

# A new measure of the resilience for networks of funds with applications to socially responsible investments\*

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## Abstract

This paper provides a novel resilience measure of a family of networks in terms of stability of its community structure. To this aim, we assign to each node a probability distribution and introduce an exogenous shock as a lump sum perturbing its left tail. Then, we measure the reactions of the considered networks to the occurrence of such exogenous shocks. Starting from the intuitive interpretation of the methodological proposal in the financial context, we employ it to compare portfolios of funds with different ranks in terms of the Environmental, Social and Governance score. In particular, we consider financial networks whose nodes represent funds, and edges are weighted on the basis of the capitalization due to the common components of the connected nodes. Interestingly, we find that the considered network of High ranked funds is more resilient than the corresponding network of Low ranked funds when funds are small-sized. The opposite behaviour is observed for the big-sized funds.

**Keywords:** Complex networks; Resilience; Community structure; Socially Responsible Investments; ESG criteria; Investment Funds.

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# 1 Introduction

Complex networks are mathematical frameworks that can model efficiently the presence of interactions among different units (see Albert and Barabási, 2002; Newman, 2003). The generality of the complex networks explains why they are so popular in modeling several expert systems in the context of applied sciences, ranging from text analysis and linguistics (see, e.g., Cinelli et al., 2019; Duari and Bhatnagar, 2020; Jung and Lee, 2020), geophysics (see, e.g., Boers et al., 2013; Santiago et al., 2014) and, of course, social science (see e.g., Li et al., 2021; Zareie and Sheikahmadi, 2018 and the monograph Vega-Redondo, 2007).

Among the various application areas, finance plays a prominent role. Indeed, the employment of complex networks to the business context – with a specific focus on funds – is not unusual in the scientific literature. A relevant contribution is Kuhnen (2009) which, to the best of our knowledge, is the first paper dealing with the corporate governance in mutual funds by including a network-based analysis of the interactions between funds’ advisors and directors. In this context, Butler and Gurun (2012) analyze the impact of voting behaviours of funds’ portfolio managers on executive compensation proposals in a network setting. Indeed, the quoted paper conceptualizes and compares two funds’ networks on the basis of the educational background of the related portfolio managers. In the same line of research, Cohen et al. (2008) discuss funds performance and educational background under a network perspective. Other papers adopt a rather quantitative approach. Bech and Atalay (2010) introduce the network of the federal funds market and explore its topology by using well-established complex networks measures. We also mention the interesting network-based exploration of the pension funds presented in D’Arcangelis et al. (2019) – where authors provide the main properties of such financial instruments – and D’Arcangelis and Rotundo (2019) – where a complex network modeling is carried out for emphasizing the presence of herding behavior among the agents investing in mutual funds. Nagurney and Hughes (1992) develop a model for describing the financial flow of a fund in a bipartite network setting. In so doing, the authors also provide an efficient distinction between assets and liabilities. Flori et al. (2019) analyze the relationships between mutual funds and portfolio holdings by using a bipartite network and propose an indicator that measures the degree of overlap of funds in the market. They find that funds investing in less popular assets generally outperform those investing in more popular financial instruments. Lavin et al. (2019) presents the analysis of the topology structure of the network of Chilean mutual funds; also in this case, the adopted framework is that of bipartite networks. Interestingly, the authors discuss the impact of a shock – occurring in the network structure – on funds’ overlapped portfolios.

In line with the literature, we specifically propose a view of funds’ portfolios as complex networks.

The nodes are given by the funds composing portfolios, while the arc connecting two funds is weighted on the basis of the holdings commonly shared by them. In more detail, Lavin et al. (2019) shares with us one central aspect when stating a link between exogenous shocks and overlapped portfolios. Indeed, we aim to measure the ability of the considered financial networks to absorb external events. Moreover, shocks are assumed to act on the common holdings shared by the funds by reducing the weights of the related arcs.

Complex networks represent a quite natural framework in our reference context, and the system's reaction to exogenous shocks is efficiently described through the concept of resilience measure of a complex network. In this respect, this paper introduces a novel version of the resilience measure of a financial network. To this purpose, we assume that a shock is a mass realization of the return of one of the holdings with a negative sign, so that the expected return of the shocked holding is below the one of the non-shocked holding. The weights of the arcs are given as the average percentage of the capitalization of the connected funds due to common holdings; therefore, the action of the shock is to weaken the links between those funds having the shocked holding as a common component. Notably, the definition of resilience measure is grounded on the stability of the community structure of the considered financial network. Indeed, the analysis of the communities provides a clear view of the way shocks propagate (see, e.g., Cerqueti et al., 2018, 2019; Karimi et al., 2020). In particular, we employ a particular version of the clustering coefficient of the network (see Onnela et al., 2005) to assess the presence of large (small) deviations of networks structure from absence to presence of shocks – i.e., weak (strong) resilience measure. In so doing, we are in line with a wide strand of literature employing the clustering coefficient for dealing with resilience measures (see, e.g., Cerqueti et al., 2020a,b; Cinelli et al.; León et al., 2018).

Under a methodological perspective, our paper is quite close to Gualdi et al. (2016). In the quoted paper, the authors propose a method to assess the statistical significance of the overlap between heterogeneous portfolios, which is an extremely relevant issue since portfolios' overlap is related to the strength of contagion among funds. Their approach is close to ours, in that it is based on a complex network structure. Moreover, we implicitly discuss systemic risk through a topological-based network's resilience measure. As a further remarkable similarity between the quoted paper and ours, we also use overlapping portfolios to define the adjacency matrix. However, we are not able to apply the test proposed in Gualdi et al. (2016) since we cannot evaluate the probability that the overlap between two funds is larger than the observed one – see Equation (1) in Gualdi et al. (2016). Furthermore, we point out that in our context, we do not study contagion risk. In this respect, the proposed resilience measure is grounded on the comparison between unshocked and shocked frameworks, without any

insights on the possibility of having contagion channels.

The methodological proposal is tested over different networks built using the dataset on funds provided by Morningstar Direct (MD) and Morningstar European Data Warehouse (EDW). Funds are ranked according to their Environmental Social and Governance (ESG) performances issued by Morningstar (Morningstar SUN GLOBE). One network is related to funds with high ESG scores – i.e., with investment strategies including mainly environmental, social and governance targets – while, conversely, the other is related to funds with low ESG scores. We repeat the analysis for big capitalization funds and small capitalization funds separately. In comparing the networks, we describe and discuss the relationship between the resilience of the financial networks – i.e., the stability of the considered portfolio of funds – and the entity of the pursued social impact targets.

The financial application of the proposed method is particularly relevant in the context of Socially Responsible Investment (SRI). Indeed, SRI funds incorporate firms' ESG characteristics in investment decisions. The increasing interest for such a kind of investment is testified by the value of the assets under management involved in SRI. According to the US Social Investment Foundation (2016, 2018) and the Global Sustainable Investment Alliance, 2016, 2018 reports, SRI accounts for one out of every four dollars under professional management in the United States and one out of every two dollars in Europe. SRI is also at the centre of an intense debate involving both the academy and the financial industry. What practitioners and researchers discuss is the profitability of SRI to conventional investments. Even though evidence is conflicting, the general picture emerging from the whole debate is that SRI is less remunerative than conventional investments (see e.g., Hong and Kacperczyk, 2009 and Luo and Balvers, 2017), but it is also less risky (see e.g., Bollen, 2007, Freeman, 1984, Kim et al., 2014, and Becchetti et al., 2018). Moreover, there is evidence of a relation among several funds' characteristics and their returns (see e.g., Herzel et al., 2012 and Nicolosi et al., 2014).

