

AKADÉMIAI KIADÓ



International Review of  
Applied Sciences and  
Engineering

13 (2022) 1, 54–62

DOI:  
[10.1556/1848.2021.00305](https://doi.org/10.1556/1848.2021.00305)  
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ORIGINAL RESEARCH  
PAPER



# Low cycle fatigue behavior of circumferentially notched specimens made of modified 9Cr–1Mo steel at elevated temperature

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Received: May 1, 2021 • Accepted: June 9, 2021  
Published online: August 31, 2021

## ABSTRACT

During service, notched designed components such as steam generators in the nuclear power plant usually experience fatigue damage at elevated temperatures, due to the repeated cyclic loadings during start-up and shut-down operations. Under such extreme conditions, the durability of these components is highly-affected. Besides, to assess the fatigue life of these components, a reliable determination of the local stress-strain at the notch-tips is needed. In this work, the maximum strains of circumferentially notched cylindrical specimens were calculated using the most commonly known analytical methods, namely Neuber's rule, modified Neuber's rule, Glinka's rule, and linear rule, with notch root radius of 1.25, 2.5, and 5 mm, made of modified 9Cr–1Mo steel at 550 °C, and subjected to nominal stress amplitudes of  $\pm 124.95$ ,  $\pm 149.95$ , and  $\pm 174.95$  MPa. The calculated local strains were compared to those obtained from Finite Element Analysis (FEA). It was found that all the analytical approximations provided unreliable local strains at the notch-tips, resulting in an overestimation or underestimation of the fatigue life. Therefore, a mathematical model that predicts the fatigue lives for 9Cr–1Mo steel at elevated temperature was proposed in terms of the applied stress amplitude and the fatigue stress concentration factor. The calculated fatigue lifetimes using the proposed model are found to be in good agreement with those obtained experimentally from the literature with relative errors, when the applied stress amplitude is  $\pm 149.95$  MPa, are of 1.97%, –8.67%, and 13.54%, for notch root radii of 1.25, 2.5, and 5 mm, respectively.

## KEYWORDS

circumferentially notched specimens, elevated temperatures, finite-element analysis, low cycle fatigue, modified 9Cr–1Mo steel

## 1. INTRODUCTION

In nuclear power plants, the vast majority of components experience low cycle fatigue damage during service, due to the repeated cyclic loadings that mainly occur during start-up and shut-down operations and due to the daily energy needs, which makes fatigue failure more frequent. Additionally, fatigue can cause catastrophic events in industries, because fatigue failures are brutal, sudden, and occur, most of the time, without prior warning. Studies have shown that the total mechanical failures that are caused by fatigue are ranging between 50 and 90 percent [1], which made researchers increasingly concerned about this topic. Most recently, a number of large research studies have been conducted to investigate the fatigue behavior of different materials and geometries, under various loading conditions [2–7]. In particular, Abarkan et al. [2] studied the Low Cycle Fatigue (LCF) behavior of

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smooth 316 LN austenitic stainless steel samples at room temperature using several numerical and analytical fatigue methods and they proposed new parameters to correct the inconsistency of some of the better-known low cycle fatigue analytical models used to predict the life to failure of the pressure vessels facilities.

In reality, mechanical components can be subjected to nominal stresses that exceed the yield strength of the material. This phenomenon is known as low cycle fatigue and generates significant plastic deformations in the materials. Likewise, mechanical components may operate under extremely high-constant temperature conditions. That is known as isothermal low cycle fatigue and generates thermal stresses that further reduce the service life. Beyond that, structural components frequently contain geometric discontinuities such as groove, fillet, and holes (i.e., notches) that, due to the phenomenon of stress concentration, can produce extensive plastic deformation even at applied stresses lower than the elastic limit. Besides, the presence of notches induce a multiaxial stress state in the material, which further complicates the estimation of fatigue life.

