User Selection in Reconfigurable Intelligent Surface Assisted Communication Systems

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***Abstract*—This paper presents a detailed investigation on the performance of reconfigurable intelligent surface (RIS)-assisted communication system with user scheduling. Depending on the availability of channel state information (CSI) at the RIS, two separate scenarios are considered, namely without CSI and with CSI. Closed-form expressions are derived for the ergodic capacity of the system in both scenarios. It is found that CSI has a significant impact on the performance of the system. Without CSI, the RIS provides an array gain of N, where N is the number of reflecting elements, and user scheduling provides an multi-user gain of $\log\log M$, where M is the number of users. With CSI, the RIS provides an array gain of N^2, while no multi-user diversity gain can be obtained.**

I. Introduction

With the commercialization of the 5G mobile systems, the quest for enabling technologies for the next generation mobile systems is currently underway. Reconfigurable intelligent surfaces (RIS), which consist of a large number of low-cost and passive reflecting elements, each of which can independently reflect the incident electromagnetic wave with adjustable phase shift, have recently emerged as a promising candidate, due to its desirable feature of low energy consumption and high spectral efficiency [1]–[4].

Hence, RIS-assisted communications have received considerable attention from both academia and industry. In [5]–[7], assuming perfect channel state information (CSI), the optimal phase shifts design of RIS-assisted communication systems are studied. Later on, robust design in the presence of imperfect CSI is pursued in [8]–[11]. In addition, the fundamental performance of RIS-assisted single antenna system is studied in [12], while the extension to the multiple antenna system is given in [13]. Moreover, the integration of RIS with various transmission techniques has been considered, including millimeter wave (mmWave) [14], two-way communications [15] and simultaneous wireless information and power transfer (SWIPT) [16].

While the above-mentioned works have greatly improved the understanding of the performance of RIS-assisted communications, the impact of multi-user scheduling remains unclear. Responding to this, we provide a detailed investigation on the performance of RIS-assisted communication systems with user scheduling, and characterize the achievable multi-user diversity gain. Specifically, two different scenarios are considered, depending on the availability of CSI at the RIS, namely, without CSI and with CSI. For both cases, analytical expressions for the ergodic capacity of the system are presented. Based on which, asymptotic analysis is conducted to reveal the impact of key system parameters. The findings suggest that, for the case without CSI, using RIS provides an array gain of N, where N is the number of reflecting elements, and user scheduling provides an multi-user gain of $\log\log M$, where M is the number of users. On the other hand, for the case with CSI, using RIS provides an array gain of N^2, while no multi-user diversity gain can be obtained.

Manuscript received October 9, 2020, revised December 5, 2020, accepted December 24, 2020. This work was supported in part by the National Key R&D Program of China 2019YFB1803400, the Zhejiang Provincial Natural Science Foundation of China under Grant LD21F010001 and the National Natural Science Foundation of China under Grant 61922071 and 61725104. The editor coordinating the review of this paper and approving it for publication was Dr. Khandaker. (*Corresponding author: Caijun Zhong*)

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II. System Model

We consider a RIS-assisted wireless communication system consisting of a single antenna BS and M single antenna users. The RIS is deployed between the BS and users, comprised by N ($N = N_r \times N_c$) reflecting elements arranged in a uniform planar array (UPA). Since the RIS is likely to be deployed at a position where exists line-of-sight (LoS) path to both the BS and users, Rician distribution are used to model these channels as in [10], [17]. Hence the channel from the BS to the RIS, and form the RIS to the i-th user ($i \in \{1, ..., K\}$) are given by

$$\mathbf{h}_0 = \sqrt{\alpha_0 \frac{K_0}{K_0+1}}\bar{\mathbf{h}}_0 + \sqrt{\alpha_0 \frac{1}{K_0+1}}\tilde{\mathbf{h}}_0, \tag{1}$$

$$\mathbf{h}_i = \sqrt{\alpha_i \frac{K_1}{K_1+1}}\bar{\mathbf{h}}_i + \sqrt{\alpha_i \frac{1}{K_1+1}}\tilde{\mathbf{h}}_i, \tag{2}$$