In general, to the best of our knowledge, literature on SRI is mainly focused on the risk/return profile of single assets or funds. This paper enters the debate on the existing relation between ESG investing and risk from a different perspective. Specifically, by using an approach based on complex networks theory, we deal with the analysis of High/Low ranked ESG funds' reaction to external solicitations generated by an exogenous shock. Our main result shows that for Small size funds, the network of High ranked funds shows a higher resilience measure than that of Low ranked funds. On the other hand, the opposite behaviour is observed for the Big size funds. Moreover, the differences in the resilience measure between the High and Low ranked networks are amplified for stronger shocks. Along the same strand of literature is Cerqueti et al. (2021) where the authors show that networks of High ESG ranked funds are also less permeable to contagion from fire-sales spillover than networks of

Low ESG ranked funds.

Our findings add new insights into the existing literature on the risk associated with SRI. Indeed, using a network approach, funds are analyzed as a whole. We do not consider the risk/return profile of an asset or fund stand-alone. We account for the strength of their interconnections as measured by the share of assets in common, and we analyze how an external shock perturbs this interdependence. Hence our measure is related to the risk associated with the whole network. Socially Responsible funds extend their portfolio composition towards non-mainstream assets. Therefore, this is the first attempt to measure whether the greater heterogeneity in portfolio composition of the segment of High ESG ranked funds may reduce the impact of an external shock.

The paper is organized as follows. Section 2 outlines the complex network model. To provide a more intuitive interpretation of the resilience measure, we present the network by adopting a financial perspective. Section 3 introduces the concept of resilience measure of the considered financial network. Section 4 contains the setting of the real data-driven experiments and presents and discusses the empirical results. The last section offers some conclusive remarks.

## 2 The financial network model

We present the complex network model used to describe the context of interconnected financial funds we deal with.

We consider  $n$  funds and collect them in a set  $V$ . We assume that funds may be interconnected so that  $V$  represents the network's set of nodes. Funds are given by combinations of holdings, and we collect all the holdings of all the funds in a unique set  $\mathcal{A}$ . With an intuitive notation, we say that  $a \in i$  when the fund  $i \in V$  also includes  $a \in \mathcal{A}$  as one of its holdings. Thus, we can interpret any fund as the collection of its holdings and, at the same time, any holding as an element of some funds. This said one can define the two sets

$$\mathcal{A}_i = \{a \in \mathcal{A} : a \in i\}, \quad \forall i \in V \quad (1)$$

and

$$V_a = \{i \in V : i \ni a\}, \quad \forall a \in \mathcal{A}. \quad (2)$$

Clearly,

$$\bigcup_{i \in V} \mathcal{A}_i = \mathcal{A}; \quad \text{and} \quad \bigcup_{a \in \mathcal{A}} V_a = V.$$

Given  $i, j \in V$ , the arc  $(i, j)$  of the network models the presence of a link between the funds  $i$  and  $j$ . The arc  $(i, j)$  does exist when the connected funds  $i$  and  $j$  share some common holdings of  $\mathcal{A}$ ,

i.e.  $\mathcal{A}_i \cap \mathcal{A}_j \neq \emptyset$ . The entity of the connections is measured through appropriately defined weights. We denote the weight of the arc  $(i, j)$  by  $w_{ij}$ . The weight  $w_{ij}$  can be constructed as follows. First, denote by  $w_{ij}^{(i)}$  and  $w_{ij}^{(j)}$  the normalized percentage of the market capitalization (*TNA*) due to the components belonging to the set  $\mathcal{A}_i \cap \mathcal{A}_j$  of fund  $i$  and of fund  $j$ , respectively.

Specifically, we introduce  $Cap_{\mathcal{A}_i \cap \mathcal{A}_j}^{(k)}$  as the market capitalization of the fund  $k$  due to assets belonging to the set  $\mathcal{A}_i \cap \mathcal{A}_j$ . Then,

$$w_{ij}^{(i)} = \frac{Cap_{\mathcal{A}_i \cap \mathcal{A}_j}^{(i)}}{Cap^{(i)}}, \quad w_{ij}^{(j)} = \frac{Cap_{\mathcal{A}_i \cap \mathcal{A}_j}^{(j)}}{Cap^{(j)}},$$

where  $Cap^{(k)}$  is the total market capitalization of fund  $k$ . Then, define

$$w_{ij} := \theta_i(j)w_{ij}^{(i)} + \theta_j(i)w_{ij}^{(j)}, \quad (3)$$

where  $\theta_i(j), \theta_j(i) \in [0, 1]$  and  $\theta_i(j) + \theta_j(i) = 1$ . The weights  $\theta$ 's are computed on the basis of the market capitalization of the individual funds

$$\theta_i(j) = \frac{Cap^{(i)}}{Cap^{(i)} + Cap^{(j)}}, \quad \forall i, j \in V.$$

By construction  $w_{ij} \in [0, 1]$  for each  $i, j \in V$  is the total market capitalization of the assets in common, normalized by the sum of the market capitalizations of funds  $i$  and  $j$ . Furthermore, the weight of the arc  $(i, j)$  is large (small) when, on average, the market capitalization in the funds  $i$  and  $j$  is largely (poorly) due to the common components of  $\mathcal{A}_i \cap \mathcal{A}_j$ . Moreover, the arcs of the network are not directed, since  $w_{ij} = w_{ji}$ , for each  $i, j \in V$ .

Equation (3) computes the weight of the link between two funds  $i$  and  $j$ . Such a formula provides two main features: on one side, if the funds share only a part of their holdings, then the strength of the link between them depends also on funds' capitalization; on the other side when the funds invest exactly in the same assets – even though with different percentages – then the weight of their link is one. This latter case describes the situation in which investing in the same holdings lets funds be highly connected.

The percentage of the market capitalization of a fund given by one of its holdings is a quantity depending on time. Indeed, since the price of the holdings of a fund evolves in time, the same amount of stocks leads to different levels of relative market capitalizations. Therefore, when needed, we will refer to  $w_{ij}(t)$  as the weight of the arc  $(i, j)$  at a given time  $t \geq 0$ .

The arcs are collected in a set  $E$  and the weights represent the entries of a squared  $n \times n$  adjacency matrix  $W = (w_{ij} : i, j \in V)$ . Time dependence also appears for  $E$  and  $W$ , so that we refer – when needed – to  $E(t)$  and  $W(t)$  to indicate the set of the arcs and the weighted adjacency matrix at time  $t$ , respectively.

By construction of arcs and weights,  $i = j$  implies that  $w_{ij} = 1$ ; moreover,  $(i, j) \notin E$  if and only if  $w_{ij} = 0$ . Therefore, we can assume that the graph  $(V, E)$  is complete with loops; the adjacency matrix  $W$  has a null value in the presence of a missing arc and the principal diagonal filled by ones.

The overall financial system of the funds is then a network  $\mathcal{N} = (V, W)$ , and also in this case, we can insert the dependence on time  $t \geq 0$  by writing  $\mathcal{N}(t) = (V, W(t))$ . Notice that also  $V$  depends, in general, on time. However, our analysis is over one-period – with  $t = 0, 1$ , as we will see below – where the set  $V$  does not change. Therefore, we prefer to refer directly to  $V$  instead of  $V(t)$  for the sake of notational simplicity.

### 3 The resilience measure of the financial network

The resilience of a network is its ability to absorb an external solicitation. Thus, to introduce the resilience measure, we need to explain and formalize what we intend with external solicitation, what the absorption of it by the network means, and how a measure of the ability of absorption should be conceptualized and interpreted.

As a premise, we state that an external solicitation – or *shock* – has a negative sense, and it represents an occurrence that can be dangerous for the financial system and destabilize it. Therefore, a high (low) resilience measure is a positive (negative) aspect of the financial system described by the network. This said resilience can be identified only after a proper definition of the financial performance of the network  $\mathcal{N}$  in its nature of a collection of funds.

#### 3.1 The measure of resilience

We here present a single period context, in which a shock occurs at time  $t = 0$  (today) and the performance of the shocked system is then measured at time  $t = 1$  (tomorrow).