An accurate estimation of the fatigue life of a notched component is dependent on the local stresses and strains values. Finite Element Analysis (FEA) is one of the most commonly used tools for predicting the local stress-strain because it is affordable and less costly and laborious than experimental evaluation. Another more frequently used methodology to assess local stress-strain is the use of the well-known local stress-strain theories, such as Neuber's rule [8], which was further modified by Topper et al. [9] for materials under cyclic loading, the Equivalent Strain Energy Density (ESED) method or Glinka's rule [10], and the linear rule [11]. These analytical approximation methods have been greatly used in the literature to estimate the maximum stresses and strains at the notch-tips for different types of loadings, materials, and specimens geometries [12–17]. However, extensive research studies have been carried out to suggest modifications in these stress-strain theories. For example, Lieb et al. [12] proposed a new formula of Neuber's rule that takes into account different notch geometries and loadings. Hoffmann and Seeger [13] extended Neuber's rule [8] to multiaxial elastic-plastic notch problems, for specimens subjected to monotonic and cyclic loads. On the other hand, Ye et al. [15] proved that Neuber's rule is only a particular case of Glinka's rule, i.e., if the plastic strain energy is neglected in Glinka's rule, it leads to the same expression given by Neuber. Besides, they proposed a new version of Glinka's equation that takes into account the heat energy dissipated during one loading cycle rather than the plastic strain energy in the local stress-strain theory. Although several research studies have demonstrated that Glinka's rule is valid for plane strain conditions [18–20], both Neuber and Glinka local stress-strain methods were originally developed for uniaxial local stress state problems [11], that is to say, for plane stress conditions such as in the case of a thin plate or a two-dimensional part. However, under a plane strain state such as that in a three-dimensional part, the stress state is biaxial and both local stress-strain

equations may give inaccurate results. In fact, for plane strain conditions, it was found that Neuber's rule provides conservative local strain results, its degree of conservatism can be reduced by using  $K_f$  instead of  $K_t$  for cyclic loadings [11]. To overcome this problem, it was suggested to be used along with a modified stress-strain relation based on plasticity equations [21]. Besides, a new version of ESED rule for a plane strain has also been proposed [22].

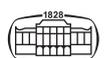
In addition, materials designed with good mechanical and low cycle fatigue properties are typically used to resist severe cyclic loadings induced mechanically as well as thermally, Modified 9Cr–1Mo (or P91) steel is the most common material used in the steam generator for the Fast Breeder Reactor (FBR), owing to its excellent oxidation resistance and LCF properties at high-temperature service [5]. Recently, several research studies have been conducted on the low cycle fatigue of 9Cr–1Mo steel smooth samples [23–26], but a few have been dedicated to the notched ones. For example, Veerababu et al. [27] conducted an investigation on the fatigue behavior of notched specimens made of 9Cr–1Mo steel at 550 °C. Based on the analytically and the numerically obtained local stress-strain results, Veerababu et al. [27] evaluated the accuracy of some of analytical approximations [8–11] in estimating the fatigue life. It was found that the predicted fatigue lives obtained by using these analytical methods were within a factor of  $\pm 16$  on the experimental fatigue lives. Therefore, they concluded that the adopted analytical stress-strain methods were not reliable to estimate the fatigue life of notched specimens made of modified 9Cr–1Mo steel, particularly for applied stresses below the material yield strength.

In the present work, the local stresses and strains have been calculated using four different analytical methods; namely the Neuber's rule [8], the modified Neuber's rule by Topper et al. [9], the equivalent strain energy density method suggested by Glinka [10], and the linear rule [11]. The obtained results were compared to those determined numerically from the FEA. Besides, the calculated fatigue lives based on the analytically obtained local strains for samples with 1.25 mm, 2.5 mm and 5 mm notch-root radius and subjected to stress amplitude levels of  $\pm 124.95$ ,  $\pm 149.95$ , and  $\pm 174.95$  MPa were compared with those obtained from the Veerababu et al. [27] experiment. Finally, by performing a regression analysis on MATLAB software [28], a simple equation was suggested to estimate the fatigue life of notched specimens made of Modified 9Cr–1Mo steel, and subjected to uniaxial low cycle fatigue at 550 °C.

## 2. MATERIALS AND METHODS

### 2.1. Experimental details and specimens geometry

A fatigue experiment was conducted by Veerababu et al. [27] on fourteen cylindrical specimens made of modified 9Cr–1Mo steel. Five of them were smooth, the geometry of which is illustrated in Fig. 1(a), and were subjected to fully reversed total strain-controlled LCF tests, at a nominal strain



