Bottom-up generative up-cycling: a part based design study with genetic algorithms

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A B S T R A C T

While describing up-cycling as a problem of fitting a set of existing/used materials into a new design, this paper utilises genetic algorithm (GA) and tree forks to exercise design in limited material inventories. It presents a bottom-up generative approach aiming to increase the applicability of up-cycling by reducing the material selectivity. The paper presents two scenarios: the first based on the tree forks being sourced from a single tree and the second utilising waste material, namely tree forks collected from a forest floor. It studies GAs incorporating material dimensions and fabrication constraints from an earlier stage of design to amplify the morphological involvement of these elements and to create a bottom-up generative system. The paper utilises waste material without a prior selection and without changing or deforming their unique geometries to minimise fabrication energy consumption. It presents a fabricated table leg structure made of ten forks.

1. Introduction

Almost 10% of annual global carbon emission is linked to the construction and demolition of buildings [1]. Finding innovative ways to use reclaimed and waste material, increasing the efficiency of the material by focusing on its interior composition, and using the minimum amount required via high customisation are some of the ways to be more sustainable in design and architecture. Up-cycling can be defined as a problem of fitting a set of existing/used materials into a new design, creating a material-availability-informed design paradigm [2]. Reducing material selectivity in up-cycling can expand its applicability by making use of more waste material. To accomplish this, this paper presents an example design system to utilise waste material, namely tree forks collected from a forest floor without a prior selection and without changing or deforming their unique geometries to minimise fabrication energy consumption. The approach involves incorporating the morphology of the found object from an earlier stage of design to use their unique geometries in a generative design environment.

While exercising design in limited material inventories, this paper identifies two possible ways to proceed: a top-down or a bottom-up approach. A top-down approach consists of preparing a target design and searching for its parts or the overall shape in an inventory. A bottom-up approach consists of identifying the possible parts through a survey of the inventory and generating design options based on the data. Both options can make use of reclaimed materials in various degrees.

However, a bottom-up approach can be more efficient, as acquiring a specific geometry in an existing inventory can be exhausting and in most cases not possible.

The paper proposes to use GAs as generative tools. GAs have been exploited in many studies [3]. Most designs and artworks have been made with GAs as a generative tool requires the algorithms to be introduced earlier in the creation process [4,5]. The presented design method of this paper introduces GAs, unique material morphologies, and fabrication constraints from an earlier stage of design to amplify their morphological involvement based on a bottom-up strategy enabling design options. The paper presents a fabricated table made of ten tree forks. The shape of the table emerges as a result of a designed process. Overall, it presents an example of a bottom-up approach which exercises design in limited material inventories and proposes up-cycling tree forks with minimised selectivity, accommodating their unique geometries.

The paper focuses on tables with relatively simple structural and functional requirements, to keep the complexity to a minimum level for testing purposes. In comparison to architectural spaces it is easier to create the fitness function of a table, based on parameters such as structural stability, leg space and useable surface area. Albeit in different scales, the objectives studied using table typology can be adapted to architecture. As such, this paper studies an upcycling process at a smaller scale for proof of concept purposes.
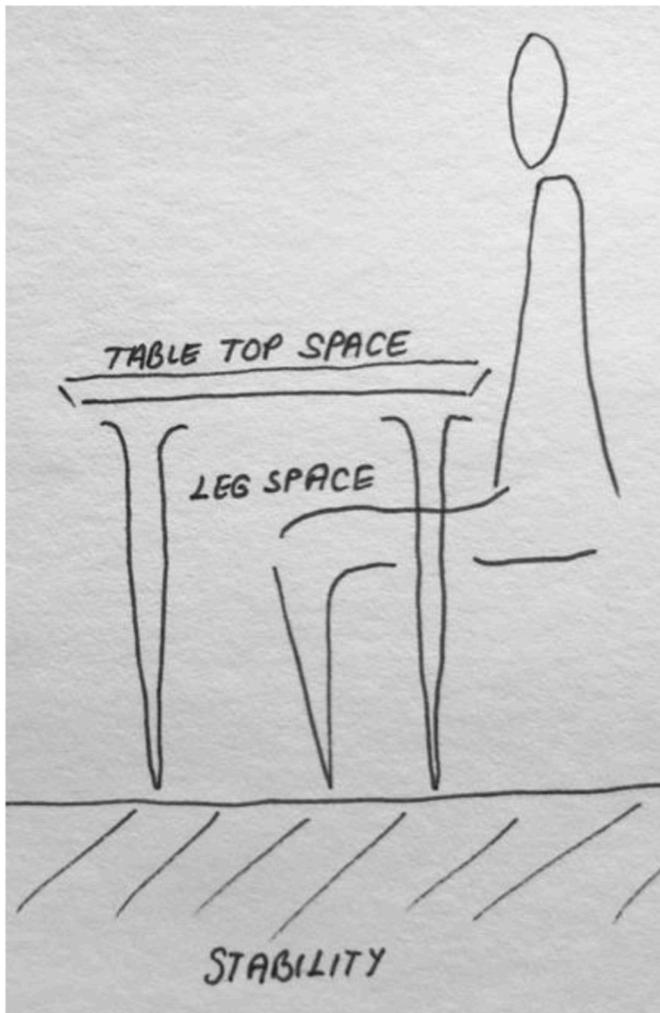
Fig. 1. Parameters of a table.