where K_0 and K_1 are the Rician K factors of $\mathbf{h}_0$ and $\mathbf{h}_i$, α_0 and α_i are the large scale path-loss coefficients, $\bar{\mathbf{h}}_0 \in \mathbb{C}^{N\times 1}$ and $\tilde{\mathbf{h}}_i \in \mathbb{C}^{1\times N}$ are the LoS components, $\tilde{\mathbf{h}}_0 \in \mathbb{C}^{N\times 1}$ and $\tilde{\mathbf{h}}_i \in \mathbb{C}^{1\times N}$ are the non-line-of-sight (NLos) components, whose elements are independently and identically distributed (i.i.d.) zero mean complex Gaussian random variables (RVs) with unit variance.

Since the RIS has the UPA, then the LoS component $\bar{\mathbf{h}}_0$ and $\bar{\mathbf{h}}_i$ can be expressed as

$$\bar{\mathbf{h}}_0 = \mathbf{a}_N^H(\phi_{AoA}, \theta_{AoA}) \quad \text{and} \quad \bar{\mathbf{h}}_i = \mathbf{a}_N(\phi_{AoD}, \theta_{AoD}), \tag{3}$$

where

$$\begin{aligned}\mathbf{a}_N(\phi,\theta) =& [1, ..., e^{j2\pi\frac{d}{\lambda}(n_r\sin(\theta)\sin(\phi)+n_c\cos(\theta))}, \\ & ..., e^{j2\pi\frac{d}{\lambda}((N_r-1)\sin(\theta)\sin(\phi)+(N_c-1)\cos(\theta))}],\end{aligned} \tag{4}$$

is the the array response of an $N_r \times N_c$ elements UPA at an azimuth (elevation) angle of ϕ_{AoA} (θ_{AoA}) and ϕ_{AoD} (θ_{AoD})

respectively. This response is with d being the antenna spacing, λ being the wavelength, ϕ_{AoA} and θ_{AoA} being the angles of arrival (AoA) to the RIS, ϕ_{AoD} and θ_{AoD} being the angles of departure (AoD) from the RIS, $0 \le n_r < N_r$ and $0 \le n_c < N_c$.

Since the user is likely to be blocked from the BS, we consider two cases. The one is Rayleigh fading, while the other is that there does not exist direct way from the user to the BS. Therefore, the channels between the BS and users can be expressed as

$$g_i = \omega \cdot \sqrt{\beta_i}\tilde{g}_i, \tag{5}$$

where $\omega = 0$ implies no direct link while $\omega = 1$ implies the existence of direct link. The elements of $\tilde{g}_i \in \mathbb{C}$ are i.i.d. complex Gaussian distribution with zero mean and unit variance, β_i captures the path-loss of the channel between the BS and the i-th user.

As in [18], we can assume that the users are clustered in a small area, and thus the distance between different users and the RIS (BS) can assume equal to d_1 (d_2). According to [19], the large scale path-loss can be modelled as the function of distance, and thus we can assume that $\alpha_i = \alpha_1$, $\beta_i = \beta_1$, $i \in \{1, ..., K\}$.

$$\alpha_0 = \frac{\phi}{d_0^l}, \alpha_1 = \frac{\phi}{d_1^l}, \beta_1 = \frac{\phi}{d_2^l}, \tag{6}$$

where d_0, d_1, d_2 are the distance from the BS to the RIS,the RIS to the user, and BS to the user respectively, l denotes the path-loss exponent, ϕ represents the path-loss normalization factor [19].

At each instant, a single user is selected for transmission. The signal received by the i-th user can be written as

$$y_i = \sqrt{P}(\mathbf{h}_i\mathbf{\Phi}\mathbf{h}_0 + g_i)s + n_i, \tag{7}$$

where P is the transmit power of the BS, $\mathbf{\Phi} = \mathsf{diag}\{e^{j\varphi_1}, ..., e^{j\varphi_N}\}$, $\varphi_n \sim [0, 2\pi)$ is the phase shift introduced by the n-th element of the RIS, $s \in \mathbb{C}$ is the normalized energy signal transmitted by BS, which satisfied $E\{|s|^2\} = 1$, $n_i \sim \mathcal{CN}(0, N_0)$ is the zero mean additive white Gaussian noise (AWGN).