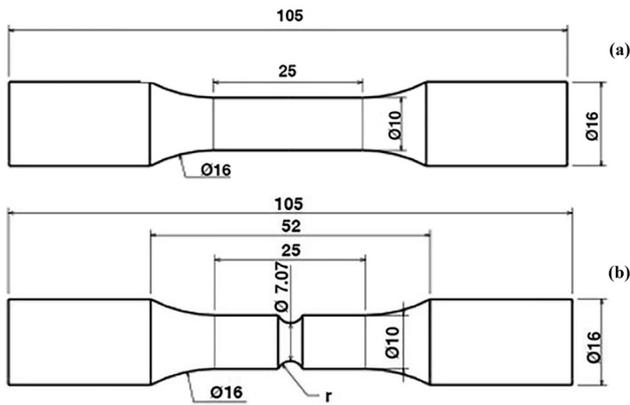


Fig. 1. Shape and dimensions of (a) smooth and (b) notched specimens ( $r = 1.25/2.5/5$  mm), all dimensions are in mm

amplitude ranging from  $\pm 0.3$  to  $\pm 0.8\%$ , at  $550^\circ\text{C}$ , and under a constant strain rate of  $3.10^{-3} \text{ s}^{-1}$ . The other nine were circumferential groove samples with three different notch

root radii, i.e., 1.25, 2.5, and 5 mm, as indicated in Fig. 1(b), and were conducted in stress-controlled mode with a frequency of 0.2 Hz, under a nominal stress amplitude of  $\pm 124.95$  MPa,  $\pm 149.95$  MPa, and  $\pm 174.95$  MPa. All the applied stress values were chosen to be lower than the yield strength of the material, which is 258.8 MPa [27], so that the plastic deformation can only occur at the root of the notch. All fatigue experiments have been performed in the air environment conditions, at a constant temperature of  $550^\circ\text{C}$ .

## 2.2. Finite Element Analysis

In order to evaluate numerically the strains at the notch-tips under different applied stresses, the notched samples were simulated using the Finite Element Analysis on the ABAQUS/Standard software [29]. The 2D-axisymmetric model has been used in this study to represent the gauge section of the samples. Prescribed pressure was introduced on the upper edge, and symmetry boundary conditions were applied along the gauge length and gauge diameter, as depicted in Fig. 2(a). The CAX4R elements were selected in

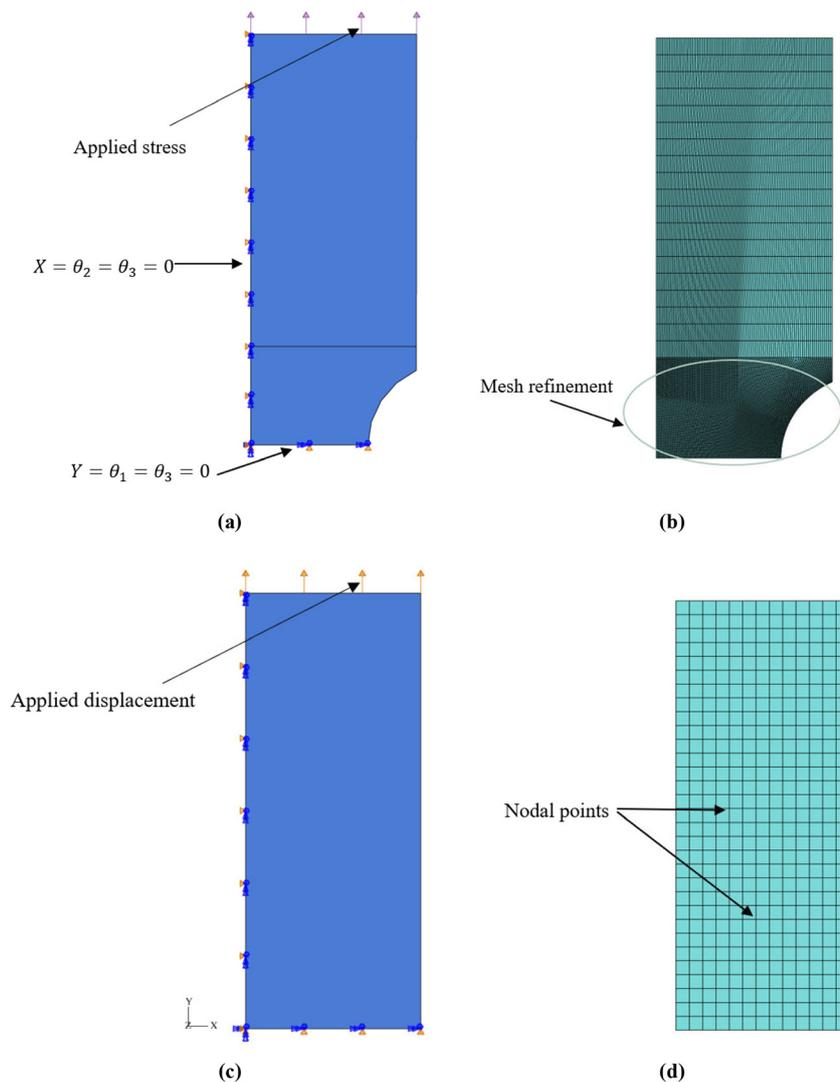


Fig. 2. FE modeling of notched and smooth parts, (a) and (c) represent the boundary conditions, and (b) and (d) represent the mesh