2. Related work

A comprehensive study on the structural use of tree forks by Ishani Desai quantifies the potential of this material form [6]. An example of a design exploring the morphological and structural potential of tree forks as a natural heterogeneous material is *Tree Fork Truss* by Zachary Mollica and Martin Self [7]. While using entire forks as construction material, in Mollica and Self's project, the forks are supported with additional straight timber elements to construct a predefined form of a truss. Zachary Mollica and Martin Self identify their ultimate challenge as to achieve construction precision with irregular and complex material geometries while following a strategy of finding the right material for the right place. While designing with existing materials, the system should be flexible enough to incorporate many shapes and sizes. In the *Tree Fork Truss* project, they had to scan 204 trees in order to harvest 25. Thus, this system is selective in matching the inventory into the desired shape.

An example of a project up-cycling tree forks as 3D spatial joints to replace steel joints in a predefined target design is by Caitlin Mueller and her Digital Structures research group [8]. Their proposal uses a combination of iterative closest point and Hungarian assignment algorithms. Similar to the Digital Structures research project, a study by Peter Von Buelow et al. focuses on the tree forks as 3D joints and utilises a parametric environment to create architectural scale proposals [9]. Both studies include additional straight pieces in combination with the fork-based joints and define a selective design environment in terms of
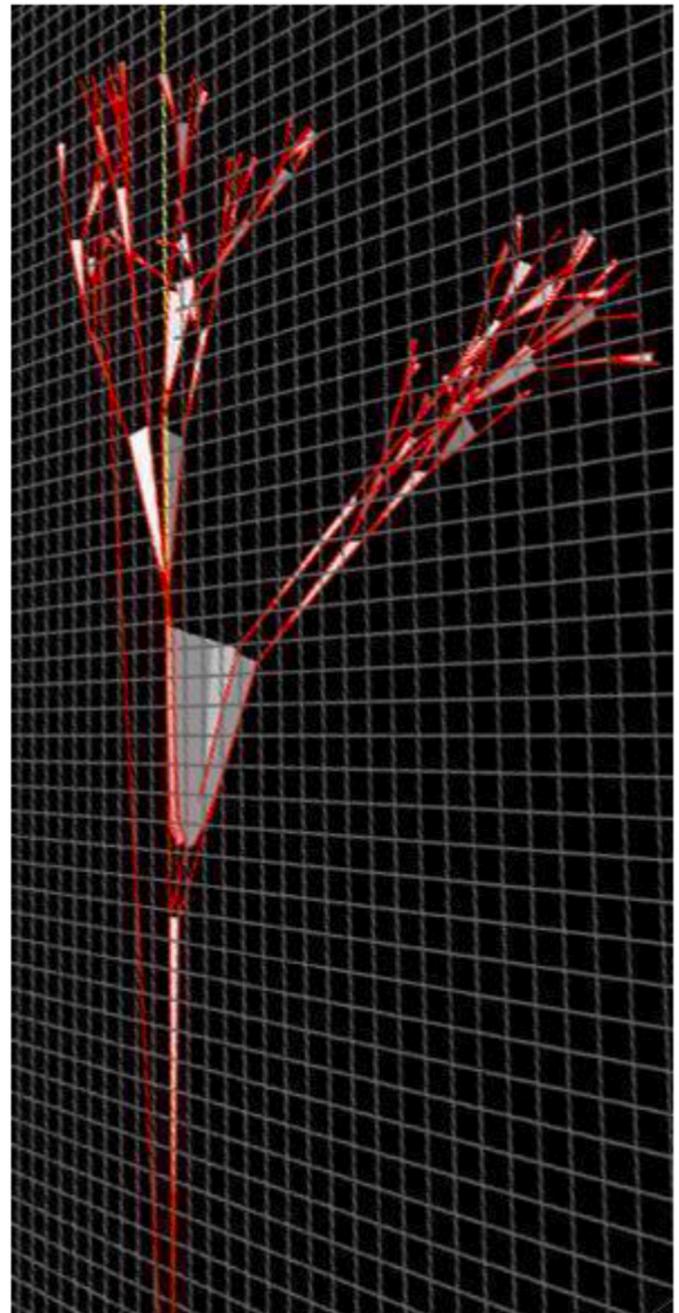
Fig. 2. L-systems tree model with tree forks identified.

matching the inventory to the design. A similar approach is studied by Lukas Allner and Daniela Kroehnert on tree forks, using their unique shapes in geometries based on mathematical models [10]. A parametric approach focusing on the use of irregular geometries of boughs to build a new structure presents results based on predefined surfaces used as a target to build complex structures [11]. In their work, Aurimas Bukauskas et al. propose using a process called form-fitting, based on the bin-packing problem, to create a desired shape from reclaimed material [12]. The above examples define the problem of up-cycling as matching an inventory to a desired shape. They use different algorithmic methods to create irregular structures from irregular material forms, while generally being selective in relation to the inventory and imposing predefined targets on their systems. Hence, the proposals require very specific shapes of reclaimed materials to create the predefined desired shapes. In contrast to these examples, this paper aims at incorporating the unique geometries of tree forks from an earlier stage of design to

