)_{\max}$ theory, the criterion for crack propagation can be expressed as,

$$\sigma(\theta)_{\max}\sqrt{2\pi r} \geq K_{IC} \tag{14}$$

where, $\sigma(\theta)_{\max}$ is obtained from the H-M model and (K_I) is experimentally determined, and r is an arbitrary value that depends on the type of element used in the FEM formulations. The value of r is 2/3 of the height of the element for triangular elements.

The direction of crack propagation is in a plane normal to the direction of greatest tension, i.e., at θ_0 such that $\tau_{r\theta} = 0$.

$$\sigma(\theta)_{\max}\sqrt{2\pi r} = \cos\left(\frac{\theta_0}{2}\right)\left[K_I \cos^2\left(\frac{\theta_0}{2}\right)\right] \tag{15}$$

5 Conclusions

Fracture parameters determined experimentally are in agreement with the existing published data. Fracture parameters were determined at different moisture contents; these values are more realistic when used in any numerical model for cracking in drying soils. LEFM seems to be the most simple fracture mechanics theory to explain cracking in soils. This is due to ease with which the LEFM parameters determination. Experimental evidences of existence of Size-effect further strengths the applicability of LEFM. The proposed model combining soil mechanics and fracture mechanics provides an ideal frame work for numerical solution of cracking in drying soils.

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