PERFORMANCE ASSESSMENT OF VOID FRACTION CORRELATIONS IN LARGE DIAMETER VERTICAL PIPE UPFLOW

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ABSTRACT

Recently, due the to increase in the production demand in many industries such as Nuclear, Oil & Gas and process industries, the requirement to migrate toward larger pipe sizes has become essential. However it is interesting to note that almost all of the earlier two phase flow research is based on small diameter pipes (D≤100mm) and experimental work on predicting the two phase flow behaviour in large diameter (D>100mm) pipes is rare. Thus, the application of methodologies/correlations/equations for predicting flow pattern, void fraction, and pressure gradient based on small diameter poses severe challenges in terms of accuracy. Specifically, the prediction of void fraction in two phase flows, as it plays a fundamental role in characterizing the distribution of the phases within the system. With large number of the void fraction correlations available in the different fields of multiphase flows, the choice for the selection of any void fraction correlation existing in the literature is bewildering.

This paper presents an assessment of the predictive capabilities of forty (40) void fraction correlations belonging to different multiphase flow industries. The assessment is performed by comparing the independent experimental data obtained from a 254mm diameter and a 12.2m high vertical pipe upflow loop using air-water as working fluid. The final assessment indicated that most of the void fraction correlations are flow regime dependent as none of the correlations successfully predicted all the four regimes (bubbly, agitated bubbly, unstable slug and churn turbulent) encountered in large diameter vertical upflow experiments.

1. INTRODUCTION

Void fraction plays a fundamental role in characterizing the distribution of the phases in two a phase flow system. With void fraction predictions further parameters, such as two phase density and viscosity are evaluated. These are required by the existing models for predicting the flow regime transitions, pressure drops and heat transfer capabilities. Generally, a design engineer needs a void fraction correlation to predict the two phase flow system behaviour before designing the actual system and/or simulating scenario related to that system. Thus the designing and reliability of any two phase model is dependent upon the prudent choice of the void fraction correlation used.

2. PREVIOUS ASSESSMENTS OF VOID FRACTION CORRELATIONS

Though assessments of void fraction correlations have been done in past, they are still comparatively few in number with evaluation confined either to orientation or specific application (Dukler et al., 1964; Vohra et al., 1975; Friedel, 1980; Simpson et al., 1987; Chexal et al., 1991; Maier & Coddington, 1997; Manera et al., 2005 and Ghajar et al., 2007). Almost all the above reported assessments are based on the small diameter pipe flow data, with the exception of Chexal et al. (1991) and Simpson et al. (1987). In the former, a total of 115 data points of vertical upflow were tested against eight void fraction correlations specifically belonging to nuclear industry while the latter study was confined to horizontal pipe of 216mm diameter. With such limited assessments that also confined to void fraction correlations derived from smaller diameter vertical pipes experiments brings in much uncertainty. This calls for a thorough assessment of the predictive capability of void fraction correlations as no study has so far included the assessment of void fraction correlations with respect to their applicability to large diameter vertical pipe upflow. The assessment is also important in order to determine the implications of the different flow patterns occurring in the large diameter and the conventional small diameter vertical pipe.