the mesh section, and a mesh refinement technique was applied near the notch-tip, as shown in Fig. 2(b), to obtain accurate numerical results. To validate the numerical model, a FE simulation was also conducted on smooth samples, using the same above-mentioned numerical considerations, and this time, subjected to strain-controlled mode (i.e., prescribed displacement), as shown in Fig. 2(c) and (d). The predefined temperature was applied and fixed at 550 °C for both notched and smooth/un-notched samples. It is worthwhile to mention that the nonlinear isotropic/kinematic hardening and Ramberg-Osgood material models [30, 31] have been employed to replicate numerically the cyclic stress-strain behavior of the smooth and notched specimens respectively, more details on the numerical simulation are given in [2]. The Newton-Raphson method was used to solve the nonlinear problem. Besides, the mesh convergence study has been performed and a mesh size of 0.4 and 0.03 mm was applied on the smooth parts and near the notch-tips for the notched parts respectively. Tables 1 and 2 show the material properties of modified 9Cr–1Mo steel incorporated in ABAQUS software [29].

### 3. RESULTS AND DISCUSSIONS

#### 3.1. Finite elements results

The numerically obtained hysteresis loops of the smooth specimens subjected to ±0.3 and ±0.8% applied strain amplitudes were compared to those found experimentally by Veerababu et al. [27]. As illustrated in Fig. 3, the numerical cyclic stress-strain curves are in good agreement with the experimental ones, which indicates that the FE model is typically appropriate. Besides, the nonlinear isotropic and kinematic hardening variables [30] presented in Table 1, along with the material parameters listed in Table 2, have been used to assess the hysteresis loop of modified 9Cr–1Mo steel. It should be pointed out herein that the study of the evolution of the peak tensile stress with the number of cycles,

Table 1. Nonlinear isotropic and kinematic hardening properties of modified 9Cr–1Mo steel at 550 °C [27]

Kinematic hardening parameters				Isotropic hardening parameters	
$a_1$ (MPa)	$C_1$	$a_2$ (MPa)	$C_2$	$Q$ (MPa)	$b$
86.9	3,676.7	152.4	291.8	-100.3	4.1

Table 2. Static and cyclic properties of modified 9Cr–1Mo steel at 550 °C [27]

Young's modulus $E$ (GPa)	Yield strength $s_y$ (MPa)	Cyclic strength coefficient $K'$ (MPa)	Cyclic strain hardening exponent $n'$
163	258.8	753.8	0.146

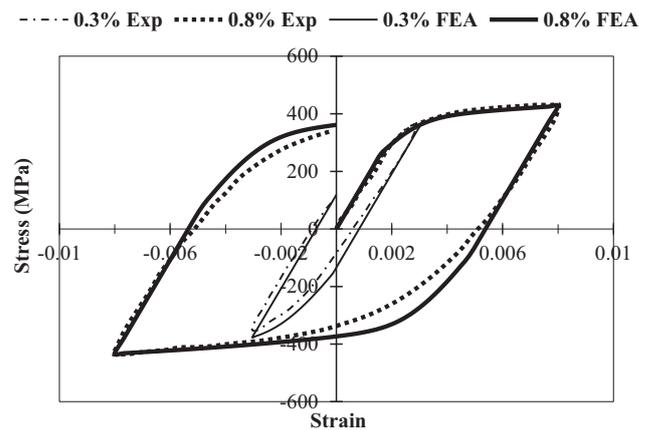


Fig. 3. Hysteresis loops obtained from FEA and those found experimentally by Veerababu et al. [27] for smooth specimens made of 9Cr–1Mo steel, under ±0.3 and ±0.8% strain amplitudes, at 550 °C

for smooth specimens, performed by Veerababu et al. [27] at 550 °C under different strain amplitudes levels has revealed that the present study used material exhibits cyclic softening until final failure for all applied strain amplitudes, except at lower nominal strain amplitude (i.e. ±0.3%) where small amount of initial hardening before softening was observed. The negative value of  $Q$  in Table 1 demonstrates well the softening phenomenon exhibited by this type of steel.