III. CAPACITY ANALYSIS

In this section, we provide a detailed analysis of the achievable capacity of the system with user-selection. We assume that there exists feedback links between the BS and users, so the BS can schedule the user with the largest channel gain. Specifically, with k-th user being scheduled, the ergodic capacity can be expressed as

$$C_k = \mathbb{E}\{\log_2(1 + \gamma_0 \cdot A_k)\}, \tag{8}$$

where $\gamma_0 = \frac{P}{N_0}$, $A_k = |\mathbf{h_k\Phi h_0} + g_k|^2$, and the index k is determined according to the following criterion:

$$k = \arg\max_{i=1,\ldots,M} A_i. \tag{9}$$

The CSI can be acquired as per the methods proposed in prior works such as [20] and [21]. In the following, we consider two separate cases depending on the availability of CSI at the RIS.

A. Without CSI

When the RIS has no access to the CSI, we can assume that the phase φ_n follows the uniform distribution. Then, we have the following result:

Theorem 1: When there is no CSI at the RIS, the phase shifts are drawn from the uniform distribution, then the ergodic capacity of the system is upper bounded by

$$C_{\mathsf{u}} = \log_2[1 + \gamma_0 \cdot (\omega^2\beta_1 + N\alpha_1\alpha_0)(\gamma + \ln(M+1)], \tag{10}$$

where γ is the Euler constant.

Proof: See Appendix A ■

Theorem 1 provides a simple closed-form expression for the ergodic capacity upper bound assuming that ϕ_n follows the uniform distribution in the case without CSI. The concise expression also facilitates the understanding of the impact of key system parameters on the ergodic capacity performance. In particular, we have the following key observations.

Remark 1: The ergodic capacity upper bound C_{u} is an increasing function with respect to N. In addition, for sufficiently large N, we have $C_{\mathsf{u}} \approx \log_2(\gamma_0(\gamma + \ln(M+1))\alpha_0\alpha_1 N)$, which implies a gain of order N on the effective SNR. The reason is that the RIS can attain the inherent aperture gain of order N by collecting more signal power.

Remark 2: The ergodic capacity upper bound C_{u} is an increasing function with respect to M. In addition, for sufficiently large M, we have $C_{\mathsf{u}} \approx \log_2(\gamma_0 \cdot (\omega^2\beta_1 + N\alpha_1\alpha_0)\ln M)$, which implies that a multi-user diversity gain of $\log\log M$ can be achieved through user scheduling.

B. With CSI

When the RIS has access to the CSI, the phase shift matrix can be properly designed to enhance the system performance, and the optimal φ_n has been reported in [10]. Capitalizing on the result of [10], the the optimal user k is selected as

$$k = \arg\max_{i=1,\ldots,M} \left(\sum_{j=1}^{N} |h_i^j||h_0^j| + |g_i| \right)^2. \tag{11}$$

Then, we have the following important result:

Theorem 2: When RIS has access to the CSI, the ergodic capacity of the system can be approximated by

$$C_{\mathsf{a}} = \int_0^\infty \log_2(1+x) \cdot M(F_{\mathsf{A}}(x))^{M-1} f_{\mathsf{A}}(x)dx, \tag{12}$$

where $F_{\mathsf{A}}(x)$ is the cumulative distribution function (c.d.f.) of RV $A_i \triangleq |\sum_{j=1}^N |h_i^j||h_0^j| + g_i|^2$ defined by

$$F_{\mathsf{A}}(x) = \frac{1}{2}\left(1 + \mathsf{erf}\left(\frac{\sqrt{x}-\mu}{\sqrt{2\sigma^2}}\right)\right) - \sqrt{\frac{\omega^2\beta_1}{\omega^2\beta_1 + 2\sigma^2}} \times e^{-\frac{\omega^4\beta_1^2(\sqrt{x}-\mu)^2}{\omega^2\beta_1+2\sigma^2}} \left[1 + \mathsf{erf}\left(\frac{\omega^2\beta_1(\sqrt{x}-\mu)}{\sqrt{2\sigma^2\beta_1(\omega^2\beta_1+2\sigma^2)}}\right)\right], \tag{13}$$