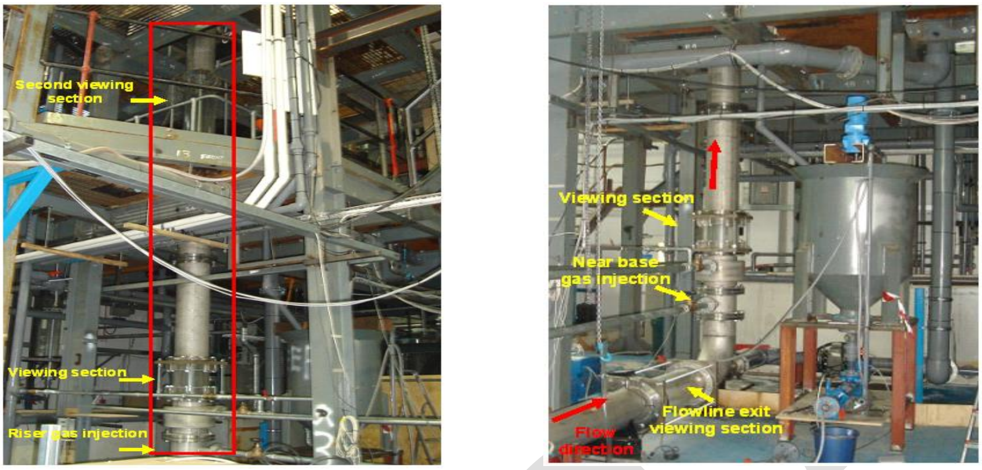
3. VOID FRACTION CORRELATIONS TESTED

In all some 40 correlations are included in this study. The accuracy of the void fraction correlations compared is evaluated in terms of average percent errors and standard deviation. Selected results are also represented graphically by using “cross plots”. Results of all correlations used in the comparison are summarized in a tabular form in later section. A separate flow regime specific table is also presented, recommending satisfactory correlations in a particular flow regime. For the sake of brevity, the correlations are not presented here and can be obtained from the original work of the researchers.

4. EXPERIMENTAL FACILITY

Figure 1 shows the large diameter facility, comprising of an air and water supply systems, horizontal flowline-vertical riser, upper plenum consisting of return tank, downcomer with return line to sump. The diameter of the loop is 254mm (nominal diameter) with horizontal pipeline of 36m and vertical riser of 12.2m length respectively. The air injection provisions are available at the upstream of the pipeline and slightly above the riser base. The pipeline-riser is equipped with special high pressure perspex viewing sections at various strategic positions in the loop to observe the flow patterns. A detailed description of the facility and measurement techniques can be found in (Ali and Yeung, 2008). However, a brief description of the relevant sensors is given here. Two differential pressure cells and a water manometer located in the riser at the height of 5m, 7m and 10m were used to deduce the void fraction. Two pressure transducers were also installed at the riser entrance and the exit. All above sensors are installed near the perspex sections so that simultaneous signal acquisition and videoing can take place. From the preliminary experiments it was deduced that under the velocity range conducted, the two phase runs were dominated by hydrostatic head. Thus, the overall and sectional void fraction from differential pressure measurements is estimated under the assumption that the differential pressure equals to static head by neglecting accelerative and frictional losses. There is good agreement between the sectional void fraction and overall void fraction. All the void fraction data has been analyzed statistically as well as visually for various flow regimes identification (Ali and Yeung, 2008). The data points (210 in number) obtained corresponds to bubbly (B), agitated bubbly (AB), unstable slug (US) and churn/froth

(C) flow regimes. The unstable slug flow was the transition region between agitated bubbly and churn/froth flow regime encountered in limited horizontal flow line runs. This can be considered as the flow regime where the flow exhibited the remains of slug flow structure (from the flowline) but was less stable as the flow transformed into churn/froth flow upon increase in gas flow rates. The existing set-up did not allow for the experiments in annular flow regime. The ranges of parameters are summarized in Table 1.



(a) (b)

Figure 1. Photographic views of large diameter riser setup (a) the vertical riser section and (b) horizontal pipeline-riser base.

Table 1. Large Diameter vertical pipe data range used in current analysis.

Test section	Fluid	Pressure	Mass flux (kg/m^2-s)
Horizontal flow line injection (air + water)	Air-water	Atmospheric	175.15 - 635
Horizontal flow line injection (air + water) & Riser injection (air)	Air-water	Atmospheric	162 - 634
Horizontal flow line injection (water) & Riser injection (air)	Air-water	Atmospheric	190 - 1100

5. RESULTS

5.1 Correlations based on Homogenous Equilibrium Model: This model exhibited greater accuracy at low void fraction only (bubbly flow). Once the transition took place from bubbly to intermittent flow, the predicted void fraction values deviated, as the flow was no longer well mixed but highly agitated (Table 2). The model is independent of the diameter of the conduit and thus can be applied as a starting point to the conditions where dispersed bubbly or dispersed droplets (or mist flows) are likely to be encountered.

Armand (1946) and *Bankoff* (1960) correlations are example of "$K\beta$" model. While former correlation takes into account of pressure only, latter attempts to correct the influence of pressure on the radial distribution of void fraction and on the flow velocities. In this comparison above models seem to improve the HEM model results. However like HEM, *Armand* correlation indicates closer results at very low void fraction only while *Bankoff* (1960) give a more reasonable prediction over the whole range.

