Similarity-based heterogeneity and cohesiveness of networked companies issuing minibonds

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Abstract

This paper adopts a complex network approach for discussing the level of heterogeneity and cohesiveness among firms that have used a particular financial instrument – the so-called minibond. The nodes of the networks represent firms, and the weight of a link is assumed to be increasing with the similarity of the corresponding nodes/firms – where similarity is intended in terms of specific economic-financial characteristics of the firms. We assess the level of heterogeneity through the strength degree and the level of cohesiveness through the clustering coefficient. The empirical experiments are based on the paradigmatic case of the Italian reality, where minibonds have been and are currently efficiently used. The analysis reveals regularities and discrepancies among firms' financial characteristics. Furthermore, the results suggest the potential identification of the main determinants of minibonds issuance.

Keywords: Complex networks, minibonds, centrality measures, heterogeneity, cohesiveness.

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1 Introduction

Complex network analysis is a very efficient mathematical tool through which it is possible to detect the presence of iterations among different agents (see [1], [2]). The nature of a complex network explains its use in many areas of applied sciences. Its versatility can be used in many scientific fields such as biological, social, geophysics, climate, brain network, and many other types. Many studies have focused on understanding how the cells of an organism interact or how certain viral diseases spread (see e.g., [3], [4] and [5]). Others have looked at social networks, identifying the most influential agents present in the social pattern (see e.g., [6] and [7]). Climate change and extreme events have been studied through complex networks (see e.g., [8] and [9]), as well as brain connections (see e.g., [10] and [11]).

Among the various fields of application, economics has seen increasing use of complex network analysis in recent years. In particular, there are many relevant contributions dealing with economics and finance in a complex network context. We mention [12] who analyses the topological characteristics of networks in the context of the stock markets. In the quoted paper, the authors use a threshold method to construct the correlation network of Chinese stocks and then study the structural properties of the network and topological stability. In [13], the authors compute the structural similarity of financial indicators using normalised Euclidean distance. They construct networks based on firm performance and analyse the topological characteristics and parameters of the networks to represent the level of similarity among firms. In [14] the authors use a network based on the similarity of financial indicators built through Pearson's correlation coefficient, providing a quantitative approach to investigate the investments and the financial management of the firm on the energy stock market. Noticeably, [15] introduces a new measure of resilience in which investment funds represent nodes, and links are weighted according to the capitalisation due to the common components of the connected nodes. Other scholars analyse the topology of networks to reconstruct a country's payment system (see [16]). Some researchers use the power of complex networks to gain insights into the role that each firm plays within a supply chain or to study the logistics and supply chain management of firms (see [17], and [18]). Others apply network theory to managerial decision-making; in this respect, Fracassi [19] proves that managers' professional and educational connections have important implications in decision-making. These social connections also have consequences for the value of firms; the quoted paper shows that more socially connected managers show better economic performance. A very recent example is [20], where the authors propose a standard multilayer network model for discussing the systemic risk of a set of interconnected insurance companies. In the context of the Chinese financial institutions, [21] build a financial network – as we do in the present paper – and explore it through the most popular nodal centrality measures, like strength degree and closeness centrality.

We share the point of view of the papers quoted above. Indeed, this paper aims to add to the strand of research related to studying complex networks applied to the economic-financial field, specifically corporate finance. In particular, we develop an empirical investigation on a specific financial instrument, the so-called minibond. Minibonds represent a group of financial securities that unlisted companies can issue. They are mainly issued by small and medium-sized enterprises (SMEs) and are a reliable alternative to traditional funding channels (for more details on such a financial instrument, see [22]). Recent contributions explore some crucial implications of introducing such an instrument in this regard. In [23], the authors show that the average credit quality of minibond issuers is higher than the average of SMEs.Therefore, they discuss the Italian market and state that the reasons companies exploit this financing channel is not the lack of alternatives but rather a series of advantages such as access to the capital market, diversification, or reducing bank dependence. In [24], scholars analyse the minibond market in Germany. The quoted paper highlights the possibility that low-quality firms can exploit this new financing channel to raise funds since rating agencies cannot effectively distinguish the quality of firms. In conclusion, high-quality firms tend to issue undervalued minibonds to signal their high quality. In [25], the authors propose a fascinating study on behalf of the European Central Bank (ECB), carrying out interesting results. First, they state that diversification of funding sources allows firms to reduce hold-up effects (see [26]) in the relationship between banks and firms, increasing bargaining power towards banks. In addition, the use of minibonds minimises companies' dependence on the banking system, although the level of financial debt increases. This suggests that firms tend to replace bank debt with market-based debt, thus keeping the cost of debt unchanged. Finally, they point out that using this new financial instrument increases total assets and fixed assets.

This paper offers a novel methodological approach for exploring the firms issuing minibonds based on complex networks theory. In particular, we consider firms as the nodes of a weighted undirected network. The links' weights are built by considering that two highly similar firms are strongly connected. The similarity is here intended in the light of several variables related to the firms' characteristics like total assets, profitability and leverage, just to mention a few. The ground of our proposal is that firms with similar features tend to show the same level of development in terms of the considered characteristics (see e.g., [13] and [27]).
More specifically, we analyse two different aspects.

First, we discuss the heterogeneity of the firms issuing minibonds in terms of the considered financial variables and how such heterogeneity varies around the issuance time.

Second, we discuss the heterogeneity in terms of the considered financial variables of the collection of firms that are adjacent to a given one. In doing so, we explore the cohesiveness of the environment surrounding firms, when similarity means strong connections. More than this, we analyse the variation of such cohesiveness around the minibond issuance date.

To pursue the targets, we investigate the networks through highly informative nodal centrality measures. In so doing, we are in the line of a large strand of research concerning network analysis and its properties. In fact, several measures allow us to understand which are the most important nodes, the types of connections and possible associations (see [28], [29], [30], [31], [32], [33], [34].

We employ the strength degree centrality and the clustering coefficient to face our problems. Indeed, the strength degree describes the homogeneity of the structure of firms' interconnections, with a clear focus on their entities. In doing so, the analysis of the strength of the nodes gives insights on the modification of the similarities among the firms – of course, in terms of their main financial variables – when also considering the minibond issuance. The clustering coefficient is a proxy of the nodes' cohesiveness and the network as a whole. Our financial setting provides information on the variation of the embeddedness of the nodes in the overall financial context when such cohesiveness is generated by similar financial variables and in the presence of minibonds issuance. In other words, we can summarise the difference between strength degree and clustering coefficient information content as follows. The strength degree is used to see if firms present similar characteristics (high strength) or dissimilar ones (low strength). The clustering coefficient is used to see if firms are close to other firms presenting similar features (high clustering coefficient) or dissimilar ones (low clustering coefficient). Therefore, a high strength degree can also be obtained with a low clustering coefficient. Indeed, a firm can be highly similar to a large set of other companies (high strength), but, at the same time, its neighbourhoods can show different characteristics (low clustering coefficient).

The explored concepts of heterogeneity and cohesiveness have the merit of describing firms as a system instead of as disaggregated entities. This approach offers a global view of the companies issuing minibonds in terms of their performances and over a time period, without preventing the exploration of the individual elements of the considered sample. Such crucial property of the complex network frameworks is not available in other methodological contexts like cluster analysis or regression models.

The methodological proposal is tested over a high-quality dataset of Italian firms issuing minibonds in 2018. The sample is taken from the annual report of Politecnico di Milano (Osservatorio Mini-Bond [35]), in which a list of 176 issuing companies in 2018 is provided. The selected financial indicators are linked to some essential characteristics in the life of the enterprise, such as size, profitability, collaterals, and age, to name just a few. In particular, we divide these characteristics into two sets. In the first one, we include features that can only be observed at the time of issuances. In the second set, we consider annual financial statements from 2016 to 2019, hence obtaining an overview of the situation before, after and at the time of issuance. In this respect, we build one network for each year and for each financial variable.