We denote by  $R_i$  the return of a given fund  $i \in V$  from time  $t = 0$  to time  $t = 1$ . Such a return is the weighted mean of the returns of the single holdings of  $\mathcal{A}_i$ , where the weights capture the percentages of the market capitalization of the fund due to the specific elements of  $\mathcal{A}_i$ .

Since  $t = 1$  is tomorrow, the elements of the set  $\mathcal{A}$  exhibit randomness in their returns. In turn, the returns  $R$ 's of the funds are random variables with a given distribution. A probability space  $(\Omega, \mathfrak{F}, \mathbb{P})$  is assumed to contain all the random quantities used in the paper.

The randomness of the returns of the components of the funds leads to randomness also for the weights  $w(1)$ 's in the weighted adjacency matrix  $W(1)$ . Indeed, the percentage of the market capitalization of the funds due to the individual holdings is strongly related to the return of the considered holdings. Therefore, the network  $\mathcal{N}(1)$  has stochastic weights on the arcs.

Consider a portfolio of the funds  $P = (x_1, \dots, x_n)$ , so that  $x_i$  is the share of capital invested in fund  $i \in V$  and  $x_1 + \dots + x_n = 1$ . We assume that short-selling is not allowed, so that  $x_i \geq 0$  for each  $i \in V$ . Such an assumption will be appreciated below.

By the perspective of network  $\mathcal{N}(1)$ , the modification of the return of one holding of  $\mathcal{A}_i \cap \mathcal{A}_j$  leads to a variation of the percentage of market capitalization due to such a holding in  $i$  and  $j$  at time  $t = 1$ , hence leading to a modification of the distribution of  $w_{ij}(1)$ . Such a variation is more evident if the network is less resilient since, in this case, it is associated with significant losses due to the downturn of the holdings of the funds.

Hence, we can say that the measure of the reaction of the financial system to the losses in the returns of the funds holdings represents the basis of the definition of the resilience measure of  $\mathcal{N}$ , and such a resilience measure depends on the specific portfolio  $P$ . For an easy notation, we will call the network  $\mathcal{N}$  when dealing with portfolio  $P$  by  $\mathcal{N}_P$ . By construction,  $\mathcal{N}_P$  can be seen as a new network, obtained by taking  $\mathcal{N}$  and adding to it the portfolio as weights  $x_1, \dots, x_n$  of the nodes  $R_1, \dots, R_n$ . Formally, we write  $\mathcal{N}_P = (V, W, P)$ . The usual notation for the dependence on time applies so that  $\mathcal{N}_P(t)$  is  $\mathcal{N}_P$  at time  $t$ .

A shock is here given by a loss in the return of one of the components of the considered funds so that it generally brings a financially negative variation of the distribution of the weights  $w(1)$ 's.

According to the arguments developed above, we employ the community structure of the network to model resilience measure. In particular, we can use the classical clustering coefficient for weighted networks (see e.g. Onnela et al., 2005) as the scientific ground for introducing the concept of resilience measure.

Given a node  $i \in V$ , the expected clustering coefficient for  $i$  at time  $t = 1$  is

$$\mathbb{E}[C_i(1)] = \mathbb{E}\left[\frac{2 \sum_{j,k \in V \setminus \{i\}, j > k} (w_{ij}(1)w_{ik}(1)w_{jk}(1))^{1/3}}{(n-1)(n-2)}\right]. \quad (4)$$

A brief explanation of the clustering coefficient is in order. Formula (4) provides the expectations of the number of existing triangles having  $i$  as one of the vertices – where triangles are here considered along with the weights on their sides – over the maximum number of theoretically existing triangles around  $i$  – all of them taken with unitary weights on all the sides. Indeed, the clustering coefficient describes the network's cohesiveness – in the sense that it is high when the adjacents of the network's nodes tend to form arcs. Thus, cohesiveness is captured by the number of the existing triples of nodes over the theoretical one.

Of course, cohesiveness is not really synonymous with community. However, there is evidence that closing triples can be a mean for generating the community structure of a network. Indeed, the



existence of a triangle around  $i$  – say, the triple composed by the nodes  $i, j, k$  – captures the evidence that  $j$  and  $k$  are not only adjacent to  $i$ , but they are also mutually connected by an arc  $(j, k)$ . In this case,  $i, j, k$  are a (small) community contributing to form the community structure around  $i$ . The absence of a connection between nodes  $j$  and  $k$  – which are taken still as adjacent to  $i$  – removes the contribution of  $i, j, k$  only from the numerator of Formula (4), hence reducing the value of  $\mathbb{E}[C_i(1)]$ . Of course, the higher the values of  $w_{ij}(1), w_{ik}(1), w_{jk}(1)$ , the higher the value of the contribution added by  $i, j, k$  to the community structure around  $i$ . In this context, it is worth mentioning the very relevant contribution of Bianconi et al. (2014). In the quoted paper, the authors state that: ”*It has been pointed out that there is a close relationship between a high density of triads and the existence of community structure*”. They elaborate on this point by observing that the concept of community – a set of nodes having strong interconnections, with a low level of links with nodes outside the set – is likely associated with the high presence of triads. Substantially, one has closed triangles more often among nodes of the same community than among those belonging to different groups (on this, see, e.g. Granovetter, 1973). This popularity of the clustering coefficient in the context of networks’ communities is witnessed by several contributions (see, e.g. Cui et al., 2014; Nascimento, 2014; Palla et al., 2005; Radicchi et al., 2004; Said et al., 2018).

We consider a weighted mean of the clustering coefficients of the nodes by including the shares of the portfolio  $P$ . Thus, the expected clustering coefficient of the network  $\mathcal{N}_P(1)$  is

$$\mathbb{E}[C_P(1)] = \sum_{i \in V} x_i \mathbb{E}[C_i(1)], \quad (5)$$

where  $\mathbb{E}[C_i(1)]$  is defined in (4). The term  $\mathbb{E}[C_P(1)]$  will be used to measure the resilience of the network  $\mathcal{N}_P$ . By definition and since short-selling is not allowed, we have  $\mathbb{E}[C_P(1)] \in [0, 1]$ . Moreover, in line with the arguments developed above, we need to conceptualize the resilience measure by considering that  $\mathcal{N}_P$  is weakly (strongly) resilient as a negative shock generates a collapse of  $\mathbb{E}[C_P(1)]$  of large (small) entity.

Now, let us implement a shock to the network. We assume that a shock is a lump sum in the left tail of the distribution of the return of a prefixed asset  $a \in \mathcal{A}$ , so that we are assuming shocks of *tail-event* types (see e.g. Taleb, 2007; Barberis, 2013 and Härdle et al., 2016). Specifically, we fix a constant  $\varepsilon < 0$  and a probability  $\pi \in (0, 1)$  such that