Fig. 3. Tree forks collected to implement in the GA.
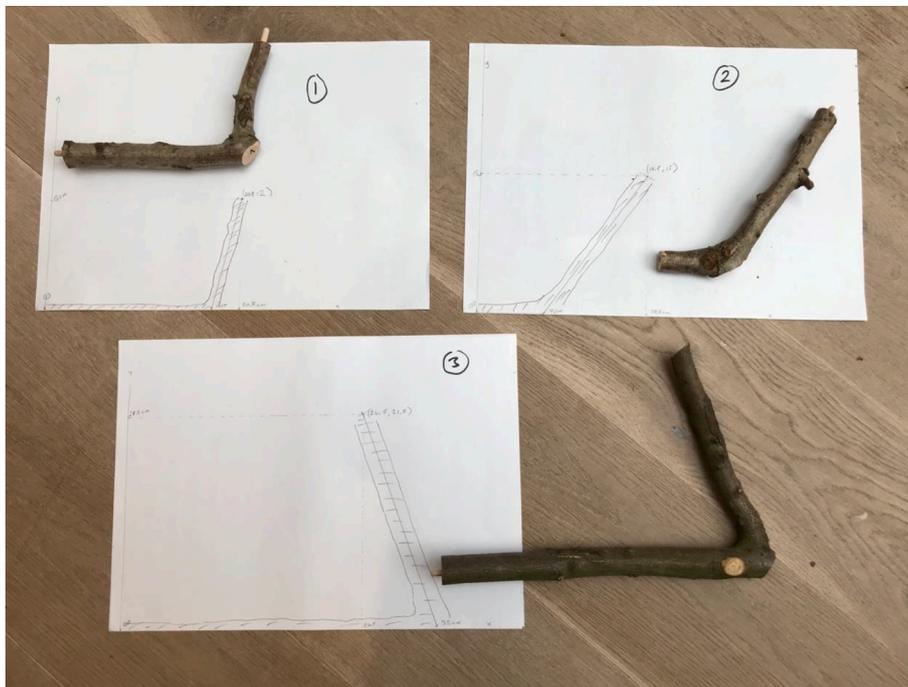
Fig. 4. The first three tree forks measured.

Table 1
The list of the tree forks collected described as three points on a plane.

	X1	Y1	Z1	X2	Y2	Z2	X3	Y3	Z3
Fork1	0	0	0	18	0	0	20.8	12	0
Fork2	0	0	0	7	0	0	17.8	15	0
Fork3	0	0	0	32	0	0	24.5	21.5	0
Fork4	0	0	0	20	0	0	29	23.5	0
Fork5	0	0	0	15	0	0	23	10	0
Fork6	0	0	0	22.3	0	0	41.5	17.3	0
Fork7	0	0	0	21	0	0	38	14.5	0
Fork8	0	0	0	18.8	0	0	24	11	0
Fork9	0	0	0	24.3	0	0	23.8	16	0
Fork10	0	0	0	24.6	0	0	44.4	20.6	0
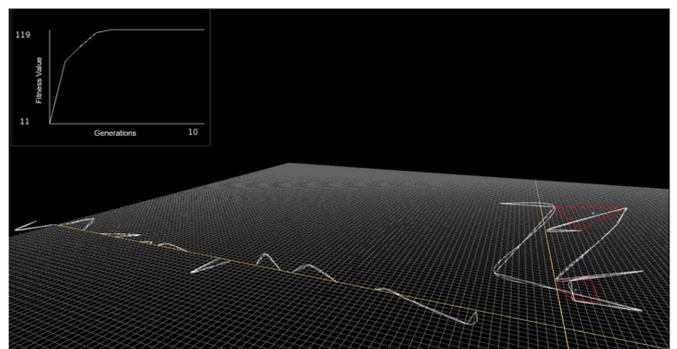
Fig. 5. Table option 1.
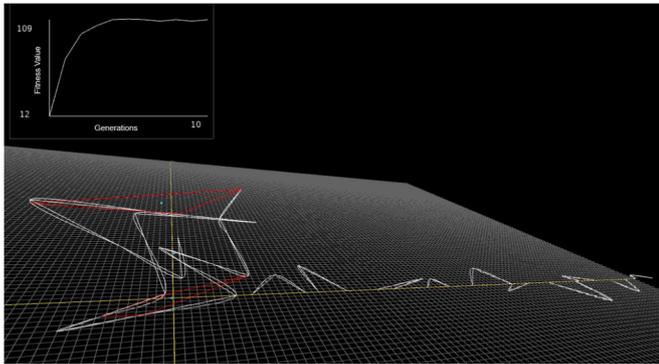
Fig. 6. Table option 2.
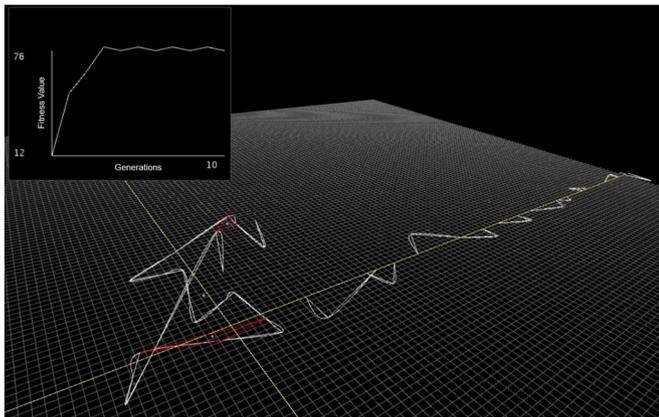
Fig. 7. Table option 3.

increase their morphological involvement while minimising selectivity.