The empirical sample is quite relevant. Indeed, most SMEs are issuing for the first time in the Italian minibond market. Due to their high degree of opacity, these companies have more difficulty accessing financial markets than larger ones (see [36]). Nevertheless, in Italy, SMEs represent 99% of all enterprises (see [37]) and are the backbone of the economy. Only a few of them have issued minibonds. Understanding the characteristics of companies that have already issued minibonds, the determinants that lead an SME to decide to issue minibond, and the financial strategies that can be implemented can help identify potential issuers and provide insights for policy makers. In addition, identifying the consequences of issuing minibonds may encourage potential issuers to take advantage of this source of debt diversification. Analysis of what happens on and around the date of issue should provide a better understanding of the value of tapping into the minibond market for SMEs.

Results go in the direction of understanding relevant features of the minibonds market. Furthermore, we obtain insights on the level of similarity of the companies issuing minibonds, the effect of such a financial strategy on the considered variables and the potential determinants of minibonds issuance, raising awareness of the decision-makers on the usefulness of such a financial instrument.

To the best of our knowledge, this is the first paper advancing the study of firms' life when introducing minibonds in the context of a complex network.

The paper is organised as follows. Section 2 presents the selected sample by explaining in detail the characteristics chosen for both sets and some descriptive statistics. Section 3 provides some preliminary information on the notation, similarity approach, and the adopted centrality measures. Section 4 reports the obtained results, while Section 5 carries out a discussion of the empirical findings. The last section 6 offers some conclusive comments and remarks.

2 Data

The theoretical network models, as specified above, are implemented through an empirical study based on datasets referring to Italian firms. The empirical data refer to a group of firms that share minibond issuance in the reference year 2018. The dataset is based on the information provided by the annual report dedicated to minibonds and published by Politecnico di Milano (see [35]). The sample includes 176 enterprises of small, medium and large size. The number of issuances of minibonds is equal to 198. To obtain the financial variables, we retrieved the financial statements of the upstream and downstream issuing firms through Bureau Van Dijk's (BVD)-AIDA, which collects a broad set of information – including the financial statements – of over $500,000$ Italian companies. The reference period is 2016-2019, thus before, at and after minibonds issuance. This phase has reduced the cardinality of the sample to 94 companies in that not all the considered firms provide complete documentation to BDV-AIDA. We have collected seven variables observed from 2016 to 2019 and five at in the issuing time[1]. For the list and the description of the considered financial variables, see Subsection 2.1. As a result, we have 33 different networks, each of them having 94 nodes (see Section 3 for the financial network model).

2.1 Firm variables

The considered variables are divided into two sets as preannounced in the Introduction. In set 1, we have the companies' characteristics at the issuance date of 2018; in set 2, we have the features that are recorded over the years 2016-2019.

In set 1 we have:

- *Age* is the number of the years from the foundation of the firms and the issuance year 2018. It is denoted by AGE_2018.
- *Amount* is the amount of the bond issued in 2018. It is expressed in millions of euro. Multiple issuances by the same company in 2018 have been aggregated. AM_18 denotes the amount issued in 2018.
- *Minibond interest rate* represents the rate at which investors are remunerated; it is expressed as a percentage. Multiple issuances in 2018 have been aggregated through a simple weighted average in which the weights

[1] Through personal communication with Giancarlo Giudici - who is the scientific director of the Mini-Bond Observatory – we have been able to add more variables at the time of issuance, such as applied interest rate, maturity and amount issued by companies in 2018.

	Maturity		
	<1	1-5	>5
First quarter	1.87	1.32	2.80
Second quarter	1.83	1.77	2.91
Third quarter	2.02	1.61	2.78
Fourth quarter	1.85	1.70	2.60
MEAN	1,89	1,60	2,77

Table 1: The table shows the values in percentage of the average interest rates applied to all the Italian companies in the quarters of 2018. The source of the data is Banca d'Italia; specifically, the data were collected from [38]. The last row is the average of the quarterly interest rates applied in 2018.

are proportional to the amount of minibonds issued. MIR_18 denotes the minibond interest rate in 2018.

- *Spread* is given by the difference between bank interest rate applied in 2018 to companies and minibond interest rate. It is expressed in percentage. To make the measure more homogeneous, we have considered three different levels of bank interest rate, as shown in table 1. Specifically, we consider maturities smaller than 1, between 1 and 5, and larger than 5 years (see [38]). We take the average interest rates charged to firms for outstanding transactions in the respective quartiles and average them. We subtract from these values the interest rate applied to the minibonds' issuance, according to their respective maturities. The spread in 2018 is denoted by SPR_18.
- *Maturity* is the number of the years from the firms' emission to the natural expiration date of the security. In the case of multiple issuances, the data are aggregated according to a weighted average, where the weights are proportional to the amount of minibonds issued. MY_18 denotes the maturity of minibond issued in 2018.

In set 2 we have:

- *Size* collects the total assets of the issuing companies; it is expressed in millions of euro. TA_yy denotes the size variable, in the year 20yy.
- *GrowthOpportunity* is given by the growth rate of sales – or sale variations; it is expressed in percentage. GO_yy stands for sales variations in year 20yy.
- *Collateral* is given by the ratio between tangible assets and total assets. It is a proxy for the guarantees offered by the issuing companies. COL_yy denotes collateral in the year 20yy.
- *Leverage* is given by the ratio between financial debts and total assets. This index expresses the company's level of indebtedness. LEV_yy is leverage variable in the year 20yy.
- *Profitability* is measured as the ratio between earnings before interests, taxes, depreciation and amortization (EBITDA) – which is one of the

main measures used to assess economic health of companies – and the total assets. It is expressed in percentage. This ratio allows to measure how profitable a company is before considering leverage. As pointed out by [39], considering EBITDA limits the impact of potential accounting manipulations. PR_yy denotes the profitability in the year 20yy.

- *Risk Firm* is proxied by the volatility of the profitability. It is measured as the absolute difference between annual profitability of a given firm i in year t and the average annual profitability of a firm i across the sample period (see [39]). FR_yy denotes the variable firm risk, in the year 20yy.
- *Liquidity* is given by the ratio between the current assets and the current liabilities. LIQ_yy denotes liquidity variable in the year 20yy.

The summary of the considered variables is given in Table 2.

VARIABLE	DEFINITION OR PROXY	UNIT OF MEASURE	REFERENCE YEARS
AGE_18	seniority	years	2018
AM_18	emissions in 2018	millions of €	2018
MIR_18	minibond interest rate	dimensionless, a percentage	2018
SPR_18	bank interest rate - MIR	dimensionless, a percentage	2018
MY_18	minibond maturity	years	2018
TA_yy	total assets in year yy	millions of €	2016–2019
GO_yy	sales growth in year yy	dimensionless, a percentage	2016–2019
COL_yy	tangible assets/total assets in year yy	millions of €	2016–2019
PR_yy	EBITDA/total assets in year yy	dimensionless, a percentage	2016–2019
FR_yy	volatility of profitability in year yy	dimensionless, a real number	2016–2019
LIQ_yy	current assets/current liabilities in year yy	dimensionless, a percentage	2016–2019
LEV_yy	financial debt/total assets in year yy	dimensionless, a percentage	2016–2019

Table 2: Variables' summary

Table 3 shows the main descriptive statistics for the considered variables over the sample of firms. We here provide a few comments on such descriptive statistics.

The variables belonging to set 1 show a controversial pattern. On average, minibond interest rates and spreads are rather stable, with standard deviations close to zero and means around 4 (MIR_18) and -2 (SPR_18). Much more scattered are the distributions of age, amount and maturity, with larger values of the standard deviation compared to the means. Regarding their distribution, we see that all the variables are leptokurtic; however, minibond interest rate, spread, and age are not too far from a normal distribution, with kurtosis and skewness close to zero. Differently, amount and maturity show positive non-negligible skewness, hence having a tail on the right.

The variables belonging to set 2 present sometimes marked deviations and quite regular behaviour in other cases.