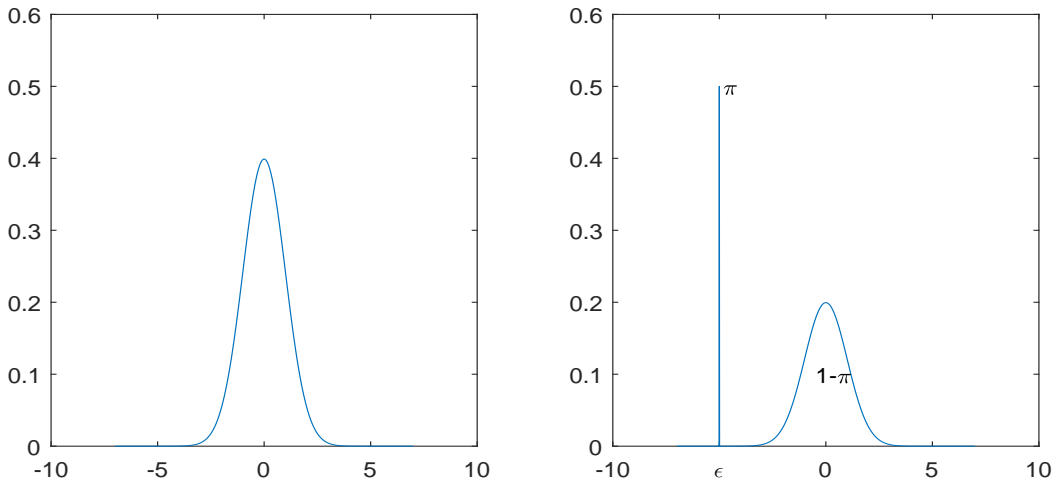
$$\mathbb{P}\left[R_{(\varepsilon, \pi)}^{(a)} \leq r\right] = \begin{cases} \mathbb{P}\left[R^{(a)} \leq r\right] (1 - \pi), & \text{if } r < \varepsilon; \\ \mathbb{P}\left[R^{(a)} \leq r\right] (1 - \pi) + \pi, & \text{otherwise,} \end{cases} \quad (6)$$

where  $R^{(a)}$  and  $R_{(\varepsilon, \pi)}^{(a)}$  are the returns of  $a \in \mathcal{A}$  in absence of shocks and when the shock described by

the lump sum with entity  $\epsilon$  and probability  $\pi$  is applied, respectively.

More specifically, the term  $\pi$  is the probability of a shock of the returns of intensity  $\epsilon$ . In this context,  $\epsilon$  represents a critical value playing the role of a highly probable lump sum affecting the left tail of the distributions of the assets' return. If  $\pi$  is high, then there is a remarkable gap in cumulative probability when passing from return below to return above  $\epsilon$ ; differently, a small value of  $\pi$  points to a rather smooth jump of the cumulative probability of returns. This said, the probability  $\pi$  is associated to the severity of the impulsive event – which is of adverse nature, see Assumption 3.1 below – affecting the returns of the assets: once the value of  $\epsilon$  is fixed, the higher the probability, the more severe the adverse effect of the negative shock. At the same time and for a fixed value of  $\pi$ , the absolute value of  $\epsilon < 0$  increases as the negative realization described by the lump sum is more relevant – hence, pointing to a more adverse event generated by the shock.

To sum up, we can say that  $\pi$  is the probability of an impulsive shock of intensity  $\epsilon$  modifying the initial distribution of an asset. To better explain the role of such a term in our context, we present the plot in Figure I, which shows an example of an impulsive negative shock of the same nature of that defined in our paper. The left panel reports the returns distribution – that for simplicity is assumed to be normal-shaped – of an asset before the shock. The right panel shows the shocked distribution which is modified by the presence of a lump sum of intensity  $\epsilon$ .



**Figure I. Action of the lump sum shock.**

On the left panel, illustration of the distribution of an unshocked return, while on the right panel we have the distribution of the same return in presence of the shock according to Formula (6).

The shock modifies the market capitalization of the funds belonging to  $V_a$  at time  $t = 1$ . Therefore,

it modifies the weights  $w_{ij}(1)$  with  $i, j \in V_a$ . In this respect, we define the shocked weighted adjacency matrix at time  $t = 1$  by  $W^{(a;\varepsilon,\pi)}(1)$  with entries  $w_{ij}^{(a;\varepsilon,\pi)}(1)$  and, according, the resulting shocked network  $\mathcal{N}_P^{(a;\varepsilon,\pi)}(1) = (V, W^{(a;\varepsilon,\pi)}(1), P)$ .

In order to proceed with the introduction of the resilience measure of the considered financial network, the lump sum with  $\varepsilon$  and  $\pi$  is assumed to be a non positive outcome for all the elements of  $\mathcal{A}$ . The following condition will stand in force hereafter:

**Assumption 3.1** *We assume that  $\varepsilon < 0$  and  $\pi \in (0, 1)$  are such that  $\mathbb{E} \left[ w_{ij}^{(a;\varepsilon,\pi)}(1) \right] \leq \mathbb{E} [w_{ij}(1)]$ , for each  $a \in \mathcal{A}$  and  $i, j \in V_a$ .*

Assumption 3.1 formalizes that the defined shock represents an adverse event in the context of the considered network. Such an assumption is crucial because the definition of the shock as a lump sum (see Formula (6)) does not automatically require that such a lump sum is an adverse event for the return of the considered asset. In fact, the relationship between the assets return and the considered lump sum-type shock depends on the particular shape of the return distribution and the choice of the value of  $\varepsilon$  and  $\pi$ . In the not unusual case that the return distribution is particularly concentrated on large losses, then even a very low value of  $\varepsilon$  could represent an improvement on the distribution. So, it is crucial to clearly state that an impulse shock is an adverse event for the return; Assumption 3.1 goes precisely in this direction.

By applying the expected clustering coefficients of (4) to the nodes of the shocked network  $\mathcal{N}^{(a;\varepsilon,\pi)}(1)$ , we can rewrite the expected clustering coefficient of the network in (5) and denote it as  $\mathbb{E} \left[ C_P^{(a;\varepsilon,\pi)}(1) \right]$ .

By hypothesis on  $\varepsilon$  and  $\pi$  and by (4) and (5), we have  $\mathbb{E} \left[ C_P^{(a;\varepsilon,\pi)}(1) \right] \leq \mathbb{E} [C_P(1)]$ .

Let us remark that the shock modifies only the market capitalization of the funds belonging to  $V_a$  at time  $t = 1$ , and not the funds outside  $V_a$ .

The comparison between  $\mathbb{E} [C_P(1)]$  and  $\mathbb{E} \left[ C_P^{(a;\varepsilon,\pi)}(1) \right]$  gives insights on the resilience of the network when the lump sum with entity  $\varepsilon$  and probability  $\pi$  is applied to the return of the component  $a \in \mathcal{A}$ .

The simplest thing to do is to consider

$$\gamma^{(a;\varepsilon,\pi)} = \frac{\left| \mathbb{E} \left[ C_P^{(a;\varepsilon,\pi)}(1) \right] - \mathbb{E} [C_P(1)] \right|}{\mathbb{E} [C_P(1)]}. \quad (7)$$

By definition,  $\gamma^{(a;\varepsilon,\pi)} \in [0, 1]$ . Moreover,  $\gamma^{(a;\varepsilon,\pi)}$  is close to one when the fall of the expected clustering coefficient from the non shocked to the shocked case is of large entity, and it is close to zero otherwise.

To assess the resilience of the network  $\mathcal{N}_P(1)$ , we need to generalize the definition in (7) to any component  $a \in \mathcal{A}$ . In doing so, we continue to keep as given the parameters of the shocking lump sum, i.e.  $\varepsilon$  and  $\pi$ .

Specifically, given  $\varepsilon < 0$  and  $\pi \in (0, 1)$  such that Assumption 3.1 is satisfied, we define the  $(\varepsilon, \pi)$ -*resilience* of the network  $\mathcal{N}_P(1)$  by

$$\Gamma^{(\varepsilon, \pi)} = \frac{1}{|\mathcal{A}|} \sum_{a \in \mathcal{A}} \gamma^{(a; \varepsilon, \pi)}. \quad (8)$$

By definition, also  $\Gamma^{(\varepsilon, \pi)} \in [0, 1]$ . A large (small) value of  $\Gamma^{(\varepsilon, \pi)}$  is associated to a weakly (highly) resilient network  $\mathcal{N}_P(1)$ . Such a concept of resilience measure lets intervene a special lump sum with parameters  $\varepsilon$  and  $\pi$  and an average of the reaction of all the shocked elements of  $\mathcal{A}$ . Moreover, the presence of a unique variation range for the  $\Gamma$ 's allows comparison exercises among different sets of funds, portfolios and shocking lump sums.