where $\mu_i = \frac{N\pi}{4}\sqrt{\frac{\alpha_0\alpha_1}{(K_0+1)(K_1+1)}} \cdot L_{1/2}(-K_0)L_{1/2}(-K_1)$, $\sigma_i^2 = N\alpha_0\alpha_1 - \mu_i^2$, $f_A(x)$ is the probability density function (p.d.f.), and $\mathsf{erf}(x)$ is the error function.

Proof: See Appendix B. ■

To gain further insights, we consider the special case when $\omega = 0$ [24].

Theorem 3: For the special case $\omega = 0$, the ergodic capacity of the system can be approximated by

$$C_{\mathsf{a1}} = \log_2 \left\{ 1 + \gamma_0 \left[\mathsf{erf}^{-1}\left(1 - \frac{2}{M}\right) \sqrt{2N\alpha_1\alpha_0} \right.\right.$$
$$\times \sqrt{1 - \frac{\pi^2 L_{1/2}^2(-K_1) L_{1/2}^2(-K_0)}{16(K_0+1)(K1+1)}} \quad (14)$$
$$\left.\left. + \frac{N\pi}{4}\sqrt{\frac{\alpha_1\alpha_0}{(K_1+1)(K_0+1)}} L_{1/2}(-K_1) L_{1/2}(-K_0) \right]^2 \right\},$$

where $\mathsf{erf}^{-1}(x)$ is the inverse error function.

Proof: See Appendix C ∎

Remark 3: The approximated ergodic capacity C_{a1} is an increasing function with respect to N. In addition, for sufficiently large N, we have $C_{\mathsf{a1}} \approx \log_2\left(\frac{\gamma_0\pi^2\alpha_1\alpha_0}{16(K_1+1)(K_0+1)} L_{1/2}^2(-K_1) L_{1/2}^2(-K_0) N^2\right)$, indicating a gain of order N^2 on the effective SNR, which is different form the case without CSI. The reason is that, with the proper design of the phase shift matrix, the RIS not only attains the inherent aperture gain of order N by collecting more signal power, but also achieves beamforming gain of order N benefit from the availability of CSI.

Remark 4: The ergodic capacity C_{a1} is an increasing function with respect to M. However, for sufficiently large M, noticing that $\mathsf{erf}^{-1}\left(1 - \frac{2}{M}\right) \approx \frac{\sqrt{\pi}}{2}[1 + \frac{\pi}{12} - (2 + \frac{\pi}{2})/M]$, it is easy to show that the impact of M on C_{a1} vanishes, i.e., there is no multi-user diversity gain. This is due to the phenomenon of channel hardening with optimal phase shift matrix, such that the variation of channels is substantially reduced, which in turn makes the benefit of user scheduling negligible.

Remark 5: The ergodic capacity estimation C_{a1} is a symmetric function with respect to the Rician factor K_0 and K_1, indicating the identical impact of the two hop channels. In addition, noticing that $\lim\limits_{K\to\infty} \frac{1}{\sqrt{K+1}} L_{1/2}(-K) = \frac{2}{\pi}$, for sufficiently large K_0 and K_1, the approximation converges to

$$C_{\mathsf{a1}} \approx \log_2(1 + \gamma_0 \cdot \alpha_1\alpha_0 N^2), \quad (15)$$

which is independent with K_0 and K_1, thereby indicating the vanishing impact of Rician factors in the asymptotic regime.

IV. Discussion and Numerical Results

In this section, we provide numerical results and simulations to validate the analytical expressions presented in the previous sections, and investigate the impact of key system parameters on the system performance. Without loss of generality, we assume $N_0 = -80$ dBm, $\phi = 1$, $l = 3$. The distances d_0, d_1 and d_2 are set to be 200, 200 and 300 meters, respectively. Furthermore, we choose the Rician factor $K_0 = K_1 = 3 + \sqrt{12}$ [22]. The azimuth and elevation AoA and AoD are set to follow the uniform distribution. To ensure that the users are in the far-field [23], we limit the maximum number of RIS elements N in the simulations to 256. All the simulation results are obtain by averaging over 10^4 independent trials.