Overall, previous studies define the problem of up-cycling as matching an inventory into a desired shape. They use different algorithmic methods, including growth, attraction and fitting, to create irregular structures from irregular material forms while generally being selective in relation to the inventory and imposing predefined targets on their systems. Hence, the proposals require very specific shapes of reclaimed materials to create the predefined desired shapes and additional pieces such as straight non-fork timber are used to compensate. In contrast to these examples, this paper aims to create a bottom-up design environment using a GA and the tree forks, which then generates design proposals with high fitness scores. Avoiding the selection process requires the material inventory to be generalised and to be used in a system that is flexible enough to accommodate different material forms. Thus, the algorithm and the shapes of the tree forks define the final form and create a design from the inventory without requiring a selection. The proposals solely consist of tree forks without using additional pieces such as straight timber.

3. Approach

The design intuition of the paper is based on utilising tree forks as repeating parts. A part is a distinguishable unit, within a fabricable size limit, and works as a whole. Parts should perform based on the interior structures of the materials, restricted by the size limits, including the minimum size to fabricate and the maximum size of material blocks in the format of logs and planks, etc. And working as a whole without breaking. Using the tree forks as parts of a design, the paper utilises GAs as generative algorithms to design a table.

While minimising selectivity, the strategy of the paper acquires a type of global description applicable to each piece in the inventory for them to be used as parts of a design in a bottom up generative process. As this description becomes more general, it enables to include more parts in the inventory. As such, the paper proposes a generic geometric description based on three 3D points that can be applied to most of the tree forks. The tree forks are the diversion points in trees where one segment of a tree meets with another. Geometrically, this information can be simplified as two vectors creating a triangular plane. The paper uses the corners of these planes as the descriptive parameters of each fork. This generalised description method applicable to most tree forks may not work in odd conditions where more than two branches emerge from a single point. The branches with these rare conditions are excluded from the scope of the study. Overall, the paper exercises design in a unitised inventory where each material form can be defined with three 3D points.

A basic definition of a table is a structure that stands at a certain height, its shape including enough space for legs, and with a load-bearing surface on top to be used for a purpose such as eating, working, etc. Based on this definition, Fig. 1 demonstrates the parameters of a table, which can be listed as structural stability, leg space and useable surface area.

This section first discusses the details of the generative GA application for a table design made of tree forks. It then presents two scenarios in terms of the source of the forks. In the first scenario, the paper aims to design a table from the tree forks of a single tree. To accomplish this, an L-system tree model is used to acquire its forks as a placeholder. In the second scenario, this dependence on a single tree is lifted and actual tree forks collected from a forest floor are measured and used in the generative design environment.

3.1. A generative GA application

Evolutionary algorithms mimic natural evolution. The GA is a type of evolutionary algorithm and is a stochastic search mechanism [13]. It was popularised by John Holland in the 1970s [14]. GAs have been widely used in architecture and engineering as an optimisation and design tool. They are efficient algorithms to explore search space and to orient design towards a direction required. An antenna that NASA developed is an example of evolved design in terms of its simple form and function. It is based on a wire bent at different angles in different locations [15]. Erwin Driessens and Maria Verstappen's work *Breed* is also an example of artwork made with GAs as a generative tool, using a minimum predefined phenotypical framework. Starting from scratch,

Fig. 8. Table options as 3D models.
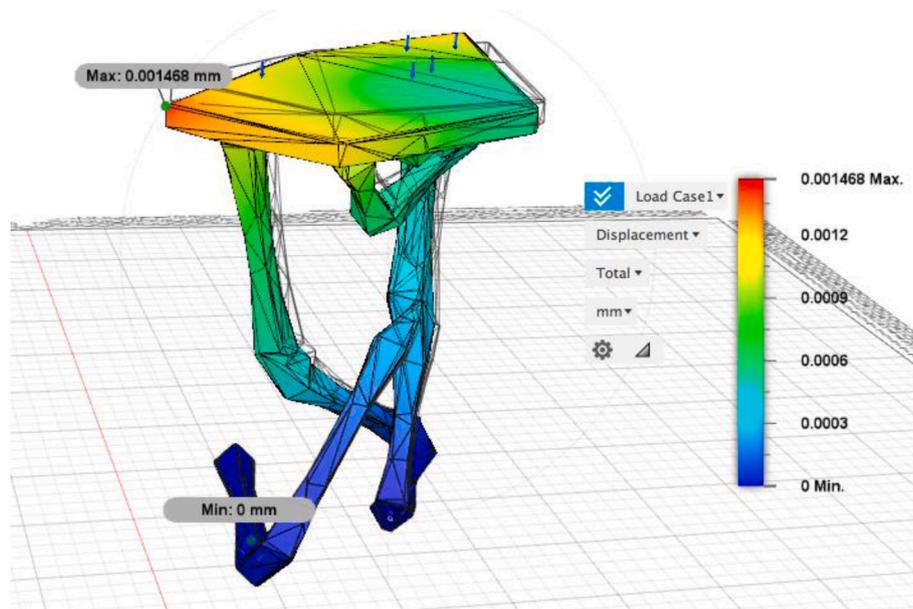
Fig. 9. Displacement diagram, 20 N force applied.
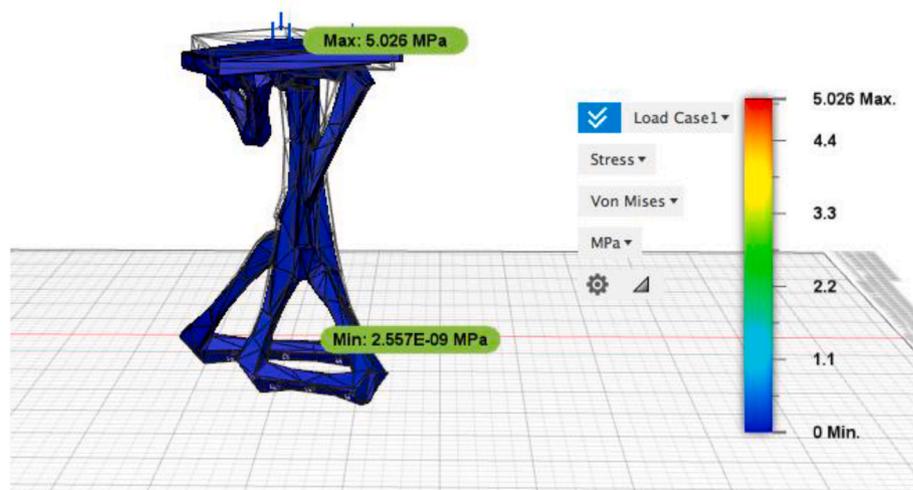
Fig. 10. Von Mises stress diagram, 20 N force applied.