It is essential to point out that our definition of resilience measure is in line with the topological-based definition of the resilience measure. Indeed – as already said above – the resilience of a network is its ability to absorb external shocks. In large part of the literature on the resilience measures, shocks are modelled by arcs or nodes removal. Then, the shocked network might change its topology by disintegrating its structure into disconnected components. In this framework, the resilience measure of the network is high when the shock does not generate the disaggregation of the network (see, e.g. the impacting contributions by Albert et al., 2000; Callaway et al., 2000; more recently, Iyer et al., 2013; Ramirez-Marquez et al., 2018; Rocco et al., 2018). Intuitively, the disintegration of a network deteriorates its communities – which are sets of nodes highly interconnected and with low connections with the external nodes.

The ground of our study and of Formula (8) is that networks' resilience can be measured by the entity of the relative decrease of the clustering coefficient of the network – being such a decrease the effect of a measurable financial shock and the clustering coefficient providing a measure of the community structure.

We point out that our definition of resilience measure can be suitably reinterpreted in terms of vulnerability (see Zio and Sansavini, 2013; Ramirez-Marquez et al., 2018), in that we advance a method based on the comparison between a performance – the expected clustering coefficient, in our case – before and after the shock.

## 4 Empirical application

In this section, we analyze four different networks. The nodes of the first two networks are funds that are High ranked for ESG compliance. The two networks differ from each other for the size of funds. The second two networks consist of nodes/funds which are Low ranked for ESG, and again they differ for the funds' size. We proceed first by introducing the dataset used to construct the networks. Then, we compute the resilience measure of the networks at hand, and we show that the network of the High ranked Small size funds is more resilient than the network of the Low ranked Small size funds. The same does not apply when comparing the networks of High and Low ranked Big size funds.

### 4.1 The dataset

We consider the cross section of the open-end equity mutual funds quoted in June 2018, provided by Morningstar Direct (MD). MD reports all data at the fund share class level, including the funds' holdings that are common across different share classes. After eliminating funds without the ISIN, consistently with Patel and Sarkissian (2017), we aggregate mutual-fund-share-class-level observations to one fund-level observation using the unique fund identifier (*FundId*) in MD. For such sub-sample, we have been then able to retrieve funds' holdings for 10421 funds from the European Data Warehouse (EDW) by Morningstar. Out of them, we keep only funds whose Total Net Asset value (*TNA*) is available. We also exclude from the sample the funds whose holdings sum to a number higher than one and those whose holdings sum to a number lower than 80%. To reduce the dimensionality of the sample and then the computational complexity, we finally trim the bottom 5% of the distribution of the funds' *TNA*. The funds removed by such a filter account only for 0.046% of the total capitalization of the sample. Moreover, such funds invest in a small number of assets (less than thirteen), so they are poorly diversified and have a very low degree of overlap with other funds. Hence, results for resilience are not influenced by such funds.<sup>1</sup> The resulting sample consists in  $N_F = 5898$  funds investing in  $N_A = 22639$  different holdings. For each fund, we normalize the holdings to sum to 1. Assets' returns and funds' *TNA* are retrieved from Refinitiv (DATASTREAM).

We rank the funds according to the sustainability level issued by Morningstar SUN GLOBE. Such a rating is based on the company level Environmental, Social and Governance (ESG) scores released by Sustainalytics and company ESG controversies. To receive a portfolio ESG score, at least 67% of the assets under management must have a company ESG score. Then funds are ranked according to 5 categories: High (*H*), Above Average (*AA*), Average (*A*), Below Average (*BA*), Low (*L*). High(Low)-

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<sup>1</sup>On the contrary, we can not trim the right tail of the fund capitalization since they account for a very large amount of market capitalization; furthermore, they are widely interconnected with all the funds in the market. As such, their impact on the results is substantial.

ranked funds are those in the top(bottom) 10% of the score distribution. Below Average funds have a score that is between the 10-th percentile and the 32.5-th percentile of the score distribution. The Average-ranked funds are those in the next 35% of the distribution. Above Average funds are ranked in the range between the 67.5-th percentile and the 90-th percentile.<sup>2</sup> Out of 5898 mutual funds, 519 are ranked as Low, 1306 as Below Average, 2150 as Average, 1368 as Above Average and 555 funds are ranked as High. For the sake of clarity, we now proceed with a brief description of the entire dataset, even if – as we will see below – we will refer for the analysis to the Low and High ranked funds.

Table I provides the main statistical indicators for the distributions of the number of assets for each fund, of the funds' total net asset value, and the annualized average daily return, respectively, across ESG categories. Table A-I reports also the t-test results for the differences of such variables across the different ESG categories. The descriptive evidence shows that funds are well diversified in terms of the number of assets for all the ESG ratings, with the Average-ranked funds having the highest numbers of assets on average. The distributions of the number of assets are positively skewed and leptokurtic (Table I, Panel A). Moreover, the High ranked funds invest on average in the lowest number of assets (see also Table A-I, Panel A, last row). Specifically, High ranked funds always appear to have less granular portfolios with respect to funds in the remaining four categories. This is due to the fact that High ranked funds tilt their portfolios towards the highest ESG ranked assets. The distribution of funds' *TNA* is relatively uniform across the ESG categories with the exception of the Low ranked funds which, on average, are smaller in size than funds in the other ESG categories (Table I, Panel B, and Table A-I, Panel B, column 1). Independently from the category, funds' *TNA* ranges from a few hundred thousand dollars to more than one billion dollars. The returns are computed daily over the last past year and are annualized for a better understanding of the figures (Table I, Panel C). Consistent with Hong and Kacperczyk (2009) and the *responsibility effect* documented by Becchetti et al. (2018), average returns decrease monotonically as we move from Low to High ranked funds (Table A-I, Panel C). As such, higher returns for Low ranked funds could be justified by their higher exposition to the stakeholder risk (Becchetti et al., 2018), to the crash-risk (Kim et al., 2014), or a combination of the two. Indeed, High ranked funds are less remunerative but also less risky. In particular, the minimum return is the highest for the High ranked funds. All the ESG categories show skewness and kurtosis values near those of a normal distribution.

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<sup>2</sup>For further details see [Morningstar Sustainability Rating](#).

**Table I. Descriptive Statistics at Fund-level Across the ESG Categories**

The table reports the number of assets for each fund across different ESG categories (Panel A), the Total Net Assets in millions of dollars (Panel B), and the annualized average daily returns in percentage (Panel C). The five ESG categories are High ( $H$ ), Above Average ( $AA$ ), Average ( $A$ ), Below Average ( $BA$ ), Low ( $L$ ).

	(1)	(2)	(3)	(4)	(5)
<b>Panel A</b>					
	$L$	$BA$	$A$	$AA$	$H$
<i>Min</i>	9	13	12	13	11
<i>Max</i>	8368	7528	4630	4495	2705
<i>Mean</i>	182.10	194.47	195.59	121.21	86.08
<i>StdDev</i>	747.27	552.56	399.93	218.86	204.04
<i>Skewness</i>	7.70	6.67	5.36	8.68	10.09
<i>Kurtosis</i>	68.71	58.48	42.30	133.63	116.76
<b>Panel B</b>					
	$L$	$BA$	$A$	$AA$	$H$
<i>Min</i>	0.34	0.33	0.31	0.31	0.34
<i>Max</i>	28595.10	138831.40	125168.20	59273.70	25016.40
<i>Mean</i>	278.88	540.23	576.94	570.96	394.58
<i>StdDev</i>	1429.51	4358.19	3558.70	2460.92	1547.70
<i>Skewness</i>	15.88	25.89	24.37	13.28	9.90
<i>Kurtosis</i>	302.00	788.42	766.68	265.90	131.86
<b>Panel C</b>					
	$L$	$BA$	$A$	$AA$	$H$
<i>Min (%)</i>	-17.64	-10.97	-24.32	-29.42	-9.01
<i>Max (%)</i>	42.10	42.49	42.34	42.10	39.80
<i>Mean (%)</i>	16.66	15.69	14.44	12.98	11.47
<i>StdDev (%)</i>	10.76	9.06	8.74	8.28	7.75
<i>Skewness (%)</i>	0.08	0.25	0.22	0.16	0.56
<i>Kurtosis (%)</i>	2.83	3.04	3.53	3.67	3.60