The variables of the group TA_yy show a high level of variability – with a large variation range and standard deviation. Compared to the mean, the standard deviations of the variables labelled by COL_yy, PR_yy and FR_yy are rather large, while the remaining variables exhibit lower variations on average. The group variables in PR_yy are more diversified than the previous

VARIABLES	MEAN	ST.D.	KURT.	SKEW.	V.C.	MIN	MAX
MIR_18	4,21	0.02	0.14	0.11	0.38	0.05	8.5
AM_18	21.95	55.43	12.03	3.52	2.52	0.40	300.00
SPR_18	-1.97	0.02	0.29	-0.17	-0.85	-6.90	1.79
MY_18	5.23	2.77	8.02	1.41	0.53	0.50	20.02
AGE_18	29.40	16.31	0.96	0.78	0.55	5.00	88.00
TA_19	733.37	5160.54	91.31	9.50	7.03	5.86	49887.23
TA_18	709.46	5042.33	91.53	9.52	7.10	5.94	48764.71
TA_17	677.15	4902.60	91.82	9.54	7.24	5.39	47436.54
TA_16	670.49	4937.41	92.19	9.56	7.36	4.65	47807.99
GO_19	3.95	0.21	10.24	2.15	5.22	-53.95	116.02
GO_18	8.55	0.28	18.47	2.52	3.32	-96.89	188.25
GO_17	124	10.86	93.92	9.69	8.72	-97.43	10535
GO_16	24.5	1.60	90.62	9.44	6.53	-34.82	1548.3
COL_19	0.21	0.19	0.84	1.14	0.94	0.00	0.85
COL_18	0.20	0.19	0.77	1.17	0.96	0.00	0.83
COL_17	0.20	0.20	1.48	1.37	0.98	0.00	0.81
COL_16	0.21	0.21	1.87	1.46	1.01	0.00	0.94
PR_19	6.09	5.28	4.88	-0.81	0.87	-18.66	22.14
PR_18	6.60	4.60	1.05	0.62	0.70	-3.17	21.47
PR_17	6.94	5.26	1.71	0.81	0.76	-3.27	23.55
PR_16	7.16	7.53	6.54	-0.43	1.05	-29.26	33.42
FR_19	2.02	2.81	7.43	2.62	1.39	0.00	15.19
FR_18	1.32	1.61	6.51	2.30	1.22	0.02	8.46
FR_17	1.50	1.87	7.97	2.60	1.25	0.00	10.07
FR_16	2.43	4.17	17.09	3.79	1.72	0.01	27.60
LIQ_19	1.42	0.80	6.99	2.20	0,56	0.34	5.23
LIQ_18	1.53	0.89	9.42	2.66	0.58	0.34	6.00
LIQ_17	1.37	0.79	8.90	2.59	0.58	0.27	5.46
LIQ_16	1.58	1.80	56.57	6.90	1.14	0.21	16.83
LEV_19	0.41	0.17	-0.61	0.10	0.41	0.07	0.82
LEV_18	0.40	0.16	-0.39	0.04	0.40	0.08	0.86
LEV_17	0.37	0.16	-0.52	0.06	0.43	0.03	0.82
LEV_16	0.54	0.24	0.88	0.30	0.45	0.04	1.38

Table 3: Descriptive statistics are respectively mean (MEAN), standard deviation (ST.D.), kurtosis (KURT.), skewness (SKEW.), coefficient of variability (V.C.), minimum (MIN) and maximum (MAX).

ones, with large variations over the years of the main statistical indicators. We also observe a significant kurtosis value for TA_yy – which stands for very spiky distributions of such a variable over the years. Moreover, we generally have different patterns over the years for some specific variables. For instance, we notice that GO_17 has a mean and standard deviation much larger than those in GO_16, which is much larger than those in GO_18 and GO_19. For what concerns the kurtosis and the skewness, GO_16 and GO_17 are around 90 and 10, respectively, while GO_18 and GO_19 are less than 20 and around 2, respectively. Also, the standard deviation of LIQ_16 is more than double that of LIQ_17, LIQ_18, and LIQ_19, and similar behaviour is observed for kurtosis (56.57 versus values between 7 and 9) and the skewness (6.90 versus values slightly greater than 2).

3 Financial network model

The methodological approach adopted for this study is grounded on complex networks, with specific reference to undirected weighted network context.

We start from a graph $G = (V, E)$, where V represents the set of n nodes and E is the set of m links connecting couples of nodes.
From now on we will denote the generic node by i, with $i \in V$ or similarly $i = 1, ..., n$, and the link (i, j) denotes the connection between nodes i and j.
In our context, the links are captured by the n-square weighted adjacency matrix $\mathbf{W} = (w_{ij})_{i,j}$, with $w_{ij} = 0$ when $(i, j) \notin E$ – i.e., there is not a link between i and j – and $w_{ij} > 0$ otherwise. The weights in $\mathbf{W}$ carry out two levels of information: on one side, a positive weight w_{ij} certifies the existence of link (i, j); on the other side, the value of the weight explains the entity of the related link; specifically, the larger the value of w_{ij}, the stronger the connection between nodes i and j. We also assume that $\mathbf{W}$ is symmetric – so that, the links are not directed – i.e., $w_{ij} = w_{ji}$, for each $i, j \in V$. The resulting network is $N = (V, \mathbf{W})$.
In our empirical investigation, the set V collects the firms of the considered sample. We denote the generic variable in set 1 (5 variables) and set 2 ($7\times4 = 28$ variables) by f, so that f can take 33 values. For each firms variable f and for year x, we build a network $N^{(f,x)} = (V; \mathbf{W}^{(f,x)})$. The weighted adjacency matrix $\mathbf{W}^{(f,x)}$ is constructed by taking the similarity of the nodes with respect to the variable f at year x as the criterion for the construction of the weights. In this respect, a preliminary step is given by the standardization of the values of each variable f over the considered companies. Such a data pre-treatment phase lets the comparability of all the considered variables be possible. With a reasonable abuse of notation, we will refer to the standardized value of the generic variable by f, hereafter.
Now, we fix a variable f. We define the distance between two companies $i, j \in V$ in terms of f as $d(i, j)$. We take that the connection between nodes i and j is strong when such firms are quite similar, which means that the distance d_{ij} is small. Thus, to capture the entity of the connection between i and j, we define

$$w_{ij}^{f,x} = \begin{cases} \frac{1}{d_{ij}+1} & \text{for } i \neq j; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

By (1), we have that the considered networks are full.
By definition of the concept of distance d, we know that $w_{ij}^{f,x}$ in eq. (1) ranges in $[0, 1]$. In particular, if $i \neq j$, we can say that the weight of (i, j) is maximum and assumes unitary value when $d(i, j) = 0$, i.e. when $f_i = f_j$. Differently, $w_{ij}^{f,x}$ decreases and approaches 0 when the connection between i and j becomes weaker – but it cannot be null when $i \neq j$. The case $w_{ij}^{f,x} = 0$ for $i = j$ means the absence of self-connections/loops.
In the empirical experiments, we take

$$d_{ij} = |f_i - f_j|, \tag{2}$$

where f_i and f_j are the values of indicator f for companies $i, j \in V$, respectively. The considered networks are explored in terms of their heterogeneity and cohesiveness. To this aim, we exploit two nodal centralities: the degree centrality measure – called strength degree for the case of weighted networks, which is the one we treat here – and the clustering coefficient.

In the following, we briefly describe such centrality measures.

The strength degree centrality of a node $i \in V$ in a weighted undirected network counts the number of the nodes which are adjacent to i, by including also the links' weights (see [40, 41]).

In the case of weighted network $N^{(f,x)}$, the strength degree centrality of a node $i \in V$ is defined as :

$$k_i = \sum_{k \in V} w_{ik}^{f,x} \tag{3}$$

According to the definition of the weights in (1), a large value of k_i means that node i is quite similar to the other companies of the network, having a small distance from them in terms of the value of the standardized financial variable f. Therefore, a network whose nodes have substantially high strength degrees appears to be homogeneous for the considered variable f. Conversely, low strength degrees are associated to marked distances among the companies – hence, leading to a more scattered and heterogeneous population.