5.2 Void fraction correlations based on slip ratio: Correlations based on the separate flow model using slip equation yielded mixed results. The correlation by *Smith* (1969) over-predicted the void fraction by +21%. The deviation was largest for bubbly flow and progressively decreases for higher flow regimes. Correlations of *Guzhov et al.* (1967) and *Premoli et al.* (1971), both take into account of mass flux (hence diameter), were found to perform superiorly than other correlations. Both correlations showed an overall mean error of +4% and +10% respectively with successful application in all flow regimes. The analytical correlations of *Chisholm* (1972) and Huq & Loth (1990) over predict the void fraction by +25% and +18% respectively. While former shows the largest deviation in the bubbly and agitated bubbly flow regimes latter indicates over prediction mainly in bubbly flow.

5.3 Void fraction correlations based on Lockhart and Martinelli parameter (X): Correlations based on Martinelli parameter showed wide variation in results. Almost all correlations did not perform very well with mostly under predicting *(Levy*, -30.06%, *Baroczy*, -28.19% and *Zivi*, -68.93%). The only partially successful were original *Lockhart and Martinelli* (1949) model and *Chen I & II* (1986) correlations with mean error lying close to maximum allowable limit (-24% and ±17%).

5.4 Void fraction correlations based on Drift flux model: Among all the categories, this is the most successful category. However, many of the correlations used in this category are subjected to following constraints (i) void expressions are flow regime dependent, hence are not continuous, this might give rise to numerical instabilities during computation (ii) the model are sensitive to prediction of flow patterns and any inappropriate choice of flow pattern would increase the variance of the whole model and (iii) many of the above correlations are iterative in nature, which inhibits its frequent use in comparison to direct correlations.

First correlation in this category is of *Wilson et al.* (1961), the prediction from the correlation are satisfactory with an overall mean error within -4%. *Nicklin* (1962) correlation also well predicted all flow regimes with -5%. *Zuber and Findlay* (1965) correlation indicates an overall mean error of 11.2%. *Rouhani* (1969) two correlations gave over prediction of void fraction of +11.62% and 19.9% respectively. *Rouhani and Axelsson* (1970) correlation also over-predicted the void fraction values by 21.25%. Interestingly, all last three correlations yielded lowest error in bubbly flow regime. *GE-Ramp* (1970) proprietary correlation predicted close values with mean error of +4.8%. *Dix* (1971) correlation, based on the most extensive measurements of the local void fraction over predicts by mean error of +7%. *Bonnecaze et al.* (1971) correlation indicates a mean error of -5% with applicability scope in agitated bubbly, unstable slug and churn/froth flow regimes. *Hills* (1976) suggested drift flux correlations based on different velocity ranges shows excellent agreement with mean percent error of 1.96%. *Kocamustafagoullari and Ishii* (1985) specifically developed drift correlation for distorted cap bubbles found in large diameter pipes predicted void fraction values within +4.5% of the measured void fraction. *Hirao et al.* (1986) correlation under predicts the void fraction (-15%). *Morooka et al.* (1989) correlation by regression analysis over predicted the void fraction by +7.6%. *Maier and Coddington* (1997) drift flux type correlation showed an overall mean error of -15.38%.

Kataoka and Ishii (1987) correlation specifically developed for large diameter pipe shows the best agreement with an overall mean error of about 1.55%. The *Chexal and Lellouche* (1992) versatile correlation applicable to all co-current and counter-current flows of the steam-water, air-water and refrigerant. The void fraction predictions of this correlation for large diameter are satisfactory (within -5%). *Ghajar et al.* (2007) correlation applicable to all flow patterns and inclination angles shows a mean error of 12.54%. *Hibiki and Ishii* (2003) correlations are inlet flow regime dependent drift flux correlations for two-phase flow in a large diameter pipe at low mixture fluxes. For high mixture fluxes they recommended Ishii and Kataoka (1987) correlations. Excellent predictions are obtained by these correlations in comparison to all the correlations used with mean error of 1.75%.

