A Generalized Error Distribution-based method for Conditional Value-at-Risk evaluation

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Abstract One of the most important issues in finance is to correctly measure the risk profile of a portfolio, which is fundamental to take optimal decisions on the capital allocation. In this paper, we deal with the evaluation of portfolio's Conditional Value-at-Risk (CVaR) using a modified Gaussian Copula, where the correlation coefficient is replaced by a generalization of it, obtained as the correlation parameter of a bivariate Generalized Error Distribution (G.E.D.). We present an algorithm with the aim of verifying the performance of the G.E.D. method over the classical Risk-Metrics one, resulting in higher performance of the G.E.D. method.

1 Introduction

Value-at-Risk (VaR) has become a standard measurement tool in financial risk management due to its simplicity. However, it is an unstable and numerically difficult to use method when the losses do not follow a Gaussian distribution (Ferraty & Quintela-Del-Río, 2016), which is usually the case in the analysis of financial data. Conditional VaR (CVaR, see Rockafellar & Uryasev, 2001; Acerbi & Tasche, 2002; Huang et al., 2010) has been proposed by literature as an alternative to VaR (Artzner et al., 1999). For a better calculation of the risk, one of the proposals (e.g. Malevergne & Sornette, 2003) is to model the interdependence of the returns by means of Copula functions (see McNeil et al., 2015). In this context, the problem can be split

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into two separate parts: first, to identify the marginal distributions of the returns of the single assets; second, to identify the specific copula which is more appropriate for representing the dependence structure of the returns (see Sklar, 1959).

In Section 2, we introduce the quantitative ingredients of the study, with the main definitions. The proposed methodological setting for calculating the Conditional Value-at-Risk of a generic portfolio is presented in Section 3. Finally, in Section 4 some conclusive remarks are given.

2 The Generalized Error Distribution and the G.E.D. Copula

The Generalized Error Distribution (G.E.D.) family was introduced by Subbotin (1923) and has been employed by various authors with different names and parameterizations (see e.g. Box & Tiao, 1973; Chiodi, 1986; Agrò, 1999; Mineo, 2007; Bottazzi & Secchi, 2011). A parameterization of the G.E.D. density function for a random variable X is:

$$f(x; \mu, \sigma_p, p) = \frac{1}{2\sigma_p p^{1/(p)} \Gamma(1 + 1/p)} exp(-\frac{1}{p} |\frac{x - \mu}{\sigma_p}|^p) \text{ for } -\infty < x < \infty \quad (1)$$

where $\mu = E(X)$ is the location parameter, $\sigma_p = [E|X - \mu|^p]^{1/p} > 0$ is the scale parameter, $p > 0$ is the shape parameter and Γ is the Euler Gamma function.

The density of a generic G.E.D. distribution is unimodal, symmetric and, for $p > 1$, bell-shaped. As particular cases we obtain the Laplace distribution ($p = 1$), the Normal ($p = 2$) and the Uniform ($p \to \infty$); for values of $1 < p < 2$ we obtain leptokurtic densities and for values of $p > 2$ we obtain platikurtic densities. Thus, the G.E.D. represents a generalization of a large set of distributions, allowing for a better description of financial data.

A bivariate copula is a function $C : [0,1]^2 \to [0,1]$ whose main interest in the field of probability is that it associates univariate marginal distributions to their joint ones (Sklar, 1959). We are here interested in the bivariate Gaussian Copula, which is defined as:

$$C(u, v|\rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\rho^2)}} exp\{\frac{-(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)}\} drds, \quad (2)$$

where Φ^{-1} is the inverse of Gaussian distribution function, (u, v) uniform independent random variables generated from (X,Y) random variables, and $\rho \in [-1, 1]$ is a parameter representing the Pearson's correlation coefficient associated to the bivariate normal. The G.E.D. Copula for a generic random vector (X, Y) is obtained by replacing the parameter ρ by the Generalized Correlation Coefficient $\rho_p \in [-1, 1]$, introduced by Taguchi (1974) and estimated as follows:

$$\rho_p = \frac{codisp^{(p)}(X, Y)}{\sigma_p(X)\sigma_p(Y)}, \text{ with } -1 \le \rho_p \le 1, \quad (3)$$

where

$$|codisp^{(p)}(X,Y)|^p = |E[(Y-\mu_Y)|X-\mu_X|^{p-1}sign(X-\mu_X)]| \cdot |E[(X-\mu_X)|Y-\mu_Y|^{p-1}sign(Y-\mu_Y)]|,$$

$$\sigma_p(X) = [E|X-\mu_X|^p]^{1/p}, \qquad \sigma_p(Y) = [E|Y-\mu_Y|^p]^{1/p}$$

and μ_X and μ_Y are the expected values of X and Y, respectively. The parameters μ, p and σ_p could be estimated e.g. by using the Lp_{min} method (Giacalone, 1996; Giacalone & Richiusa, 2006).

3 The methodology

We here discuss theoretically how to compute the CVaR in a G.E.D. framework. Real world applications on financial data are available upon request.
We start from two sets of consecutive observations of the returns of two assets, assumed to be empirically distributed according to a G.E.D.:

$$X = \{x_1, \ldots, x_n\}, \qquad Y = \{y_1, \ldots, y_n\}.$$

For what concerns the stochastic dependence between X and Y, we propose a G.E.D. copula model.
The fundamental steps in the algorithm for the computation of the CVaR of a portfolio consisting of (α, β) are as follows:

0. we fix a portfolio (α, β), with $\alpha + \beta = 1$;
1. estimation of the parameters μ, p, σ_p in (1) for the two series of returns X and Y. In accord to the notation used above, we will denote the parameters as $\mu_X, p_X, \sigma_{p,X}$ and $\mu_Y, p_Y, \sigma_{p,Y}$;
2. estimation of the ρ_p parameter of the G.E.D. copula by using formula (3), with $p = \alpha p_X + \beta p_Y$;
3. generation of couples (x,y), which are the realization of the double stochastic variable (X,Y) having G.E.D. marginals of item 1. and stochastic dependence described by the G.E.D. copula with ρ_p of item 2.;
4. construction of the realizations of the returns of portfolio $P = \alpha X + \beta Y$ and of its empirical distribution;
5. computation of the Value-at-Risk of P at a confidence level $(1-c)\%$;
6. computation of the Conditional Value-at-Risk of P at a confidence level $(1-c)\%$.

The algorithm above is repeated for all the considered portfolios, given by $\alpha = 0.01 : 0.01 : 0.99$ and $\beta = 1 - \alpha$.

4 Conclusions

The present paper is part of market risk calculation methods, whose purpose is to support risk managers' decision making processes. Among the different methods proposed in the literature for calculating Value-at-Risk, we took the well-known RiskMetrics into account. We introduced the new G.E.D. method and proposed the G.E.D. Copula as a generalization of the Gaussian Copula. Moreover, we introduced the Generalized Correlation Coefficient of norm p that, for the $p = 2$ case, equals the classic Bravais-Pearson correlation coefficient. We then presented an algorithm with the aim of verifying the performance of the new method over the classical RiskMetrics one. The problem of whether CVaR-G.E.D. can constitute a valid generalization of CVaR-R.M. is still debatable, and an empirical analysis would provide more insights on this relevant topic.

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