The (local) clustering coefficient C_i of a node $i \in V$ concerns the level of embeddedness of i in the surrounding environment of its adjacents – in the whole network, in our context. It is the ratio between the existing triangles formed by the links around i and the hypothetical ones, where triangles are evaluated by including also the weights of the links. Therefore, the local clustering coefficient of a node ranges between 0 and 1, being close to 1 when the cohesiveness around i is strong and close to 0 in the opposite case of weak embeddedness.

For a weighted network $N^{(f,x)}$, the clustering coefficient C_i for a node $i \in V$ is defined as follows (see [42]):

$$C_i = \frac{1}{(n-1)(n-2)} \sum_{j,k} \left(w_{ij}^{f,x} w_{jk}^{f,x} w_{ki}^{f,x} \right)^{\frac{1}{3}}. \tag{4}$$

4 Results

Tables 4 and 5 report the descriptive statistics of strength degree and clustering coefficient in all the considered networks, respectively.

VARIABLES	MEAN	ST.D.	KURT.	SKEW.	V.C.	MIN	MAX	V.% M.
TA_16	87.89	9.47	56.13	-7.20	0.11	8.73	90.01	
TA_17	87.77	9.55	54.52	-7.11	0.11	8.74	89.94	-0.13
TA_18	87.58	9.62	52.91	-7.00	0.11	8.75	89.81	-0.22
TA_19	87.47	9.66	52.09	-6.95	0.11	8.75	89.73	-0.12
PR_16	54.96	10.36	2.34	-1.63	0.19	16.11	63.48	
PR_17	52.28	8.83	1.98	-1.52	0.17	24.17	60.07	-4.87
PR_18	51.35	8.34	1.37	-1.32	0.16	23.33	59.38	-1.78
PR_19	53.05	9.04	2.81	-1.67	0.17	16.60	60.56	3.31
LIQ_16	68.53	11.03	10.71	-2.99	0.16	9.75	75.52	
LIQ_17	58.35	11.44	3.44	-1.88	0.20	15.40	67.34	-14.86
LIQ_18	58.13	11.23	3.49	-1.84	0.19	15.84	67.16	-0.38
LIQ_19	55.86	10.06	4.21	-2.01	0.18	16.49	63.65	-3.90
LEV_16	51.15	7.55	2.59	-1.48	0.15	21.91	58.42	
LEV_17	49.98	5.86	2.94	-1.65	0.12	26.32	55.52	-2.28
LEV_18	50.21	6.67	0.95	-1.04	0.13	25.92	57.45	0.45
LEV_19	49.82	6.16	0.90	-1.30	0.12	29.90	55.38	-0.77
GO_16	81.94	9.06	47.48	-6.30	0.11	8.77	85.65	
GO_17	89.40	8.48	90.96	-9.47	0.09	8.70	90.87	9.11
GO_18	60.18	11.91	3.23	-1.83	0.20	12.70	69.31	-32.68
GO_19	56.70	10.76	3.30	-1.81	0.19	14.64	65.25	-5.79
FR_16	65.11	12.93	5.29	-2.39	0.20	13.35	72.77	
FR_17	58.88	11.14	4.74	-2.32	0.19	17.46	65.78	-9.57
FR_18	58.01	11.37	2.45	-1.76	0.20	17.88	66.24	-1.47
FR_19	61.09	12.81	3.00	-1.95	0.21	16.86	69.68	5.31
COL_16	54.32	9.63	2.24	-1.68	0.18	22.18	62.13	
COL_17	53.74	9.22	2.31	-1.74	0.17	24.36	61.06	-1.08
COL_18	53.12	8.88	1.46	-1.47	0.17	22.64	60.83	-1.14
COL_19	52.69	8.35	2.26	-1.71	0.16	22.87	59.10	-0.80
AM_18	74.02	15.81	6.42	-2.77	0.21	16.01	80.95	
AGE_18	51.17	6.82	5.36	-2.16	0.13	21.11	56.52	
MIR_18	50.63	7.31	1.93	-1.55	0.14	27.21	56.79	
MY_18	56.29	10.73	1.33	-1.30	0.19	14.81	64.82	
SPR_18	50.94	7.91	1.04	-1.20	0.16	25.15	59.06	

Table 4: Summary statistics for the strength degree of the considered variables. The presented descriptive statistics are mean (MEAN), standard deviation (ST.D.), kurtosis (KURT.), skewness (SKEW.), coefficient of variability (V.C.), minimum (MIN), maximum (MAX), and mean's variation (V. % M.).

As shown in Table 4, looking at the means, we notice that the variables in TA_yy have large values that remain constant over the period considered, around 87. The other variables have lower values of the mean; we mention PR_yy (between 51 and 55), LIQ_yy (between 55 and 69), LEV_yy (between 49 and 52), FR_yy (between 58 and 66) and COL_yy (between 52 and 55). Interestingly, GO_yy has a large variation range (with extremes 56.70 and 81.94) over the years. Regarding the variables in set 1, AM_18 has a value of the mean (about 74) that is larger than the others, with AGE_18 (about 51), the MIR_18 (about 50), MY_18 (about 56) and SPR_18 (about 50).

If we look at the standard deviation, we notice that its value remains concentrated in a narrow set, ranging in the majority of the cases between 8 and 10, with a few upper and lower deviations from this interval. The minimum

VARIABLES	MEAN	ST.D.	KURT.	SKEW.	V.C.	MIN	MAX	V.% M.
TA_16	0.94	0.08	64.78	-7.76	0.09	0.20	0.96	
TA_17	0.94	0.08	62.86	-7.65	0.09	0.20	0.95	-0.14
TA_18	0.93	0.09	61.23	-7.54	0.09	0.20	0.95	-0.23
TA_19	0.93	0.09	60.29	-7.49	0.09	0.20	0.95	-0.13
PR_16	0.57	0.08	3.15	-1.78	0.14	0.26	0.63	
PR_17	0.54	0.07	2.31	-1.60	0.12	0.32	0.60	-5.17
PR_18	0.53	0.06	1.67	-1.39	0.12	0.32	0.59	-1.83
PR_19	0.55	0.07	3.61	-1.80	0.12	0.26	0.61	3.53
LIQ_16	0.72	0.09	15.07	-3.48	0.12	0.20	0.78	
LIQ_17	0.61	0.09	4.49	-2.10	0.15	0.25	0.67	-15.95
LIQ_18	0.61	0.09	4.73	-2.08	0.14	0.26	0.67	-0.40
LIQ_19	0.58	0.08	5.26	-2.20	0.13	0.26	0.64	-3.94
LEV_16	0.53	0.06	2.58	-1.49	0.11	0.31	0.58	
LEV_17	0.52	0.04	2.43	-1.50	0.09	0.34	0.56	-2.52
LEV_18	0.52	0.05	0.74	-1.01	0.10	0.34	0.57	0.53
LEV_19	0.52	0.05	0.70	-1.23	0.09	0.37	0.56	-0.75
GO_16	0.87	0.08	58.83	-7.16	0.09	0.20	0.90	
GO_17	0.96	0.08	92.21	-9.56	0.08	0.20	0.97	9.31
GO_18	0.63	0.09	4.71	-2.09	0.15	0.23	0.70	-34.13
GO_19	0.59	0.08	4.45	-2.02	0.14	0.24	0.65	-6.00
FR_16	0.68	0.10	6.69	-2.62	0.15	0.24	0.74	
FR_17	0.61	0.09	5.69	-2.48	0.14	0.27	0.67	-9.99
FR_18	0.60	0.09	3.34	-1.95	0.15	0.27	0.66	-1.83
FR_19	0.64	0.10	3.84	-2.13	0.16	0.27	0.70	5.64
COL_16	0.56	0.07	2.76	-1.81	0.13	0.31	0.62	
COL_17	0.56	0.07	2.78	-1.85	0.13	0.32	0.61	-1.06
COL_18	0.55	0.07	1.81	-1.58	0.12	0.31	0.60	-1.31
COL_19	0.54	0.06	2.57	-1.78	0.12	0.31	0.59	-0.78
AM_18	0.77	0.13	7.14	-2.89	0.17	0.28	0.83	
AGE_18	0.53	0.05	5.42	-2.12	0.10	0.30	0.57	
MIR_18	0.52	0.05	1.89	-1.53	0.10	0.35	0.57	
MY_18	0.58	0.08	1.96	-1.43	0.14	0.25	0.65	
SPR_18	0.53	0.06	1.05	-1.21	0.11	0.33	0.59	