To estimate the local stresses and strains for notched specimens, Ramberg-Osgood [31] cyclic material model was implemented in ABAQUS software [29], and the obtained numerical results were used to evaluate the accuracy of the analytical approximations. That will be thoroughly addressed in Section 3.2. The resulting stress contours along the loading direction (i.e.  $S_{22}$ ) under a peak applied stress of ±145.95 MPa and for different notch root radius of 1.25, 2.5 and 5 mm are depicted in Fig. 4.

#### 3.2. Local stress-strain estimation

The strains at the notch-roots were determined by the mean of the following local stress-strain theories:

- The original Neuber’s rule [8];
- The modified Neuber’s rule by Topper et al. [9];
- The ESED method/Glinka’s rule [10];
- The linear rule [11].

The stress and strain ranges at the notch-tips  $\Delta\sigma$  and  $\Delta\epsilon$  are related to the applied stress and strain ranges  $\Delta S$  and  $\Delta\epsilon$ , respectively, by the following Neuber equation [8]:

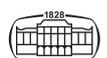
$$\Delta\sigma\Delta\epsilon = K_t^2\Delta S\Delta\epsilon \tag{1}$$

where  $K_b$  is the elastic stress concentration factor.

In the case when the applied stress is elastic, the  $\Delta S$  and  $\Delta\epsilon$  are related by Hook’s law as follows:

$$\Delta\epsilon = \Delta S/E \tag{2}$$

where  $E$  is the material Young’s modulus. Substituting (2) in (1), one can obtain the following expression:



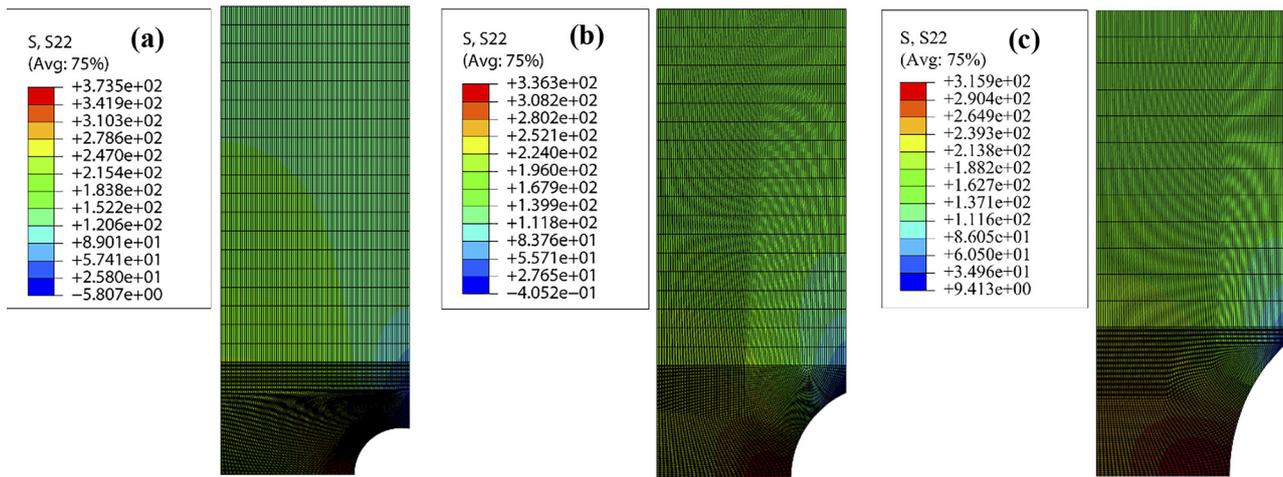


Fig. 4. The vertical stress contours due to maximum applied stress of  $\pm 145.95$  MPa in the first quarter cycle and for notch root radii of (a) 1.25 mm, (b) 2.5 mm, and (c) 5 mm at 550 °C

$$\Delta\sigma\Delta\varepsilon = \frac{(K_t\Delta S)^2}{E} \tag{3}$$

Thus, the local strains can be determined by using Eq. (3), along with the following cyclic Ramberg-Osgood relation [31].