## 4.2 Network analysis

In the following, we consider four different networks: the network of the High ranked Big size funds  $\mathcal{N}^{HB}$ , that of the Low ranked Big size funds  $\mathcal{N}^{LB}$ , that of High ranked Small size funds  $\mathcal{N}^{HS}$  and finally the network of the Low ranked Small size funds  $\mathcal{N}^{LS}$ . The High(Low)-ranked Big(Small) size funds are the funds in the top(bottom) 10% of the High(Low)-ranked funds'  $TNA$  distribution. We compare the  $(\varepsilon, \pi)$ -resilience of  $\mathcal{N}^{HB}$  with that of  $\mathcal{N}^{LB}$ , and the  $(\varepsilon, \pi)$ -resilience of  $\mathcal{N}^{HS}$  with that of  $\mathcal{N}^{LS}$  for different values of parameters  $\varepsilon$  and  $\pi$ .  $\mathcal{N}^{HB}$  consists of 55 nodes/funds investing in 4492 assets while  $\mathcal{N}^{LB}$  has 52 nodes/funds investing in 14322 assets. The actual investment universe of the funds in  $\mathcal{N}^{HB}$  shares 4252 assets with the universe of allocation of the funds in  $\mathcal{N}^{LB}$ . The network  $\mathcal{N}^{HS}$  has 55 nodes/funds investing in 1839 assets, while  $\mathcal{N}^{LS}$  consists of 52 nodes/funds investing in 2347 assets. The two networks share 716 assets.

Such comparisons are motivated as follows: High ESG funds invest in a subset of the market consisting of the assets with the highest ESG ranks. Hence, we expect that High ESG funds form a stronger community structure than Low ESG funds. For this reason, it is interesting to compare

the resilience – to be intended as the stability of the community structure – of the network of High and Low ESG funds. It is worth noticing that we compare the network of High ranked funds against that of Low ranked funds, rather than the network of Socially Responsible funds against the entire universe of funds, since for a conventional fund it is not possible to dissect its responsibility level due to data limitations, which generally affects all the studies on such a topic. Additionally, we perform the analysis separately for the Big size funds and the Small size funds to disentangle the effect of the size from the ESG information. In so doing, two scopes are pursued. On one side, we include in the analysis a further source of information, which gives more insights on the discrepancies between the resilience measure of the High and Low ranked funds; by the other side, we remove the computational complexity of the analysis on the entire networks, which leads to a substantial computational intractability of the problem.

As in the previous subsection, we briefly provide some descriptive statistics of the main funds' variables used for the analysis, namely the number of assets for each fund, the funds' TNA value, and the annualized average daily return across the networks analyzed.

Table II shows that the Low ranked Big size funds invest on average in a higher number of assets with a higher dispersion than the High ranked Big size funds (Panel A, column 1-2). The opposite is observed for the Small size funds (Panel A, columns 3-4). However the difference of the average number of assets is significant only for the Big size funds networks (Table A-II, Panel A). In terms of  $TNA$ , the Low ranked funds on average have lower capitalization than the High ranked funds (Table II, Panel B) and the difference is significant only for the Small funds (Table A-II, Panel B). As concerning the financial profitability of the funds, the returns indicate that the High ranked Big size funds, on average, perform similarly to the Low ranked Big size funds with a lower dispersion and a positive skewness (Table II, Panel C, columns 1-2). On the other hand, the High ranked Small size funds have lower returns than the Low ranked Small size funds (Table II, Panel C, columns 3-4) and the difference in terms of returns between the two is also significant (Table A-II, Panel C).

All the networks are dense. The nodes of  $\mathcal{N}^{HB}$  are connected by 885 links (59.60% out of 1485 possibilities).  $\mathcal{N}^{LB}$  has 660 links (49.85% out of 1326 possible connections). For the small size funds networks, we have 554 active links in  $\mathcal{N}^{HS}$  (37.31% out of 1485 possibilities) and 344 links in  $\mathcal{N}^{LS}$  (25.94% out of 1326 pairs of funds). We note that there are more connections for the High ranked funds than for the Low ranked ones and for the Big size funds than for the Small size funds. Moreover, for the Small size funds, the links connecting the nodes of the High ranked funds network weigh more than the links of the corresponding Low ranked funds network, meaning that portfolios of High ranked



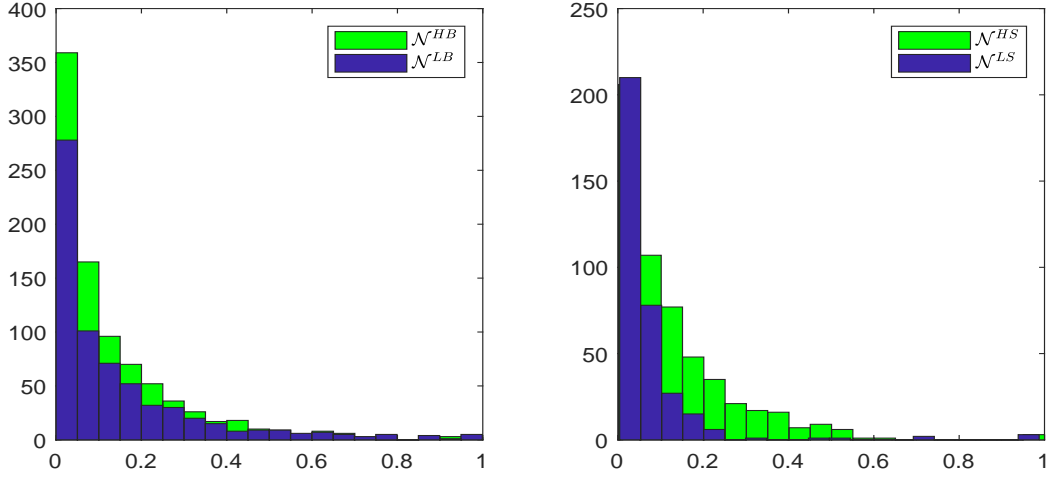
**Table II. Descriptive Statistics at Fund-level Across the Networks**

The table reports the number of assets for each fund across different networks (Panel A), the Total Net Assets in millions of dollars (Panel B), and the annualized average daily returns in percentage (Panel C).  $\mathcal{N}^{HB}$  and  $\mathcal{N}^{LB}$  represent the networks formed by High and Low ranked Big size funds,  $\mathcal{N}^{HS}$  and  $\mathcal{N}^{LS}$  represent the networks formed by High and Low ranked Small size funds.

	(1)	(2)	(3)	(4)
<b>Panel A</b>				
	$\mathcal{N}^{HB}$	$\mathcal{N}^{LB}$	$\mathcal{N}^{HS}$	$\mathcal{N}^{LS}$
<i>Min</i>	19	15	14	15
<i>Max</i>	2705	8368	392	192
<i>Mean</i>	232.84	997.63	61.42	59.65
<i>StdDev</i>	563.57	2031.19	74.39	39.31
<i>Skewness</i>	3.72	2.33	3.47	1.72
<i>Kurtosis</i>	15.50	7.60	14.88	5.42
<b>Panel B</b>				
	$\mathcal{N}^{HB}$	$\mathcal{N}^{LB}$	$\mathcal{N}^{HS}$	$\mathcal{N}^{LS}$
<i>Min</i>	718.14	488.50	0.34	0.34
<i>Max</i>	25016.40	28595.10	3.20	2.27
<i>Mean</i>	3059.10	2139.21	1.55	1.20
<i>StdDev</i>	4044.78	4092.41	0.93	0.59
<i>Skewness</i>	3.46	5.46	0.35	0.30
<i>Kurtosis</i>	17.45	35.14	1.81	1.88
<b>Panel C</b>				
	$\mathcal{N}^{HB}$	$\mathcal{N}^{LB}$	$\mathcal{N}^{HS}$	$\mathcal{N}^{LS}$
<i>Min (%)</i>	-3.71	-12.27	-9.01	-6.18
<i>Max (%)</i>	39.80	33.08	30.30	40.70
<i>Mean (%)</i>	13.39	14.81	11.24	16.04
<i>StdDev (%)</i>	9.07	9.86	6.53	10.73
<i>Skewness (%)</i>	0.43	-0.22	0.32	0.27
<i>Kurtosis (%)</i>	3.47	3.10	4.69	2.78