they use cell division, growth and mutation embedded in a GA to accumulate and generate the shape of their sculptures through generations [16]. A series of table designs by P. J. Bentley is an example of a generative GA system demonstrating 20 different table designs constructed with similar types of pieces [17]. This paper uses a custom-made GA written in Python, which enables design exploration in a similar way to that explored in Caitlin T. Mueller and John A. Ochsendorf's paper on interactive, search space exploration-based GAs for designers [18].

The algorithm starts with 100 randomly created individuals with 10 tree forks (a pre-given number which can be changed for different configurations). In scenario 1, the sequence of the forks is randomly generated. In scenario 2, the sequence is pre-given based on the thicknesses of the forks to match the end faces. The angles between the forks are randomly generated. Once their fitness functions have been calculated, the 20 best individuals are selected from the initial population. Later, randomly paired parents from the best performing 20 produce their offspring to create the next generation of 100 individuals. The GA of the paper identifies each tree fork as a gene and each table consists of

ten tree forks/genes. The genotype to phenotype mapping is omitted for simplification purposes. Following is the description of the genes directly used in the cross-over operation.

$$p1 = (x1, y1, z1)$$

$$p2 = (x2, y2, z2)$$

$$p3 = (x3, y3, z3)$$

$$gene = (p1, p2, p3)$$

$$ind = (gene1, \ldots, gene10)$$

The fitness function consists of the sum of three values; the number of points supporting the table top, the number of points touching the ground and the inverted distance between these points to the centre of gravity. Hence, the individuals with higher fitness scores have more points supporting the table top, touching the ground and are structurally more stable due to their increased balance.