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \tag{4}$$

where  $K'$  and  $n'$  are the cyclic strength coefficient and the cyclic strain hardening exponent of the material, respectively. Substituting (4) in (3), the following expression can be obtained:

$$\frac{\Delta\sigma^2}{E} + 2\Delta\sigma\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} = \frac{(K_t\Delta S)^2}{E} \tag{5}$$

Thus, for a given nominal stress range value  $\Delta S$ , the local stress range  $\Delta\sigma$  can be calculated from Eq. (5) and the local strain  $\Delta\varepsilon$  from Eq. (4).

Topper et al. [9] proposed an alternative to Neuber’s rule, for fatigue problems by using the fatigue stress concentration factor  $K_f$  rather than the elastic stress concentration factor  $K_t$ , where  $K_f$  can be determined from the following equation [32]:

$$K_f = 1 + \frac{K_t - 1}{1 + a/r} \tag{6}$$

$r$  is the notch root radius, and  $a$  depends on the material properties [32]:

$$a = 0.0254\left(\frac{2070}{S_u}\right)^{1.8} \tag{7}$$

where  $S_u$  is the ultimate tensile strength in MPa, and it is equal to 340 MPa, at 550 °C [27]. Thereby, for notch root radius of 1.25, 2.5, and 5 mm, the values of the  $K_t$  obtained from FEA are: 3.91, 3.13 and 2.62, respectively, and those of  $K_f$  found from Eq. (6) are: 3.02, 2.69 and 2.43, respectively.

The elastic strain energy density equation proposed by Glinka [10], for the elastic applied stress, is defined as:

$$\frac{\Delta\sigma^2}{E} + \frac{4\Delta\sigma}{n' + 1} \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} = \frac{(K_t\Delta S)^2}{E} \tag{8}$$

Thus, for a given nominal stress range  $\Delta S$ , one can obtain the local stress range  $\Delta\sigma$  from Eq. (8), and the local strain  $\Delta\varepsilon$  from Eq. (4).

In the case of elastic applied stress, the linear rule [11] relates the local strain range  $\Delta\varepsilon$  to the nominal stress range  $\Delta S$  in the following way:

$$\Delta\varepsilon = \frac{K_t\Delta S}{E} \tag{9}$$

Therefore, for a given  $\Delta S$ , one can easily calculate  $\Delta\varepsilon$  by using Eq. (9).

The calculated local strains using the aforementioned analytical approximations were then compared with those obtained from the FE simulations. Table 3 represents the relative error between the calculated analytical local strains and those obtained numerically, for the notch root radius of 1.25, 2.5, and 5 mm, under  $\pm 124.95$ ,  $\pm 149.95$ , and  $\pm 174.95$  MPa nominal stress amplitudes. Compared to the FE results, the original Neuber’s rule [8] overestimates the local strains for all notch root radius, under all stress amplitude levels, except at higher notch root radius, i.e., 5 mm, and under higher applied stress amplitudes, namely at  $\pm 174.95$  MPa with a relative error of -10.75%. Thus, the Root Mean Square Error (RMSE) of local strains calculated using Neuber’s rule is 0.20%. The modified Neuber’s rule [9] provides conservative local strain estimation for all samples, except for samples with higher notch root radius, namely 5 mm, and under lower applied stress amplitudes, i.e.,  $\pm 124.95$  and  $\pm 149.95$  MPa with a relative error of 7.32 and 3.57%, respectively. Using this method, the RMSE of the local strains is 0.12%. As shown from the same table, the linear rule [11] underestimates the local strains for all notch root radius and under all applied stress amplitudes. The



Table 3. Relative error and RMSE values between the elastic-plastic local strains obtained from the analytical methods with those found from FEA for different applied stress amplitudes at 1.25 mm, 2.5 mm, and 5 mm notch root radius