Fig. 1 plots both the exact ergodic capacity and upper bound without CSI when $N_r = N_c = 16$ with different M. It can be readily observed that the upper bound remains sufficiently tight over the entire range of SNRs. In addition, the ergodic capacity increases when the number of users M becomes larger, due to the benefit of multi-user diversity gain.

Fig. 2 compares the capacity performance between random phase matrix and optimal phase matrix for different M. Intuitively, the ergodic capacity of the system with optimal phase matrix is strictly higher than that of the system with random phase matrix. Moreover, the performance gap between the two cases narrows down with the increase of user number M, mainly due to the fact that the case with random phase matrix can exploit the multi-user diversity gain.

Fig. 3 depicts the impact of Rician factor K and number of reflecting elements N on the ergodic capacity of the system with CSI. It can be observed that, a larger N leads to higher capacity, due to the array gain of N^2. Also, stronger LoS paths also improve the ergodic capacity. However, the capacity gain is rather insignificant and quickly vanishes for large K, as predicted by the analytical results.

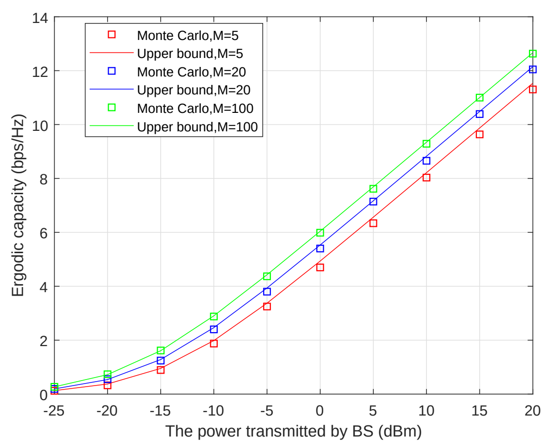

Fig. 1. Ergodic capacity without CSI under different M, with $N_r = N_c = 16$.

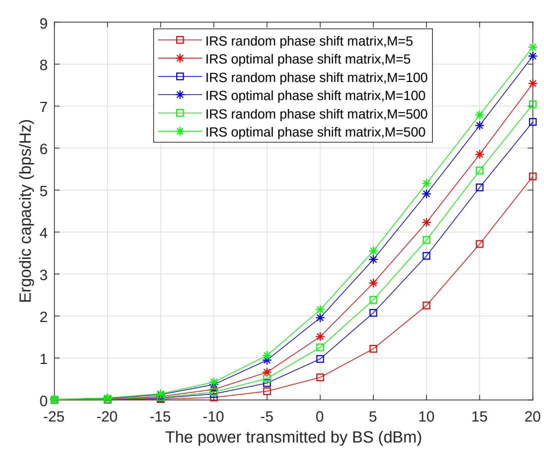

Fig. 2. Performance gain versus RIS phase shift matrix under different M, with $N_r = N_c = 16$.

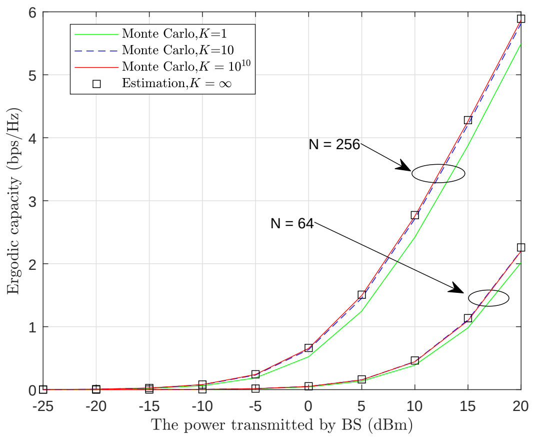

Fig. 3. Impact of Rician factor K on ergodic capacity with CSI and without direct channel under different N, with $M = 20$.