The cross-over operation divides the genes of the parents from a
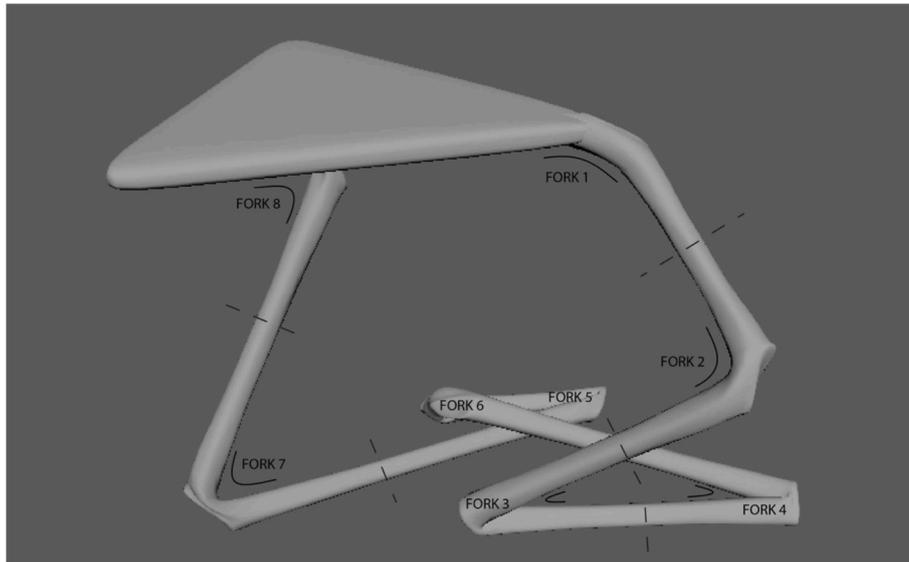
Fig. 11. A table with its forks marked.
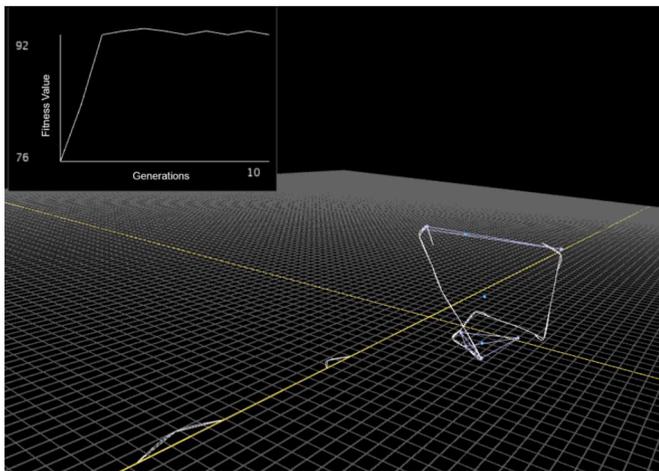
Fig. 12. Converged table option 1 with 92 fitness value.
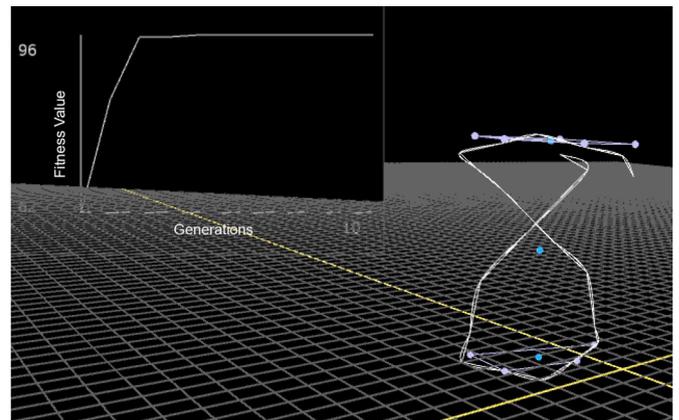
Fig. 14. Converged table option 3 with 96 fitness value.
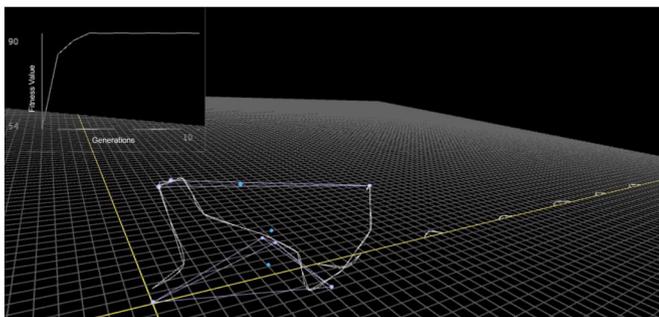
Fig. 13. Converged table option 2 with 90 fitness value.
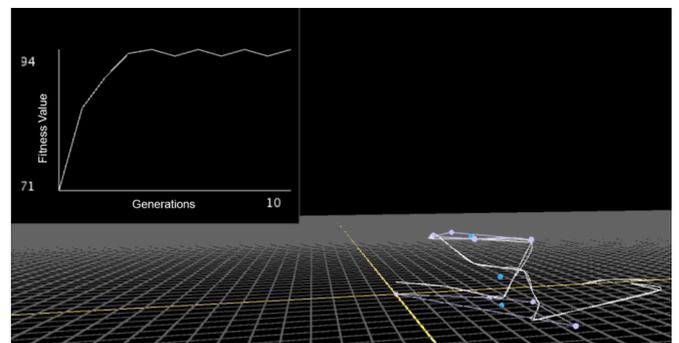
Fig. 15. Selected option for fabrication.

randomly selected point into two parts. It creates the offspring by merging the first part of the first parent (s1) with the second part of the second parent (s2). In detail, s2 is rotated and moved to be attached to the end of s1. The direction of vector (p2,p3) of the last gene of s1 matches to the direction of vector (p1,p2) of the first gene of s2. This process guarantees to link s1 and s2 together.

Once, the vector directions of s1 and s2 are matched, the new

configuration is likely to have a suboptimal structure in terms of objective function due to several factors including the random dis-positioning of the centre of gravity. In order to have a faster conver-gence, we use a heuristic that finds the optimal angle between s1 and s2 to bring the configuration to a better starting point in the objective space. This heuristic search runs at each cross-over operation and rotates s2 18 times (20° at each rotation) around its matched vector axis to

Fig. 16. Wooden dowel joint experiment on two thin tree forks.

calculate the fitness value at each step. This is the same fitness function as the main GA. The rotation angle with the highest fitness score is selected as the final configuration. Due to this heuristic search proceeding in each cross-over operation, the resulting GAs converge into an option in under five generations.

3.2. Scenario 1: A table from tree forks of a single tree

Recognising up-cycling as a problem of fitting a set of existing/used materials into a new design, the first scenario is based on tree forks sourced from a single tree, which brings size, number and shape constraints to the inventory of forks. To explore the potential of a customised GA in such a scenario, an L-systems algorithm, a well-known method to create mathematical models of trees, is written to generate a medial axis model of a tree as a place holder. Once a tree has been created and its forks identified, the information is converted into a list containing all the tree forks with their original positions in relation to the tree. Fig. 2 shows a medial axis tree model built with L-systems. Its forks are marked in white. The lengths of the branches per fork are initially randomised and later defined by the GA in the process of finding fitter configurations.