Table 5: Summary statistics for weighted clustering coefficient of the considered variables. The presented descriptive statistics are mean (MEAN), standard deviation (ST.D.), kurtosis (KURT.), skewness (SKEW.), coefficient of variability (V.C.), minimum (MIN), maximum (MAX), and mean's variation (V. % M.).

value is recorded for the variable LEV_17 (5.86) and the maximum for AM_18 (15.81). The coefficient of variation states a rather low volatility; such an indicator reaches its maximum value for AM_18 and FR_19 (0.21). This shows that the variability is low, and the means can be considered a good indicator. Finally, looking at the kurtosis and symmetry values, we can see that none of the strength degree distributions behave similarly to a normal distribution. In fact, all the variables are leptokurtic and asymmetric with left-tail– with cases of exceptionally large kurtosis, like the variables TA_yy (between 52.09 and 56.13) and GO_16 and GO_17 (with values 47.48 and 90.96, respectively). However, it is worth specifying that many distributions do not suffer from a pronounced kurtosis, being in several cases in the presence of a value close to 0 – like for the variables LEV_18, LEV_19 and SPR_18. Even some variables have a level of skewness not too far from zero – examples are LEV_18 and SPR_18.

Table 5 shows the descriptive statistics related to the clustering coefficient. By looking at the column of the mean, we notice a very high value for the variables TA_yy (with values 0.93 and 0.94, so rather close to 1). For the other features we observe smaller values: PR_yy (between 0.53 and 0.59), LIQ_yy (between 0.58 and 0.72), LEV_yy (between 0.52 and 0.53), FR_yy (between 0.61 and 0.69), COL_yy (between 0.54 and 0.56). As in the previous case, we want to emphasize the large range shown by GO_yy (between 0.59 and 0.96, with an increasing trend from 2016 to 2019).

The variables in set 1 have an average clustering coefficient of 0.77 for AM_18, 0.53 for AGE_18, 0.52 for MIR_18, 0.58 for MY_18 and 0.52 for SPR_18.

As for the case of strength degree centrality, we have a relatively small standard deviations for all the considered variables – concentrated for the most part between 0.05 and 0.09 – with the maximum value recorded in AM_18 (0.13) and the minimum value for the variable LEV_17 (0.04). Interestingly, the maximum and minimum values are taken by the same variables observed in Table 4. So is the maximum value of the coefficient of variation, which is still recorded by AM_18 (about 0.17). These values confirm the low variability of the values of the variables over the years for the clustering coefficient – hence, confirming the pattern observed in the case of strength degree. By looking at the various levels of kurtosis and skewness, we observe also in this case a substantial departure from the normal distribution – all the distributions are leptokurtic and asymmetric with left-tail – even with some variables having kurtosis close to zero (e.g., LEV_18, LEV_19 and SPR_18) or skewness not far from zero (e.g., LEV_18 and SPR_18). Thus, we infer a regularity of the variables close to a normal distribution. We point out that kurtosis assume exceptionally large value for TA_yy (between 60.29 and 64.78) and GO_16 and GO_17 (with values 58.83 and 92.21, respectively) – and also in this we find an analogy between strength degree and clustering coefficient.

To provide a global view of the connections between the statistics of strength degrees and clustering coefficients, see Figures 1–4. Such figures compare the means and standard deviations of the 33 variables of interest.

Figure 1 shows how the relationship between the means of the strength degrees and the clustering coefficients is almost perfectly linear. This finding supports that highly homogeneous companies in the selected sample are embedded in a robust and cohesive community.

Figure 2 compares the standard deviations of the two reference nodal centrality measures. Also, in this scenario, we find a confirmation of a statistically satisfactory linear relation between such measures. Nevertheless, this linearity is less evident than in the case of the means.

Figures 3 and 4 compare the means and the standard deviations of strength degrees and clustering coefficients, respectively. The two plots are very similar; in both of them, we see a marginally decreasing volatility as the means of the centrality measures increase.

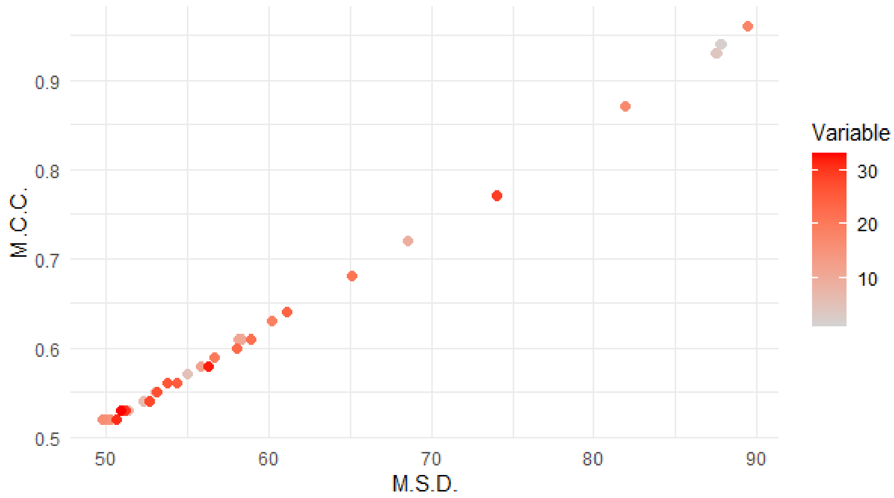

Figure 1: On the x-axis we have the strength degrees' means (M.S.D.) and on the y-axis the clustering coefficients' means (M.C.C.) for the 33 variables considered.

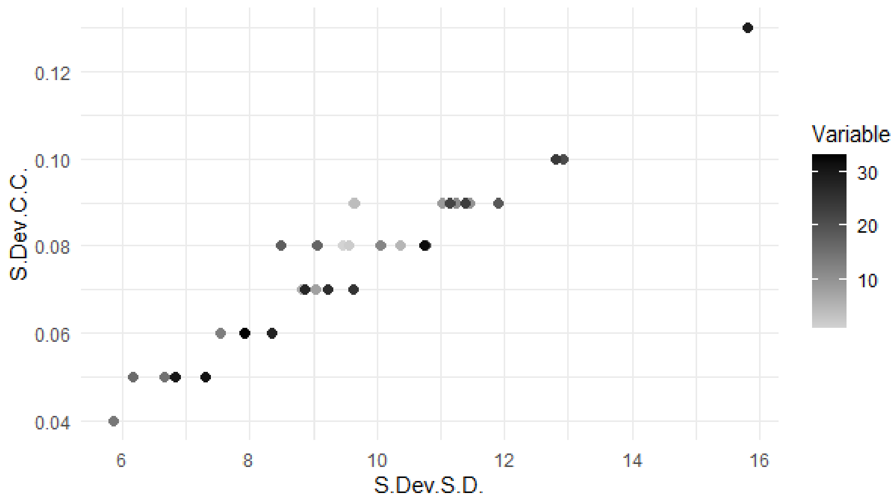

Figure 2: On the x-axis we have the strength degrees' standard deviations (S.Dev.S.D.) and on the y-axis the clustering coefficients' standard deviations (S.Dev.C.C.) for the 33 variables considered.