Notch root radius (mm)	Nominal stress amplitude (MPa)	Maximum local strain range %					Relative error % = $\left(\frac{\Delta\epsilon^{Analytical}}{\Delta\epsilon^{Numerical}} - 1\right)100\%$				
		FEA	Neuber rule	Modified Neuber rule	ESED method	Linear rule	Neuber rule	Modified Neuber rule	ESED method	Linear rule	
1.25	124.95	0.68	0.92	0.61	0.72	0.60	35.29	-10.29	5.88	-11.76	
	149.95	0.89	1.25	0.82	0.93	0.72	40.45	-7.87	4.49	-19.10	
	174.95	1.29	1.61	1.05	1.16	0.84	24.81	-18.60	-10.08	-34.88	
2.5	124.95	0.52	0.64	0.51	0.53	0.48	23.08	-1.92	1.92	-7.69	
	149.95	0.68	0.86	0.67	0.69	0.58	26.47	-1.47	1.47	-14.71	
	174.95	1.01	1.11	0.87	0.84	0.67	9.90	-13.86	-16.83	-33.66	
5	124.95	0.41	0.49	0.44	0.43	0.40	19.51	7.32	4.88	-2.44	
	149.95	0.56	0.65	0.58	0.54	0.48	16.07	3.57	-3.57	-14.29	
	174.95	0.93	0.83	0.74	0.66	0.56	-10.75	-20.43	-29.03	-39.78	
RMSE % = $\sqrt{\frac{\sum_{i=1}^9 (\Delta\epsilon^{Numerical} - \Delta\epsilon^{Analytical})^2}{9}}$ %							0.20	0.12	0.12	0.24	

relative errors range from -2.44 to -39.78% and the RMSE value is 0.24%. The obtained local strains by the ESED method [10] are non-conservative for all notch root radius, under lower nominal stress amplitude levels. For instance, the relative error is 5.88% when the notch root radius is 1.25 mm, and the applied stress is ±124.95 MPa. Whereas at higher nominal stress amplitudes, the method results in conservative local strain calculations. For example, the relative error is -29.03% for samples whose notch root radius is 5 mm and the applied stress is ±174.95 MPa. Using this method, the RMSE is the same as that obtained in the modified Neuber’s rule [9].

As it is observed, the modified Neuber’s rule [9] always gives lower local strain results compared with the original Neuber’s relation [8]. That is because the values of fatigue stress concentration factor  $K_f$  are always smaller than those of the elastic stress concentration factor  $K_t$ . As anticipated, the original Neuber’s rule leads to higher local strains compared to the ESED approximation [10] that is due to the presence of a factor of  $2/(n'+1)$  in Eq. (8) as compared with Eq. (5). One can note that local strain values obtained from different analytical methods show no consistency when compared to each other and to the FEA results.

### 3.3. Fatigue life estimation

The fatigue life  $N_f$  was assessed for various notch root radius, under different stress amplitudes using Eq. (10), which was found by regression analysis on five experimental low cycle fatigue life of smooth samples, for each applied strain range [27].

$$\Delta\epsilon = A(N_f)^l \tag{10}$$

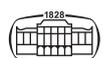
where  $A$  and  $l$  are material parameters, and are 0.2191 and -0.4165, respectively [27].

The fatigue lives calculated from Eq. (10), by using the previously obtained analytical local strains were compared to those obtained experimentally by Veerababu et al. [27]. As shown in Fig. 5, the ESED method developed by Glinka [10]

underestimates the fatigue lives at all stress amplitude levels when the notch root radius is 1.25 mm. Thus, under ±124.95 MPa, the relative error is -54.08%. Moreover, this approximation method gives conservative fatigue life results when the applied stresses are small i.e., ±124.95 MPa, for 2.5 and 5 mm notch root radius. Otherwise, it leads to a non-conservative fatigue life prediction, with a maximum relative error of 88.53%, for samples with 5 mm notch root radius, and subjected to ±174.95 MPa. The original Neuber’s rule [8] leads to a conservative fatigue life estimation at both 1.25 and 2.5 mm notch root radius, under all applied stress amplitudes. However, it results in a non-conservative fatigue life prediction when the notch root radius is 5 mm and under ±149.95, ±174.95 MPa. As an illustration, the relative error is -40.82% for a notch root radius 2.5 mm and is 7.66% when the notch root radius is 5 mm, and the applied stress is ±174.95 MPa. Besides, the modified Neuber’s rule [9] underestimates the fatigue life at all nominal stress amplitudes for 1.25 mm notch root radius, and at lower applied stress amplitudes, i.e., ±124.95 MPa for 2.5 and 5 mm notch root radius. Otherwise, it overestimates the fatigue life. For example, the relative error is -32.34% at 1.25 mm and under ±124.95 MPa and is 43.87% at 5 mm under ±174.95 MPa. For all notch root radius, the linear rule [11] gives a conservative fatigue life estimation for an applied stress amplitude of ±124.95 MPa and non-conservative results for nominal stress amplitudes of ±149.95 MPa and ±174.95 MPa. For instance, the relative error at 5 mm notch root radius is -29.52% under a nominal stress amplitude of ±124.95 MPa, and 175.89% under ±174.95 MPa.