The table proposal consists of ten tree forks attached to each other using perpendicular cuts to the longitudinal direction of the branches. The initial assumption is that all tree forks at the same level of branching will have similar thicknesses. Therefore, these can be put together without too much variation in thicknesses around the joints – the end faces where one tree fork touches another to attach – to create a table. Thus, the GA of this scenario favours fork sequences with similar branching levels connecting to each other. Once a list of tree forks has been created, the GA can generate table options using this list as an input.

The proposed GA includes idiosyncratic material information as a list of points. The fitness function includes a fabrication constraint by minimising thickness variations of the joints by identifying and selecting forks at the same level of branching and seeking the best possible configuration to function as a table. Other objectives of stability and

Fig. 17. Fabricated table with ten tree forks collected from a forest, put together based on the instructions generated by the GA.

table top area – are integrated into the same fitness function.

3.3. Scenario 2: A table from tree forks collected from A forest floor

In the second scenario, the material inventory consists of actual tree forks collected from a forest. In contrast with the first scenario, these forks do not necessarily need to be from the same tree. The advantages of this approach include minimal waste through using wood already discarded on a forest floor without needing to harvest from a living tree, and the material potentially being dryer than freshly cut timber. Fig. 3 shows ten pieces of tree forks collected and ready to implement in the GA. The dimensions of each fork are recorded based on their inferred medial axis models, as three points on a plane: start, middle and finish. Fig. 4 shows the first three forks and their marked dimensions.

Once all ten forks have been measured and recorded as a list as shown in Table 1, the information is implemented in the GA. The planar shapes of the tree forks are recorded as three 3D points to incorporate the rotational operations of later stages in the GA. Due to the varying thicknesses and the diameters of their end faces where a tree fork supposedly touches another, the GA of this second scenario is modified to include a sequence order for the forks as a user input, aiming to put those forks with similar diameters one after the other.

4. Investigating and visualising

Computational power can limit the ability of GAs to calculate and
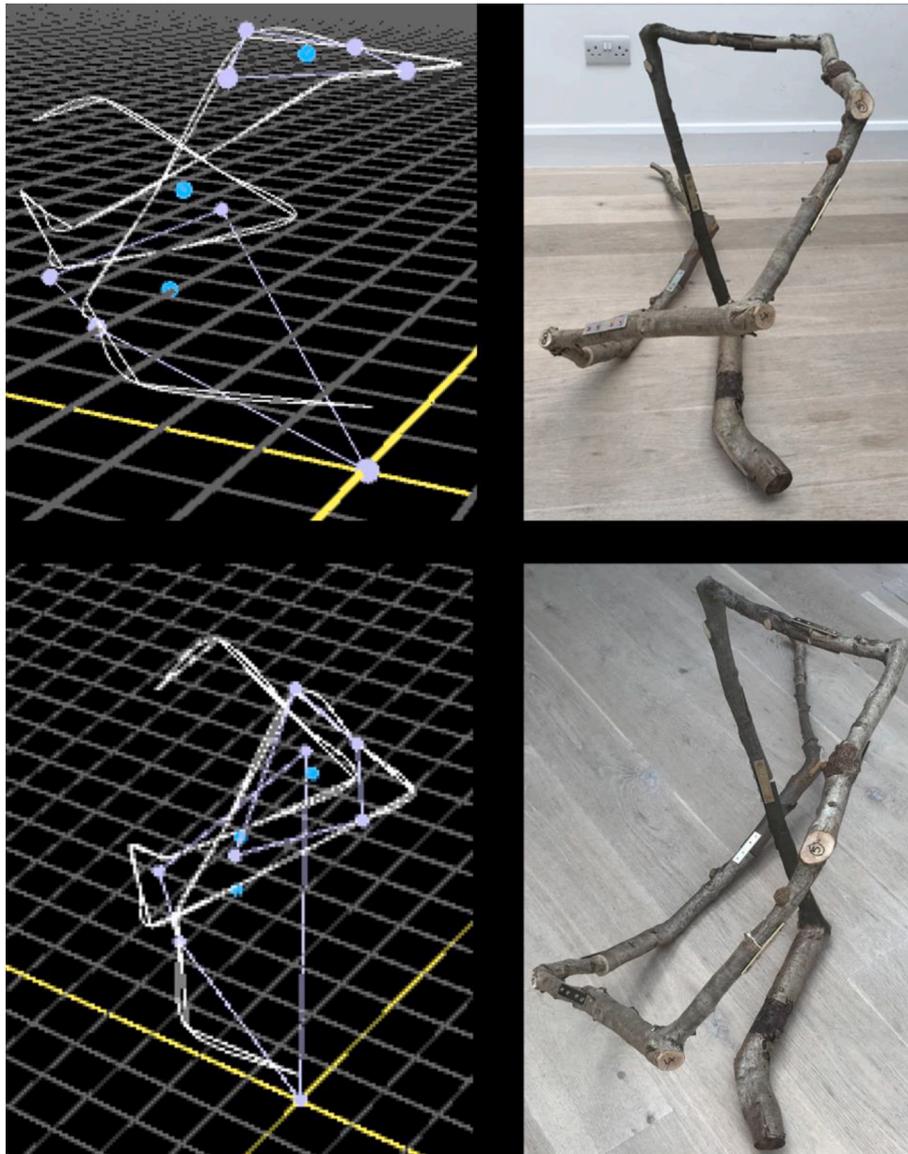
Fig. 18. Fabricated table with ten tree forks collected from a forest, put together based on the instructions generated by the GA.

process complex phenotypes and genotypes over large numbers of generations. This paper uses medial axis models, which are based on lines and points, as simple representations of phenotypes, describing the skeleton of the geometries. In the following figures, the centre of gravity, the centre of all the points touching the floor and the centre of the table top are marked as blue dots. The bottom red frame identifies the anchor points to the floor and the top red frame identifies the anchor points to the table top surface.