V. Conclusion

This work has studied the ergodic capacity of RIS-assisted communication systems with user selection. Depending on the availability of the CSI at the RIS, two separate cases are considered. Closed-form expressions are derived for the achievable capacity in both cases. The findings of the paper suggest that, an array gain of N and N^2 can be achieved for the case without CSI and with CSI, respectively. In addition, a multi-user diversity gain of $\log\log M$ can be realized through user scheduling when CSI is not available. However, the impact of user scheduling on the case with CSI is rather insignificant. For further works, it is of great interest to extend the analysis to the more general setups with multiple antennas at the BS.

Appendix A
Proof of Theorem 1

Conditioned on the pubic link $\mathbf{h}_0$, the RV $\mathbf{h}_i\mathbf{\Phi}\mathbf{h}_0 = \sum_{j=1}^{N} h_i^j e^{j\varphi_j} h_0^j$ is the sum of N i.i.d RVs. When the number of the RIS units is sufficiently large, the central limit theorem can be invoked, hence we have

$$\mathbf{h}_i\mathbf{\Phi}\mathbf{h}_0 \sim \mathcal{CN}(\mu_1, \sigma_1^2), \tag{16}$$

where $\mu_1 = N \cdot \mathbb{E}[h_i^j e^{j\varphi_j} h_0^j] = 0$ and

$$\begin{aligned}\sigma_1^2 &= \sum_{j=1}^{N} \mathbb{E}[(h_i^j e^{j\varphi_j} h_0^j - \mu_1)(h_i^j e^{j\varphi_j} h_0^j - \mu_1)^*] \\ &= \|\mathbf{h}_0\|_2^2 \alpha_1.\end{aligned} \tag{17}$$

Since the sum of independent normal distributions still follows the normal distribution, we have

$$\mathbf{h}_i\mathbf{\Phi}\mathbf{h}_0 + g_i \sim \mathcal{CN}(0, \omega^2\beta_1 + \|\mathbf{h}_0\|_2^2\alpha_1). \tag{18}$$

Hence, the square of the amplitude A_i is exponentially distributed with mean $1/(\omega^2\beta_1 + \|\mathbf{h}_0\|_2^2\alpha_1)$.

Since the c.d.f. of the exponential RV is a convex function, invoking the result of [26], the conditional ergodic capacity with user scheduling can be upper bound by

$$C_{\mathsf{u}|\mathbf{h_0}} \leqslant \log_2(1 + \gamma_0 q_{e^{\gamma}(M+1)}), \tag{19}$$

where $q_m = F_{\mathsf{A}}^{-1}(1 - \frac{1}{m})$.

Then, invoking the Jensen's inequality, we have

$$C_{\mathsf{u}} = \log_2(1 + \gamma_0(\omega^2\beta_1 + \mathbb{E}[\|\mathbf{h}_0\|_2^2]\alpha_1)(\gamma + \ln(M+1))).$$

To this end, noticing that $\|\mathbf{h}_0\|_2^2$ follows the non-central chi-square distribution, it is easy to obtain

$$\mathbb{E}[\|\mathbf{h}_0\|_2^2] = N\alpha_0, \tag{20}$$

which completes the proof.

Appendix B
Proof of Theorem 2

When the number of the RIS units is sufficiently large, the central limit theorem can be invoked, hence, $X_i \triangleq \sum_{j=1}^{N} |h_i^j||h_0^j|$ follows the normal distribution, i.e., to get the following approximation:

$$X_i \sim \mathcal{N}(\mu_i, \sigma_i^2), \tag{21}$$

where $\mu_i = \frac{N\pi}{4}\sqrt{\frac{\alpha_0\alpha_1}{(K_0+1)(K_1+1)}} \cdot L_{1/2}(-K_0)L_{1/2}(-K_1)$ and $\sigma_i^2 = N\alpha_0\alpha_1 - \mu_i^2$.