5.5 Empirical void fraction correlations: Few empirical void fraction correlations were also included in this analysis. These include *Spedding & Chen (1981)*, *Spedding & Chen I & II* (1984), *Neal and Bankoff* (1965) and some flow regime specific empirical correlations widely applied in Oil & Gas industries namely *Hagedorn and Brown* (1965), *Duns and Ross* (1963), *Beggs and Brill* (1973) and *Mukherjee and Brill* (1985).

Spedding & Chen (1981) correlation was not able to yield satisfactorily results, surprisingly, the correlations of *Spedding & Chen I & II* (1984) performed very well with the closest best prediction in all flow regimes with mean error of -2.9% and -8.5%. *Neal and Bankoff* (1965) correlation overall under predicts by mean error of -22% with largest deviation in bubbly flow. *Hagedorn & Brown* (1965), *Duns & Ross* (1963), *Beggs & Brill* (1973) and *Mukherjee & Brill* (1985) are in actual pressure gradients prediction methods. *Hagedorn & Brown* (1965) correlation based on average properties of the phases, over predicted the void fraction values strongly showing largest mean error (+50%). *Duns & Ross* (1963) correlation specifically developed for vertical flows, over predicts the void fraction with an overall mean error +15%. *Beggs & Brill* (1973) method of pressure gradient applicable to any inclination indicated a mean percent error of 21.42%. *Mukherjee & Brill* (1985), a modification of Beggs & Brill method predicted a mean error of 16.54%. In all the Oil & Gas correlations, large inconsistencies in flow regimes predictions were also seen especially for agitated bubbly and churn/froth flow regime as they were identified as slug flow.

5.6 Based on two fluid model: OLGA is 1D, extended two-fluid model that is extensively used multiphase simulation tool in Oil & Gas industries. The code is broadly applied to simulate the various scenarios such as start-up and shutdown transients, terrain slugging etc. In current analysis the OLGA clearly indicates the pipe diameter affects, the results are +30% in mean error. The flow regime predictions in the riser section are also in contradiction with experimental results for agitated bubbly flow regime and churn turbulent flow regime.

The below comparison (see Table 2) of the void fraction correlations with experimental data suggest that the closest predicting void fraction correlations are *Kataoka and Ishii* (1987), *Hibiki and Ishii* (2003), *Hills* (1976), *Spedding & Chen* (1984), *Guzhov et al.* (1967), *Wilson et al.* (1961), *Kocamustafagoullari and Ishii* (1985) and *Chexal and Lellouche* (1992). Figure 2 indicates their results on the cross plots. It is to be noted that in the above correlations *Kataoka & Ishii* (1987), *Hibiki & Ishii* (2003) and *Hills* (1976) are specifically for large diameter application. Note that *Hibiki & Ishii* correlation also performed very well at very low void fraction with few others while all the rest yielded a strong under prediction in this range. Wherever approximate averages are required, *Premoli et al.* (1971) and *Guzhov et al.* correlations are recommended on the basis of their simplicity. Based on the results of the individual flow regime assessments, a set of table is developed (see Table 3) that recommends correlations based on their performances in the individual flow regimes.

Table 2. Void fraction correlation used in this study.