funds are more overlapped than portfolios of Low ranked funds. On average, the weight for the High ranked funds is 0.13, while that for the Low ranked funds is 0.07. On the other hand, for the Big funds, the links connecting the High ranked funds weigh as those connecting the Low ranked funds. On average, the weight is 0.13 for the High ranked funds and 0.14 for the Low ranked funds. This is shown in Figure II that reports the histograms of the non null entries of the adjacency matrix (3) for  $\mathcal{N}^{HB}$  and  $\mathcal{N}^{LB}$  (left panel), and for  $\mathcal{N}^{HS}$  and  $\mathcal{N}^{LS}$  (right panel). This evidence is explained by the fact that the High ranked funds tilt their investments toward the assets with the very best ESG ratings. For the Small funds, such a feature gives higher weights to the links among the High ranked funds. On the other hand, for the Big funds, this effect is compensated by the fact the Big funds, managing large amount of assets, invest in a larger segment of the market. Then the adjacency matrices for the two networks of Big funds are more similar to each other.

We now compute the resilience measure of the networks considered. We associate to any network a



**Figure II. Histograms of the non null entries of the adjacency matrices**

$\mathcal{N}^{HB}$  and  $\mathcal{N}^{LB}$  represent the networks formed by High and Low ranked Big size funds,  $\mathcal{N}^{HS}$  and  $\mathcal{N}^{LS}$  represent the networks formed by High and Low ranked Small size funds. The adjacency matrices are defined according to equation (3).

vector of weights  $x$  whose entries are proportional to the  $TNA$  of the funds in the network as described in Section 3. First, we have to evaluate the expected clustering coefficient.

Let  $Cap^{(i,a)}$  be the holding of fund  $i$  in asset  $a$  at time  $t = 0$ , that is the capitalization of fund  $i$  invested in asset  $a$ . To compute the expected clustering coefficient at time  $t = 1$ , as in Equation (4), one has to update the matrix of holdings. Let  $R^{(a)}$  be the return of asset  $a$  at time  $t = 1$ . The updated  $(i, a)$ - element of the holding matrix at time  $t = 1$  is

$$Cap^{(i,a)}(1) = Cap^{(i,a)}(1 + R^{(a)})$$

The holding matrix is then used to compute the  $(i, j)$ -element  $w_{ij}(1)$  of the adjacency matrix at time  $t = 1$  according to Equation (3). Hence, to compute the expected clustering coefficient in Equation (4), one should simulate returns and then take the sample mean of the random sample. The problem with this procedure is that the covariance matrix of the assets' returns, needed for Monte Carlo simulation, is singular. The reason is that we have more than 22,000 assets and 250 daily observations. In general, with  $N_A$  assets and  $T$  observations where  $N \gg T$ , the covariance matrix is singular with  $N - T + 1$  null eigenvalues. To avoid Monte Carlo simulations, we assume that the correlation between assets' returns is zero. In this case, to compute the expected clustering coefficient, we can use the assets' expected returns rather than the returns. Indeed, it is useful to stress that the expected clustering coefficient is the expectation of terms containing products of returns. Therefore,

when the correlation is zero, we evaluate these terms by computing products of expected returns rather than the expectation of the products of returns. To assess the impact of such an approximation, we also evaluate the expected clustering coefficient accounting for correlation by using the method of historical simulations. Such a method consists in considering each daily observation as a random realization from the empirical joint distribution of assets' returns. Then the expected clustering coefficient is computed by evaluating the clustering coefficient each day (using the observed returns at each day) and then taking the sample mean.

Table III reports results of the expected clustering coefficient, for the High/Big, High/Small, Low/Big and Low/Small categories, when the correlation is neglected (Panel A) and when the correlation is accounted for (Panel B), together with the corresponding 95%-confidence intervals (in parenthesis).

**Table III. Clustering Coefficients  
Across the Networks**

The table reports the expected clustering coefficient defined in Equation (5) for the networks of Big/Small High/Low ranked funds when the correlation among assets is neglected (Panel A) and when it is accounted for (Panel B).

Panel A	Big	Small
<i>High</i>	0.0393386	0.0141491
<i>Low</i>	0.0294797	0.0027401
Panel B	Big	Small
<i>High</i>	0.0393381 (0.0393322 - 0.0393441)	0.0141489 (0.0141474 - 0.0141505)
<i>Low</i>	0.0294794 (0.0294776 - 0.0294812)	0.0027400 (0.0027394 - 0.0027406)

It is possible to see that including correlation affects the fifth/sixth significant digit, and the result without correlation is included in the confidence interval of the corresponding case with correlation. Hence the zero correlation approximation does not impact substantially on the results. It overcomes the Monte Carlo simulation issues and drastically reduces the computational complexity of the problem at hand.

The results in Table III show that there is heterogeneity among clustering coefficients across the different responsibility levels of the funds. Specifically, the expected clustering coefficient is higher for the High ranked funds' networks than for the Low ranked ones. The reason is that High ranked funds diversify their portfolios across the same stocks. Indeed, in line with Joliet and Titova (2018), responsible funds appear to discriminate stocks based on their ESG scores, among other factors, and they privilege firms with higher ESG scores. Hence, they reduce their potential opportunity set since holdings with lower scores represent the majority of the stocks (Becchetti et al., 2018).

Moreover, Big funds networks have higher expected clustering coefficients than Small funds networks because Big funds are more interconnected. Finally, the difference of the expected clustering coefficient between the High and the Low ranked funds is higher for the Small size funds than for the Big size funds, reflecting the characteristics of the adjacency matrices already described by Figure II.

To gauge the resilience of different networks to an external shock, we consider shocks of different intensities by setting  $\varepsilon = -25\%$ ,  $-50\%$ ,  $-75\%$ ,  $-100\%$ , and probabilities  $\pi = 0.25, 0.5, 0.75, 1$ . Then we compute the expected clustering coefficient after shock. We first consider the updated holding of fund  $i$  at time  $t = 1$ ,  $Cap_{(\varepsilon, \pi)}^{(i, a)}(1)$ , obtained when the return of asset  $a$  is shocked according to the distribution in Equation (6)

$$Cap_{(\varepsilon, \pi)}^{(i, a)}(1) = Cap^{(i, a)}(1 + R_{(\varepsilon, \pi)}^{(a)}).$$

Then the  $(i, j)$ -element of the shocked adjacency matrix  $W^{(a; \varepsilon, \pi)}(1)$  is again computed by means of Equation (3) using the updated holdings after shock. Under the assumption of zero correlation, the expected clustering coefficient depends only on the expected returns of the assets. From Equation (6), the expected return for the shocked asset is

$$E[R_{(\varepsilon, \pi)}^{(a)}] = (1 - \pi)E[R^{(a)}] + \pi\varepsilon,$$

where  $E[R^{(a)}]$  is the pre-shocked expected return.

As expected, since we shock one asset at a time and we have thousands of assets, the expected clustering coefficient after the shock is very close to that before the shock. Therefore, to improve the readability of results, we normalize the resilience measure in such a way that the smallest value of the resilience measure as in Equation (8) is set to 0 while the highest one is set to 1. The results are reported in Table IV.

**Table IV. Re-scaled resilience measure across the networks**

The table reports the resilience measure levels across the different intensity shocks ( $\varepsilon$ ) and probabilities ( $\pi$ ).