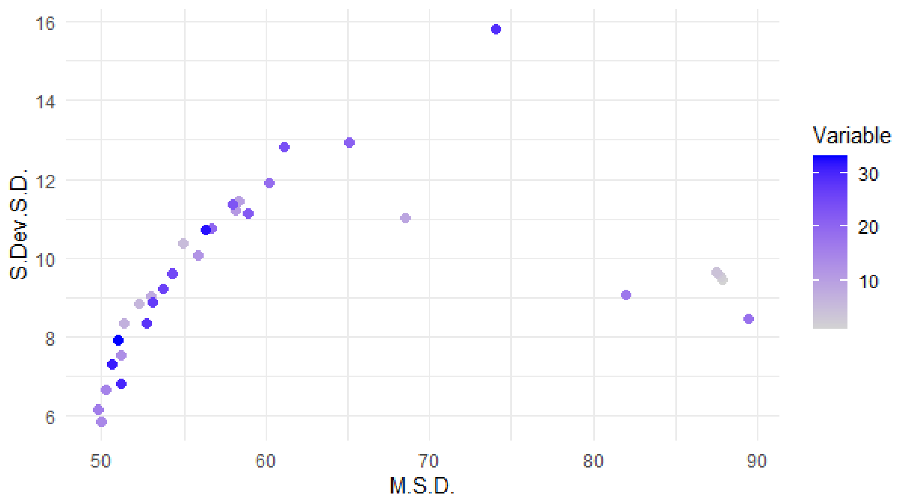

Figure 3: On the x-axis we have the strength degrees' means (M.S.D.) and on the y-axis the strength degrees' standard deviations (S.Dev.S.D.) for the 33 variables considered.

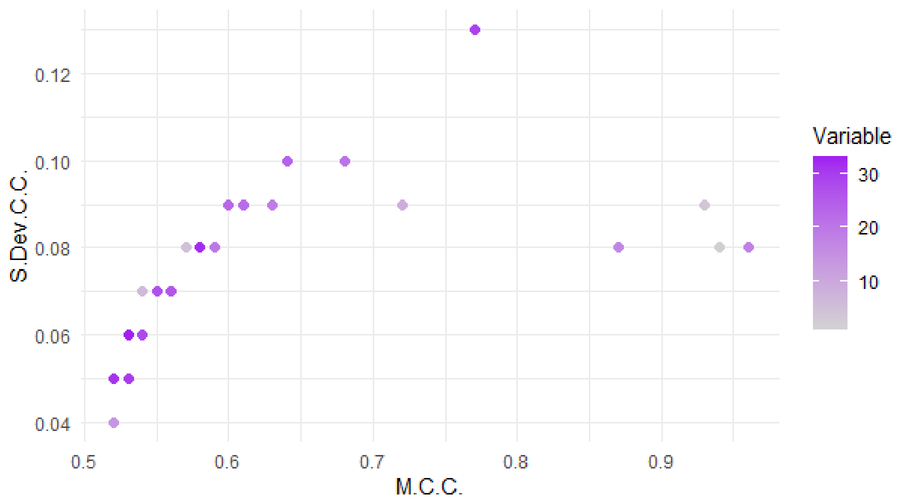

Figure 4: On the x-axis we have the clustering coefficients' means (M.C.C.) and on the y-axis the clustering coefficients' standard deviations (S.Dev.C.C.) for the 33 variables considered.

5 Discussion of the results

Results have a clear financial interpretation.

The mean strength degree from 2016 to 2019 related to TA_yy remains almost unchanged. This means that companies were very similar in terms of total assets before, at and after the issuance of minibonds. Therefore, minibond issuances did not lead to a disjointness of the homogeneity found in size. Specifically, the introduction of this instrument does not lead to a disarticulation (heterogeneity) of the network. This result goes in the direction of the outcomes of [25], where the authors find a general increase in the total assets of issuing companies in the years following minibonds issuance. Moreover, the oscillation of homogeneity for the standard deviation is relatively small in all four years compared to the mean. By observing the level of skewness and kurtosis, we do not notice significant fluctuations in their values over time. There is a remarkably large kurtosis – always greater than 50 – and negative skewness. A large kurtosis witnesses a very high concentration around the mean and thick tails. If we also consider the strong negative skewness – around -7 – we see a very sharp strength degree distribution, with a very elongated and thick left tail. The most straightforward reason behind the high kurtosis could be the existence of outliers, underlining the presence of some firms with very different sizes from the others. This outcome is reasonable; it is in line with our sample, which includes SMEs and a few large companies.

Also, for the variables PR_yy, all the descriptive statistics remain relatively stable over time. The main differences between this variable and TA_yy are the smaller networks' homogeneity level and the larger oscillations. We can say that the issuance of minibonds does not lead to changes in terms of homogeneity/heterogeneity of networks for the profitability variable over time. This general stability may be due to the possibility that the minibonds-based investments could be not associated with immediate significant changes in profitability. We notice a positive but small kurtosis and a negative skewness in this case for all the years. The result is a more pointed distribution with an elongated and thicker left tail. In this case, the kurtosis might be generally smaller also because we do not report outliers in the sample for this variable.

Regarding the proxy of Liquidity (LIQ_yy), we find a rather high mean level of homogeneity that decreases over time, moving from 68.53 in 2016, with a jump to 58.35 in 2017 and ending to 55.86 in 2019. Therefore, we have a reduction of the mean in the quadriennium of around 19%. This underlines a tendency of the sample firms to an increasing level of heterogeneity. In other words, on average, the firms that use this financial instrument present subsequently increasing dishomogeneous levels of liquidity. We can affirm that this trend may result from a strategic approach. For example, some companies might decrease this ratio a priori to subsequently transform short-term debt into long-term debt by issuing minibonds. In contrast, others might decide to issue this short-term financial instrument because they have excellent liquidity. Indeed the minibond maturity can be short-, mid-and long-term. On the other hand, if we look at the level of homogeneity, we note that firms are not so similar according to LIQ_yy. Our distribution is always leptokurtic and has negative skewness.

LEV_yy denote the lowest mean value of homogeneity according to the strength degree centrality measure. In fact, in 2019, the average level is around 49. If we look at the trend over time, we see substantial stability. We can deduce from this result that minibonds do not affect the level of heterogeneity of firms in the time span considered. Considering the previous result, we can say that issuing companies tend mainly to replace outstanding debt and leave their level of indebtedness unchanged. This seems to be in line with the regulator's target of decreasing the dependence of Italian firms on the banking system (see [25]). For Leverage, we find a negative skewness and leptokurtic distribution of node centralities, but this distribution is quite close to the normal one. In fact, we have the lowest value of kurtosis and skewness.

Looking at the next proxy (GO_yy), we see that it undergoes the most significant variation in node mean homogeneity. The Growth opportunity registers a mean strength value of around 89 in 2017 and drops to a minimum of 56 in 2019. The relative reduction recorded between the maximum and minimum value is equal to 37%. Thus, the average sales growth value in the years preceding the issuance is very similar among firms, changing markedly in the year of the issuance and afterwards. Therefore the effect of this financial instrument would seem to have an important impact on the growth of average sales. The economic interpretation of this result suggests a particular specification. In fact, the decline of homogeneity underlines different business strategies. On the one hand, some companies may pursue highly risky projects, such as mergers and acquisitions. On the other hand, we can have less risky targets, such as working capital funding or debt restructuring (see, e.g. [43]). Also, in this case, the average fluctuations of the strength degrees are not so high – even if they present larger values in 2018 and 2019, i.e. at and after the issuance. The level of kurtosis recorded in 2016 and 2017 suggests that there are anomalous values of sales growth in the mentioned years.

For the proxy of firm risk (FR_yy), the average level of homogeneity is medium-high in 2016; in fact, it is around 66. Therefore, firms have a similar overall level of risk. We record a reduction in the year preceding the issuance of around 10%. This outcome might have a double interpretation. The financial instrument of minibond has become necessary for some firms because they are experiencing an increase in the risk level, or some entrepreneurs have rebalanced their firm to face the capital market in a more relaxed way. In 2018 we see stability in terms of heterogeneity, while in 2019, we observe an increase in homogeneity, which suggests that minibonds tend to let nodes be more similar in terms of the risk level. An additional interpretation concerning the low level of homogeneity is suggested by [24], who argue that non-creditworthy firms may be hidden in the set of minibonds issuers. Also, in this case, the standard deviation is not very high, leading to an acceptable level of diversification by maintaining at the same time a reliable degree of stability of the results.