Category	Correlation	References	Mean error (%)	Standard Deviation (%)
Homogenous void fraction model (α=β)	HEM model	Neil & Kazimi (1989)	52.67	19.59
The "α=$K\beta$" forms.	Armand (1950)	Spedding et al. (1998)	27.17	16.32
	Bankoff (1960)	Neil & Kazimi (1989)	16.46	16.52
Some commonly used slip ratio (*S*) relations.	Chisholm (1972)	Chisholm (1972)	-27.48	9.89
	Smith (1969)	Thom (2004)	21.38	14.30
	Premoli et al. (1971)	Hewitt (1982)	10.11	10.54
	Guzhov et al. (1967)	Vohra et al. (1975)	3.13	16.82
	Huq & Loth (1992)	Huq & Loth (1992)	17.63	13.60
Based on Lockhart and Martinelli parameter (*X*).	Lockhart & Martinelli (1949)	Butterworth (1975)	-24.07	9.66
	Levy (1960)	Thom (2004)	-30.06	10.21
	Zivi (1964)	Butterworth (1975)	-68.93	10.49
	Baroczy (1966)	Butterworth (1975)	-28.39	9.37
	Chen (1986) I & II	Spedding & Spence (1989)	-17.11 & 16.54	12.38 & 22.17
Based on empirical correlations.	Neal & Bankoff (1965)	Spedding & Spence (1989)	-22.48	21.79
	Spedding & Chen (1981)	Spedding & Spence (1989)	-34.94	8.76
	Spedding & Chen (1984) - I & II	Spedding & Chen (1984)	-2.59 & -8.54	16.19 & 25.37
Based on drift flux model, mostly from Nuclear industry.	Wilson et al. (1961)	Kataoka & Ishii (1987)	-3.98	14.57
	Nicklin (1962)	Nicklin (1962)	-5.41	11.93
	Zuber & Findlay (1965)	Thom (2004)	11.19	9.27
	Rouhani (1969) - I & II	Spedding & Spence (1989)	19.93 & 11.62	11.53 & 10.08
	Rouhani & Axelsson (1970)	Thom (2004)	21.25	11.96
	GE Ramp (1970)	Tecdoc-1203	4.79	12.81
	Dix (1971)	Neil & Kazimi (1989)	-6.90	19.34
	Bonnecaze et al. (1971)	Spedding et al. (1998)	-5.40	11.93
	Hills (1976)	Hills (1976)	1.96	10.81
	Kocamustafagoullari & Ishii (1985)	Hibiki & Ishii (2003)	4.45	10.85
	Hirao et al. (1986)	Hirao et al. (1986)	-14.86	13.34
	Kataoka & Ishii (1987)	Kataoka & Ishii (1987)	1.55	10.40
	Morooka et al. or Toshiba (1989)	Morooka et al. (1989)	7.63	13.29
	Chexal and Lellouche (1992)	Chexal & Lellouche (1992)	-4.53	19.62
	Maier & Coddington (1997)	Coddington et al. (2002)	-15.38	15.47
	Hibiki & Ishii (2003)	Hibiki & Ishii (2003)	1.75	8.78
	Ghajar et al. (2007)	Ghajar et al. (2006)	12.54	11.06
Based on popular Oil & Gas industry.	Hagedorn & Brown (1959)	Brill & Mukherjee (1999)	52.63	19.61
	Duns & Ross (1963)	Brill & Mukherjee (1999)	15.62	13.86
	Beg & Brill (1973)	Brill & Mukherjee (1999)	21.42	50.73
	Mukherjee & Brill (1985)	Brill & Mukherjee (1999)	16.54	17.18
Two fluid model	OLGA-S	Scandpower (2000)	30.26	11.51