$\varepsilon$		Big Size				Small Size			
		-25%	-50%	-75%	-100%	-25%	-50%	-75%	-100%
$\pi = 0.25$	<i>High</i>	0.0096	0.0233	0.0372	0.0512	0.0324	0.0693	0.1066	0.1445
	<i>Low</i>	0.0000	0.0041	0.0082	0.0123	0.0406	0.0859	0.1319	0.1786
$\pi = 0.50$	<i>High</i>	0.0234	0.0512	0.0796	0.1088	0.0694	0.1445	0.2219	0.3020
	<i>Low</i>	0.0041	0.0123	0.0207	0.0293	0.0860	0.1787	0.2750	0.3757
$\pi = 0.75$	<i>High</i>	0.0373	0.0797	0.1237	0.1701	0.1068	0.2220	0.3433	0.4742
	<i>Low</i>	0.0082	0.0207	0.0337	0.0473	0.1321	0.2751	0.4284	0.5987
$\pi = 1$	<i>High</i>	0.0513	0.1088	0.1702	0.2553	0.1447	0.3021	0.4743	0.7415
	<i>Low</i>	0.0123	0.0293	0.0473	0.0721	0.1790	0.3760	0.5989	1.0000

Table IV shows a difference in terms of resilience measure among the High and Low-ranked funds within the Small and Big size categories. We enter the details. Consider Small funds. Here, our results show that the network of the High ranked funds is more resilient than the network of the Low ranked ones, with a resilience coefficient that is consistently lower for the different combinations of shock intensity and probability considered. Now, take the Big funds. In this case, we observe the opposite behaviour. The network of the Low ranked funds is more resilient than its High ranked counterpart.

We provide a possible explanation behind such a result. High ranked funds generally have a higher idiosyncratic portfolio composition than Low ranked funds. They are also less exposed to systematic risk due to a reduction in the stakeholder risk (see Becchetti et al., 2015). This feature should make the network of the High ranked funds more resilient to external shocks than the network of the Low ranked funds. This is what we observe for the Small funds. Such a behaviour is not observed for Big funds since these funds are intrinsically highly exposed to systematic risk due to a large amount of Assets Under Management (AUM). As such, for the Big funds, the idiosyncratic part of the portfolio plays a less important role in the computation of the resilience measure. Moreover, the difference in resilience levels appears to be more pronounced for increasing levels of adverse shocks. Such an effect is expected since a bigger shock perturbs more the network, thus amplifying the impact on the resilience measure.

## 5 Conclusions

This paper proposes a new concept of resilience measure based on the destabilizing effect of the shock on the community structure of a network, measured through the clustering coefficient. The theoretical proposal is validated through an empirical analysis performed on a high-quality dataset, which is particularly appropriate for testing the methodology. Specifically, we take into account the interrelation between funds as measured by the share of assets in common and analyze how an external shock impacts it. Using a worldwide cross-section of investment funds, we find that the networks of Small funds tilting their portfolio through more responsible assets present a higher degree of resilience to different severity and occurrence probability of exogenous shocks compared to the network of Small funds investing in less responsible assets. The opposite behaviour is observed for the Big funds.

Using a network approach, we study the market of funds as a whole. In so doing, we pursue two targets: by one side, we advance the theory of expert systems by dealing with the relevant theme of the resilience measure of the complex networks; by the other side, we add a new insight on the riskiness associated with Socially Responsible funds. Indeed in an interconnected market, the risk is mediated by common investments. On top of that, funds with high ESG scores also invest in a segment of the

market that is not mainstream, exploring a niche of the market that can have a positive effect on their risk. Hence, we here elaborate on whether the heterogeneity in portfolio composition of the High ranked funds may lead to a reduction of the impact of an external shock.

Interestingly, we can use other criteria for dealing with resilience. In this respect, the concept of avalanche is one possibility to define a criterion for assessing the resilience of a complex network. In the context of avalanches, one has that shocks propagate from individual parts of the network to the entire structure through patterns with increasing strength. In the present paper, we are pretty far from this approach by avoiding the evolutionary view of the effect of the shocks and preferring a more impulsive perspective. A shock manifests itself as a negative realization of an asset's return, weakening the interconnections among the funds in the network. In turns, such an action of the shock makes the network itself less cohesive. Therefore, the adopted perspective does not foresee an evolution of the network - which would imply the presence of amplified effects of the shock on all the network nodes, i.e. an avalanche - but rather a change in the state of the network. Such a change can be well measured by comparing the cohesiveness properties of the nodes before and after the shock. Undoubtedly, the avalanche approach is valuable and allows us to measure the long-run effects of an exogenous shock. The exploration of the long-run effects is not the target of the paper. Our approach will enable us to quickly measure the effect of the negative change in the assets' return across the entire network structure by avoiding waiting for the shock to propagate across the whole context, as in the case of avalanches. We plan to explore the context of avalanches in our framework of resilience.

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## Appendix: Statistical tests on the differences across the ESG categories and networks

**Table A-I. t-Test on the differences across the ESG categories**

The table reports the t-test results on the differences across the five ESG categories of the number of assets (Panel A), the Total Net Assets in millions of dollars (Panel B), and the annualized average daily returns in percentage (Panel C). The five ESG categories are High (*H*), Above Average (*AA*), Average (*A*), Below Average (*BA*), Low (*L*). The t-statistics are reported in square brackets. The differences are computed as column-minus-row. \*\*\*, \*\* and \* denote 1%, 5%, and 10% significance respectively.

	(1)	(2)	(3)	(4)
<b>Panel A</b>				
	<i>L</i>	<i>BA</i>	<i>A</i>	<i>AA</i>
<i>BA</i>	-12.37 [-0.34]			
<i>A</i>	-13.49 [-0.4]	-1.12 [-0.06]		
<i>AA</i>	60.89* [1.83]	73.26*** [4.47]	74.38*** [7.11]	
<i>H</i>	96.02*** [2.83]	108.39*** [6.17]	109.51*** [8.96]	35.13*** [3.35]
<b>Panel B</b>				
	<i>L</i>	<i>BA</i>	<i>A</i>	<i>AA</i>
<i>BA</i>	-261.35** [-1.92]			
<i>A</i>	-298.06*** [-3.01]	-36.71 [-0.26]		
<i>AA</i>	-292.07*** [-3.19]	-30.72 [-0.22]	5.98 [0.06]	
<i>H</i>	-115.7 [-1.27]	145.65 [1.06]	182.36* [1.81]	176.37* [1.89]
<b>Panel C</b>				
	<i>L</i>	<i>BA</i>	<i>A</i>	<i>AA</i>
<i>BA</i>	0.97* [1.81]			
<i>A</i>	2.22*** [4.37]	1.25*** [3.99]		
<i>AA</i>	3.68*** [7.04]	2.71*** [8.06]	1.46*** [4.99]	
<i>H</i>	5.19*** [9.02]	4.22*** [10.2]	2.97*** [7.83]	1.51*** [3.79]

**Table A-II. t-Test on the differences  
across the networks**

The table reports the t-test on the differences across the networks of Big/Small High/Low ranked funds in terms of the number of assets (Panel A), the Total Net Assets in millions of dollars (Panel B), and the annualized average daily returns in percentage (Panel C). The t-statistics are reported in square brackets. \*\*\*,\*\* and \* denote 1%, 5%, and 10% significance.

	(1)	(2)
<b>Panel A</b>		
	<i>Big</i>	<i>Small</i>
<i>Low – High</i>	764.80*** [2.62]	-1.76 [-0.15]
<b>Panel B</b>		
	<i>Big</i>	<i>Small</i>
<i>Low – High</i>	-919.89 [-1.17]	-0.35** [-2.35]
<b>Panel C</b>		
	<i>Big</i>	<i>Small</i>
<i>Low – High</i>	1.41 [0.77]	4.81*** [2.78]