Finally, we have COL_yy. The mean values of centrality are practically unchanged over time. We record a value of 54.32 in 2016, which decreases slightly and constantly to 52.69 in 2019. Such levels of homogeneity are not extremely large, suggesting the presence in the sample of firms that can offer very different guarantees. The average fluctuations around the mean are not so high also in this case, and the distributions over the years present a small kurtosis and a small negative skewness.

We now focus on the networks built on the characteristics in set 1, with the variables collected at the time of issuance 2018. The highest level of homogeneity is recorded with AM_18 – about 74 – with a large standard deviation. This goes in the direction of highlighting that there are companies with very different sizes in the sample. Moreover, this suggests that minibonds could be a test for novel financing strategies for some companies (using this instrument for the first time) or a way to make themselves known to the financial markets by issuing amounts of minibonds. As for the network built on the AGE_18, we observe a low level of homogeneity, suggesting that this instrument is transversal – i.e., it is used by both young and old companies. The last characteristics we observe in Table 4 are the interest rate of the minibonds (MIR_18), maturity (MY_18) and spread (SPR_18). These three characteristics are interconnected, especially MIR_18 and SPR_18. Therefore, it is intuitive and expected to see that the average centrality values of MIR_18 and SPR_18 are almost the same. In all these features, we notice a rather weak homogeneity showing that the issuance strategies and the risk of the individual firm (as also seen in the FR_yy feature in 2018) are rather heterogeneous.

Now, let us offer some details on the clustering coefficient in Table 5. As said above, this nodal centrality measure is within a range of $[0, 1]$, unlike the strength degree centrality. It expresses the ability of any couple of adjacents of a node i for a triangle with i. In other words, this network indicator allows underlining the influence of each node in generating links between the nodes close to it. Looking at the characteristics from 2016 to 2019, we realize that some patterns are somehow similar to those of strength degree centrality over the years. This might be explained in part because our network is full – even if the presence of the weights leads to remarkable discrepancies between strength and clustering coefficient.

We note some evident similarities among some specific variables. For example, the network constructed on the size (TA_yy) variables shows a very high and constant cluster coefficient over the observed period. This underlines that the 94 firms are already very cohesive and interconnected in terms of total assets before the minibond is issued and remaining so afterwards. Therefore, this result confirms the inability of this financial instrument to create significant shocks to the size of a company.

More noticeable effects are found for the Liquidity (LIQ_yy) and Growth opportunity (GO_yy) variables, in which the average level of interconnection after issuance is significantly reduced by 20% and 38% respectively. This confirms that the most significant effects on companies are realized in these two characteristics. Therefore, not only are firms less related to each other (as shown by strength degree), but nodes lose their ability to create strong links to their neighbours in terms of similarity with respect to the considered variable.

Although presenting different levels of clustering coefficient, all the other variables remain almost constant over time, creating networks and interconnections with a stable level of cohesiveness. Therefore, the use of minibonds does not lead to and generate destabilization of networks. The cohesiveness for LEV_yy, PR_yy and COL_yy present mean levels of medium entity, around 0,5 over time. The mean clustering coefficient of FR_yy is always above 0,6. In all cases, the standard deviation is extremely low. Hence – as for the case of the strength degree – the four mentioned variables do not show significant changes in the context of minibonds. Such a financial instrument is not in charge of variations of the community structure around the individual nodes.

Finally, looking at the variable belonging to set 1, we record a marked closeness with the results obtained through the strength degree centrality. The interconnectedness between the nodes is relatively high for the amount (AM_18) of minibonds issued and low for age (AGE_18), coupon paid (MIR_18), and maturity (MY_18) and spread (SPR_18). Thus, AM_18 is a variable that seems to link rather well not only individual nodes but also their neighbours, while this trend is weaker for the others.

To conclude, it is legitimate to affirm the existence of homogenization or heterogenization effects of the considered financial networks when minibonds are introduced. In particular, we observe that some characteristics such as liquidity (LIQ_yy) and growth opportunity (GO_yy) undergo significant effects following the decision. This financial strategy- essential for the company's life- is not extemporaneous but rather carefully evaluated in the years preceding the issue. It allows advancing a specific hypothesis concerning potential minibonds' determinants. Variables with a high level of homogeneity and cohesiveness over the pre-issue period indicate high levels of similarity among firms in terms of the considered proxies. Therefore, we believe that TA_yy, LIQ_yy, and GO_yy can be determinants because they are those variables that present higher values of strength degree and clustering coefficient. Furthermore, we want to underline the general tendency of the variables to decrease in terms of homogeneity and cohesiveness. In other words, the firms after the issue are more different from each other and less embedded in the network. Finally, we notice that, on average, nodes/firms that issue minibonds do not have very high homogeneity levels. This leads us to suppose that the firms interested in this kind of financial instrument can be different, all of them being predisposed and targeted to diversifying their funding sources.

6 Conclusions

This paper presents a new concept of heterogeneity and cohesiveness by exploiting the concept of similarity widely used in complex network analysis. In particular, we borrow two of the primary measures of network theory, the strength degree centrality and the clustering coefficient. The theoretical proposal is validated through an empirical analysis performed on a high-quality dataset, which is particularly appropriate to test the methodology. Specifically, we looked at the level of similarity among companies that financed their activities using an innovative financial instrument, namely minibonds. In this way, we were able to detect the levels of heterogeneity and cohesion among issuing firms.

In addition, a broader temporal observation allowed us to verify the variables most influenced by this choice and identify potential determinants. Indeed we can make some conjectures obtained indirectly from our results. Proxies with a high strength and clustering coefficient in the preparatory phase suggest a number of potential determinants. Interestingly, we observe a noticeable interconnection between the strength degree and the clustering coefficient. This could be due to the networks' nature, being complete and undirected – even if the presence of weights may also suggest discrepancies.

The analysis of the degree of heterogeneity and the degree of cohesion of the five networks about the characteristics at the date of the issue shows that the minibond market is aimed at companies that differ significantly in terms of risk perceived by the market and age. On the other hand, the amounts issued are much more homogeneous. The market thus seems to address companies that differ in terms of perceived risk and life cycle but are similar in terms of the amount of minibonds they issue.

The analysis of the evolution from 2016 to 2019 of the degree of heterogeneity and the degree of cohesion of the seven networks regarding the characteristics of the 94 companies forming the nodes of these networks allows us to identify two elements. Firstly, the financial strategy implemented and, secondly, the possible determinants of the decision to issue minibonds. On average, companies seem to issue minibonds to substitute debt and thus diversify their sources of debt without changing their financial leverage. The impact of this issue on companies' liquidity is not uniform. The same is true, and to a greater extent, for growth opportunities. This means that the reasons for companies to access this market are diverse. It can be a pure financial strategy or a financial strategy combined with an economic strategy. Therefore, the minibond market seems to meet the needs of SMEs and the reasons that led the public authorities to set it up. Thus, other SMEs should use this market to improve their debt structure, diversify their sources of financing and grow. Of course, all this will be possible if investors have confidence in this market.

Beyond the clear financial implications of our study, we also observe that the versatility of our method allows scholars to apply it in several real-world instances – not only in finance.

We have one main limitation in this study based on our analysis's structural characteristics. Indeed, heterogeneity and cohesiveness are global features of the networks. Therefore, it is not possible to infer the behaviours of the individual companies from them. In this respect, we present an analysis where a modification in the homogeneity or heterogeneity of the networks over time does not provide insights into the related changes in the companies' variables. From a different perspective, the global analysis of the companies issuing minibonds is worthy because it describes a universe – hence, leading to more general policies to be implemented for modifying the overall industrial structure of a set of companies in the light of some prefixed targets.

References

[1] R. Albert and A.-L. Barabási, "Statistical mechanics of complex networks," *Reviews of Modern Physics*, vol. 74, no. 1, pp. 47–97, 2002.

[2] M. E. Newman, "The structure and function of complex networks," *SIAM Review*, vol. 45, no. 2, pp. 167–256, 2003.

[3] A.-L. Barabasi and Z. N. Oltvai, "Network biology: Understanding the cell's functional organization," *Nature Reviews Genetics*, vol. 5, no. 2, pp. 101–113, 2004.