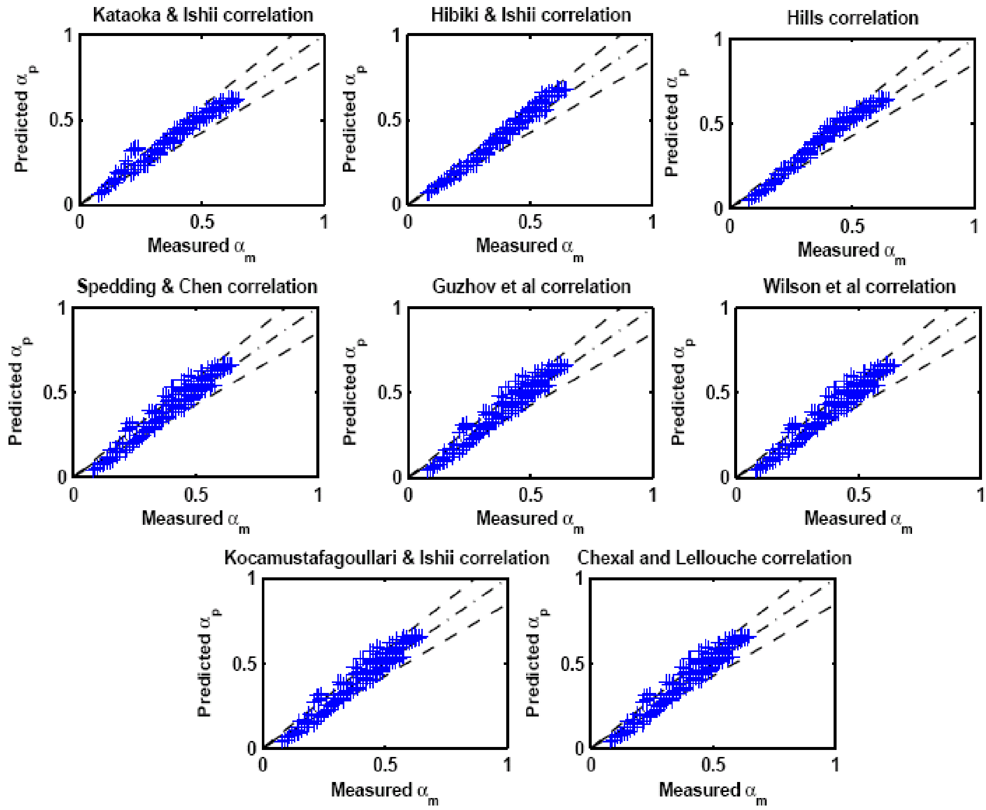


Figure 2. Comparison of the measured and predicted void fraction for selected correlations.

Table 3. Recommended correlations according to individual flow regimes.

Flow regime	Recommended correlation
Bubbly (B)	Wilson et al (1961), Duns & Ross (1963), Rouhani & Axelsson-I (1970), Rouhani-I (1969), Rouhani-II (1969), Chen-II (1986) and Hibiki & Ishii (2003).
Agitated bubbly (AB)	Wilson et al (1961), Nicklin (1962), Bonnecaze et al (1971), Chen & Spedding-II (1984), Chen & Spedding-III (1984), Chexal and Lellouche (1992), Ishii & Kocamustafagoullari (1985), Kataoka & Ishii (1987), Chen & Spedding (1998), Hibiki & Ishii (2003).
Unstable slug (US)	Bankoff (1960), Nicklin (1962), Bonnecaze et al (1971), Dix (1971), Premoli et al (1971), Hills (1976), Chen & Spedding-II (1984), Chen-III (1986), Kataoka & Ishii (1987) and Chexal and Lellouche (1992).
Churn turbulent (C)	Nicklin (1962), Bonnecaze et al (1971), Dix (1971), Chen & Spedding-I (1984), Chen & Spedding-II (1984), Chen-II (1986), Chen-III (1986), Kataoka & Ishii (1987), Hirao et al (1986), Chexal and Lellouche (1992), Maier & Coddington (1997).

6. Conclusions

An extensive assessment of some of the often cited and commonly used void fraction correlations from the different fields was performed. The results of the study indicate that many correlations have potential to perform satisfactorily within the allowable range (±30%) with the drift flux based correlations being more successful in predicting the closer results. The important implication of this assessment then is that two phase flow void fraction prediction should be based on flow regime. The correlations taking into account of this fact are closer to experimental trends with the exception of few empirical correlations.

It was also noted that most of these correlations performed well in some of the flow regimes and their performance deteriorated in the other flow regime. Thus, none of the correlation was able to predict all four flow regimes accurately. Most of the successful correlations at maximum, predicted three flow regimes from the total of four. It is seen that correlations that successfully predicted the three flow regimes (agitated bubbly, unstable slug and churn) did not predicted the bubbly flow regime accurately. While that predicting bubbly flow regime, showed acceptable trend for agitated bubbly but did not predicted unstable slug and churn flow regime satisfactorily. This trend highlights the difference in the void fraction variation behind the bubbly and rest of the flow regimes. Thus, in the conditions where the likely prevailing flow pattern is known prior to the design or simulation stage, the appropriate void fraction prediction will be closer to the true values than the values of randomly selected. It is also worth mentioning that that many correlations belonging to Nuclear Industry are closer in prediction while none of the Oil & Gas industry correlations succeeded in predicting the void fraction under ±10%. It is anticipated that this study will help in providing the guidance to specialists in the field of Nuclear, Oil & Gas and other industries where the data pertaining to large diameter pipes is scarce and direct selection of an appropriate void fraction correlation is difficult.

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