4.1. Scenario 1

Based on scenario 1, Figs. 5–7 demonstrate the three different table options, including their fitness value/generation diagrams and the tree forks used to create these options. The forks identified are from the same tree, generated using an L-system. The diagrams of the options generally show a learning curve, converging smoothly.

In this first scenario, the table options are designed to be fabricated from the forks of an L-system-based mathematical model of a tree. Each option uses approximately 30% of the tree forks available and only one table can be produced per tree. The forks to be discarded are generally unsuitable on account of their thickness – being either too thick or too

thin in comparison with the rest of the forks selected. Fig. 8 demonstrates three converged table options converted from medial axis models to 3D geometries. Structural analysis of an option is prepared using Autodesk Fusion 360 software. A 20 N equally distributed force, which corresponds to 2.0394 kg of weight, is applied on the table along its top surface. The stress is calculated based on the von Mises yield criterion [19]. The maximum MPa is 5.026 and maximum displacement is 0.001 mm. The stress is distributed evenly throughout the model. The results of the structural analysis demonstrate that the fitness function is working efficiently. Consequently, there are no areas under intense stress which could result in a fraction or break, the weights generally being distributed equally (see Fig. 10) (see Fig. 9).

Fig. 11 shows a close-up render of a table, explaining how the forks are aligned to function as a table. In summary, this study explores the possibilities of generating table designs using tree forks obtained from a single tree model.

4.2. Scenario 2

Scenario 2 utilises the tree forks collected from a forest floor. It measures the forks to create a list of points. The sequence of the forks is

Fig. 19. Fabricated table with ten tree forks collected from a forest, put together based on the instructions generated by the GA.

defined by the user, based on their thicknesses. Once the GA has converged into an option, it produces information to aid the fabrication process, such as the angles between the forks to attach to each other. Figs. 12–14 demonstrate three table options with their fitness value/ generation diagrams generated by the GA.

Running the same GA several times reveals different table options converging on different areas in the search space. This approach is common in the generative use of GAs as an option making tool to explore the search space [20–22]. Some proposals include similar features, such as a cross-like move towards the middle of the table (Fig. 14) or a tail-like feature to provide additional stability (Fig. 13). Fig. 15 demonstrates the selected table proposal to fabricate, which includes a cross and a tail feature.

4.3. Fabrication

To attach the tree forks to each other, the paper initially experiments with wooden dowel joints. To test this joint type and to see the dimension limits, two thin tree forks are used. First, with the help of a drill, two holes are prepared and a 6 mm wooden dowel is used to put the forks together. Based on the mock-up shown in Fig. 16, this joint type proved to be not very strong and prone to twisting. To increase stability, two steel mending brackets are proposed per connection. Figs. 17 and 18 demonstrate the fabricated table-leg structure without the top surface. As shown in Fig. 19, the final design is stable and can carry a table-top.

5. Conclusion and future work

Using the ten tree forks put together to create a table design based on the instructions generated by the GA, this study demonstrates two examples of approaches of a generative design system. The first scenario aims to solve the problem of using tree forks of a single tree, bringing size, number and shape constraints to the inventory of forks and exercising design in limited material inventories. The second scenario focuses on up-cycling forks collected from a forest floor, which would otherwise be wasted. In future studies, the difference between inventories can be quantified and their effects on convergence rates can be measured. Both approaches utilise the unique geometry of the forks based on a series of instructions enabling the final shape. Furthermore, the GAs of both approaches converges into similar results with similar time frames. Overall, the paper presents examples of systems that minimise waste by using discarded wood pieces. The proposed system utilises the shapes of the forks as they are. Hence, the problem of designing a functioning piece of furniture from 10 different found tree forks is solved with a GA.

The tree fork measurement method of the paper utilises the unique fork geometries without the need for them to be scanned, despite this being less precise. In future, various description methods to generalise tree forks can be studied to fully explore their relationship with the inventory selectivity. The morphology of the proposals emerges from the evolutionary process of seeking to find the best configuration for the tree forks to perform as a table. In the fabricated table design, the idiosyncratic qualities of timber are integrated into the design process from the early stages, while enhancing their morphological involvement. For

future work, different joint options should be tried to create stronger tables. The design could be further improved by including the specific structural performance of different types of tree forks, including tension and compression, in the fitness value calculations. Further benefits of the proposed generative bottom-up approach can include to accurately calculate the minimum amount of material required.

The proposed up-cycling strategy focuses on a unitised form of waste which can be defined and measured with three 3D points. While increasing the applicability of up-cycling, the paper constraints the scope of the inventory into units. As a future work, increasing the amount of information per unit and experimenting with variable unit definitions should be discussed. Furthermore, the study could be developed further to create architectural scale solutions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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