[4] L. Basnarkov, "SEAIR epidemic spreading model of COVID-19," *Chaos, Solitons & Fractals*, vol. 142, no. 110394, 2021.

[5] C. J. Silva, G. Cantin, C. Cruz, R. Fonseca-Pinto, R. Passadouro, E. S. Dos Santos, and D. F. Torres, "Complex network model for COVID-19: Human behavior, pseudo-periodic solutions and multiple epidemic waves," *Journal of Mathematical Analysis and Applications*, no. 125171, 2021.

[6] W. Li, H. Zhu, S. Li, H. Wang, H. Dai, C. Wang, and Q. Jin, "Evolutionary community discovery in dynamic social networks via resistance distance," *Expert Systems with Applications*, vol. 171, no. 114536, 2021.

[7] A. Zareie and A. Sheikhahmadi, "A hierarchical approach for influential node ranking in complex social networks," *Expert Systems with Applications*, vol. 93, pp. 200–211, 2018.

[8] N. Boers, B. Bookhagen, N. Marwan, J. Kurths, and J. Marengo, "Complex networks identify spatial patterns of extreme rainfall events of the South American monsoon system," *Geophysical Research Letters*, vol. 40, no. 16, pp. 4386–4392, 2013.

[9] J. F. Donges, Y. Zou, N. Marwan, and J. Kurths, "The backbone of the climate network," *EPL (Europhysics Letters)*, vol. 87, no. 48007, 2009.

[10] D. S. Bassett and E. Bullmore, "Small-world brain networks," *The Neuroscientist*, vol. 12, no. 6, pp. 512–523, 2006.

[11] L. K. Gallos, H. A. Makse, and M. Sigman, "A small world of weak ties provides optimal global integration of self-similar modules in functional brain networks," *Proceedings of the National Academy of Sciences*, vol. 109, no. 8, pp. 2825–2830, 2012.

[12] W.-Q. Huang, X.-T. Zhuang, and S. Yao, "A network analysis of the Chinese stock market," *Physica A: Statistical Mechanics and its Applications*, vol. 388, no. 14, pp. 2956–2964, 2009.

[13] X. Xi, X. Gao, J. Zhou, H. Zheng, J. Ding, and J. Si, "Uncovering the impacts of structural similarity of financial indicators on stock returns at different quantile levels," *International Review of Financial Analysis*, vol. 76, no. 101787, 2021.

[14] X. Xi and H. An, "Research on energy stock market associated network structure based on financial indicators," *Physica A: Statistical Mechanics and its Applications*, vol. 490, pp. 1309–1323, 2018.

[15] R. Cerqueti, R. Ciciretti, A. Dalò, and M. Nicolosi, "A new measure of the resilience for networks of funds with applications to socially responsible investments," *Physica A: Statistical Mechanics and its Applications*, no. 126976, 2022.

[16] S. R. de la Torre, J. Kalda, R. Kitt, and J. Engelbrecht, "On the topologic structure of economic complex networks: Empirical evidence from large scale payment network of Estonia," *Chaos, Solitons & Fractals*, vol. 90, pp. 18–27, 2016.

[17] S. P. Borgatti and X. Li, "On social network analysis in a supply chain context," *Journal of Supply Chain Management*, vol. 45, no. 2, pp. 5–22, 2009.

[18] C. R. Carter, L. M. Ellram, and W. Tate, "The use of social network analysis in logistics research," *Journal of Business Logistics*, vol. 28, no. 1, pp. 137–168, 2007.

[19] C. Fracassi, "Corporate finance policies and social networks," *Management Science*, vol. 63, no. 8, pp. 2420–2438, 2017.

[20] G. P. Clemente and A. Cornaro, "A multilayer approach for systemic risk in the insurance sector," *Chaos, Solitons & Fractals*, vol. 162, no. 112398, 2022.

[21] W.-Q. Huang and D. Wang, "Systemic importance analysis of Chinese financial institutions based on volatility spillover network," *Chaos, Solitons & Fractals*, vol. 114, pp. 19–30, 2018.

[22] Osservatorio Mini-Bond, "Report Italiano sui mini-bond," *Politecnico di Milano*, no. 1, pp. 1–82, 2015.

[23] E. I. Altman, M. Esentato, and G. Sabato, "Assessing the credit worthiness of Italian SMEs and mini-bond issuers," *Global Finance Journal*, vol. 43, no. 100450, 2020.

[24] M. Mietzner, J. Proelss, and D. Schweizer, "Hidden champions or black sheep? The role of underpricing in the German mini-bond market," *Small Business Economics*, vol. 50, no. 2, pp. 375–395, 2018.

[25] S. Ongena, S. Pinoli, P. Rossi, and A. Scopelliti, "Bank credit and market-based finance for corporations: The effects of minibond issuances," *CEPR Discussion Paper*, no. 1315, 2021.

[26] S. A. Sharpe, "Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships," *The Journal of Finance*, vol. 45, no. 4, pp. 1069–1087, 1990.

[27] R. Cerqueti, M. Cinelli, and L. F. Minervini, "Municipal waste management: A complex network approach with an application to Italy," *Waste Management*, vol. 126, pp. 597–607, 2021.

[28] A. Bavelas, "Communication patterns in task-oriented groups," *The Journal of the Acoustical Society of America*, vol. 22, no. 6, pp. 725–730, 1950.

[29] L. C. Freeman, "A set of measures of centrality based on betweenness," *Sociometry*, vol. 40, no. 1, pp. 35–41, 1977.

[30] P. Bonacich, "Factoring and weighting approaches to status scores and clique identification," *Journal of Mathematical Sociology*, vol. 2, no. 1, pp. 113–120, 1972.

[31] M. E. Newman, "Assortative mixing in networks," *Physical Review Letters*, vol. 89, no. 208701, 2002.

[32] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world'networks," *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.

[33] T. Opsahl, F. Agneessens, and J. Skvoretz, "Node centrality in weighted networks: Generalizing degree and shortest paths," *Social Networks*, vol. 32, no. 3, pp. 245–251, 2010.

[34] L. C. Freeman, "Centrality in social networks conceptual clarification," *Social Networks*, vol. 1, no. 3, pp. 215–239, 1978.

[35] Osservatorio Mini-Bond, "Report Italiano sui mini-bond," *Politecnico di Milano*, no. 5, pp. 1–84, 2019.

[36] E. F. Fama, "What's different about banks?," *Journal of monetary economics*, vol. 15, no. 1, pp. 29–39, 1985.

[37] ISTAT, "Censimento permanente delle imprese," 2019. https://www.istat.it/it/censimenti-permanenti/imprese.

[38] Banca D'Italia, "Banche e istituzioni finanziarie: Condizioni e rischiosità del credito per settori e territori," 2018. https://www.bancaditalia.it/pubblicazioni/condizioni-rischiosita/2018-condizioni-rischiosita/index.html.

[39] A. D'Amato, "Capital structure, debt maturity, and financial crisis: Empirical evidence from SMEs," *Small Business Economics*, vol. 55, no. 4, pp. 919–941, 2020.

[40] A. Barrat, M. Barthelemy, R. Pastor-Satorras, and A. Vespignani, "The architecture of complex weighted networks," *Proceedings of the National Academy of Sciences*, vol. 101, no. 11, pp. 3747–3752, 2004.

[41] T. Opsahl, V. Colizza, P. Panzarasa, and J. J. Ramasco, "Prominence and control: The weighted rich-club effect," *Physical Review Letters*, vol. 101, no. 168702, 2008.

[42] J.-P. Onnela, J. Saramäki, J. Kertész, and K. Kaski, "Intensity and coherence of motifs in weighted complex networks," *Physical Review E*, vol. 71, no. 065103, 2005.

[43] S. Boccaletti, E. Rossi, and M. Rossolini, "How can SMEs signal their quality and growth orientation to the market? An analysis of the cost of Italian corporate mini-bonds," *Journal of International Financial Management & Accounting*, 2022. https://doi.org/10.1111/jifm.12157.