We now look into the distribution of RV $A_i = (X_i + |g_i|)^2$. Using the fact that X_i and $|g_i|$ are independent, the c.d.f. of A_i can be computed as

$$F_{\mathsf{A}}(x) = \frac{1}{2}\left(1 + \mathsf{erf}\left(\frac{\sqrt{x}-\mu}{\sqrt{2\sigma^2}}\right)\right) - \sqrt{\frac{\omega^2\beta_1}{\omega^2\beta_1 + 2\sigma^2}} \, e^{-\frac{\omega^4\beta_1^2(\sqrt{x}-\mu)^2}{\omega^2\beta_1+2\sigma^2}}\left[1 + \mathsf{erf}\left(\frac{\omega^2\beta_1(\sqrt{x}-\mu)}{\sqrt{2\sigma^2\beta_1(\omega^2\beta_1+2\sigma^2)}}\right)\right]. \tag{22}$$

In general, the RV X_i are correlated due to the common factor $\mathbf{h}_0$. However, in the asymptotic large N regime, they becomes uncorrelated. Specifically, the person product-moment correlation coefficient of X_m and X_n can be calculated via

$$\begin{aligned}\rho_1 &= \frac{\mathbb{E}[X_mX_n] - \mu_m\mu_n}{\mathbb{D}[X_m]\mathbb{D}[X_n]} \\ &= \frac{N\alpha_0\alpha_1\frac{\pi L_{1/2}^2(-K_1)}{4(K_1+1)}(1 - \frac{\pi L_{1/2}^2(-K_0)}{4(K_0+1)})}{N\alpha_0\alpha_1 - N^2\alpha_0\alpha_1\frac{\pi^2 L_{1/2}^2(-K_0)L_{1/2}^2(-K_1)}{16(K_0+1)(K_1+1)}}.\end{aligned} \tag{23}$$

Then, dividing both the numerator and denominatar by $N^2\alpha_0\alpha_1$, we have

$$\rho_1 = \frac{\frac{\pi L_{1/2}^2(-K_1)}{4N(K_1+1)}(1 - \frac{\pi L_{1/2}^2(-K_0)}{4(K_0+1)})}{\frac{1}{N} - \frac{\pi^2 L_{1/2}^2(-K_0)L_{1/2}^2(-K_1)}{16(K_0+1)(K_1+1)}}. \tag{24}$$

As N becomes large, we have $\lim_{N\to\infty}\rho_1 = 0$. Since X_m and X_n are normal RVs, they are independent as long as they are uncorrelated.

Having established the independency of X_i, it is easy to show that the c.d.f. of $A_{\mathsf{max}} = \max\limits_{i=1,\ldots,M} A_i$ can be expressed as

$$F_{\mathsf{Amax}}(x) = [F_{\mathsf{A}}(x)]^M, \tag{25}$$

which completes the proof.

APPENDIX C
PROOF OF THEOREM 3

When $\omega = 0$, the p.d.f and the c.d.f of the RV A_i can be expressed by

$$f_A(x) = \frac{1}{2\sqrt{2\pi x\sigma^2}} \exp\left[-\frac{(\sqrt{x}-\mu)^2}{2\sigma^2}\right], \tag{26}$$

and

$$F_A(x) = \frac{1}{2}\left[1 + \mathsf{erf}\left(\frac{\sqrt{x}-\mu}{\sqrt{2}\sigma}\right)\right], \tag{27}$$

where $\mu = \frac{N\pi}{4}\sqrt{\frac{\alpha_0\alpha_1}{(K_0+1)(K_1+1)}} \cdot L_{1/2}(-K_0)L_{1/2}(-K_1)$ and $\sigma^2 = N\alpha_0\alpha_1 - \mu^2$. Then, it is easy to verify that

$$\lim_{x\to\infty} \frac{1-F_A(x)}{f_A(x)} = 2\sigma^2. \tag{28}$$

Capitalizing on the result of [27], it can be shown that $\max(A_i) - l_k$ converges to the distribution $e^{-e^{-x/C}}$ when the M grows large, where $l_k = F_{\mathsf{A}}^{-1}(1-\frac{1}{M}) = (\mu + \mathsf{erf}^{-1}(1-\frac{2}{M})\sqrt{2}\sigma)^2$.

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