### 3.4. The proposed model

As stated earlier, the previous analytical approximations provided unreliable local strains estimation and lead thus to an incorrect fatigue life prediction for modified 9Cr-1Mo notched steel under 550 °C. Hence, a new mathematical model was suggested for this type of material, under the same test condition, to rectify the discrepancies between the



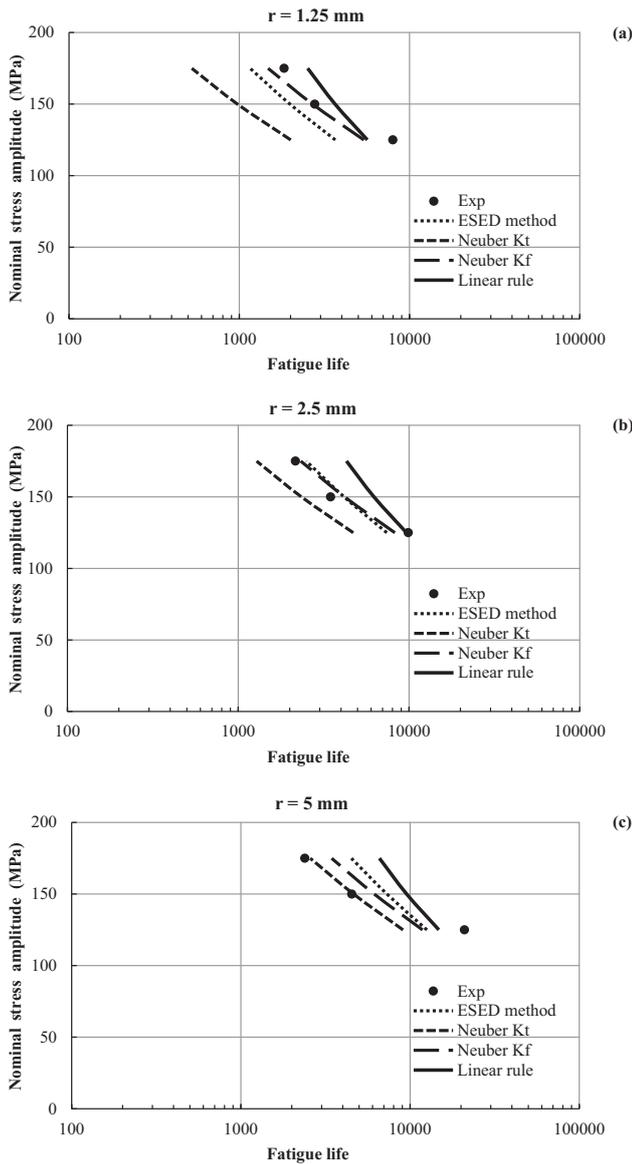


Fig. 5. Comparison of the estimated fatigue life with those obtained experimentally by Veerababu et al. [27] under different stress amplitudes for (a) 1.25 mm, (b) 2.5 mm, and (c) 5 mm notch root radii

experimental and the analytical results. The proposed model contains only three variables, namely the number of cycle to failure  $N_f$ , the nominal stress amplitude  $S_a$ , and the fatigue stress concentration factor  $K_f$ , and fulfills the following condition:

$$f(X) = \text{minimize} \left\{ \sqrt{\frac{\sum_{i=1}^n (N_{f,experimental_i} - N_{f,estimated_i})^2}{n}} \right\} \quad (11)$$

where  $X = x_1, x_2, x_3, x_4, x_5, x_6$  are the parameters of the proposed model, and  $n$  is the number of experimental data points.

Based on a regression analysis performed in MATLAB software [28], the mathematical model was developed, and

the fatigue life can be easily calculated using the following expression:

$$N_f = S_a^{-10.05} \exp(-181 \log(K_f) - 8.386K_f^2 + 110.6K_f) + 10.55S_a \quad (12)$$

The mathematical model, which was obtained based on nine different data point for notched samples given in [27] and compared against the same test data, was found to provide excellent fits to the experimental fatigue life, with an R-square of 0.9987. The obtained fatigue lives by using Eq. (12), for different notch root radius, under nominal stress amplitudes of  $\pm 124.95$ ,  $\pm 149.95$ , and  $\pm 174.95$  MPa are represented in Fig. 6. One can conclude that the proposed

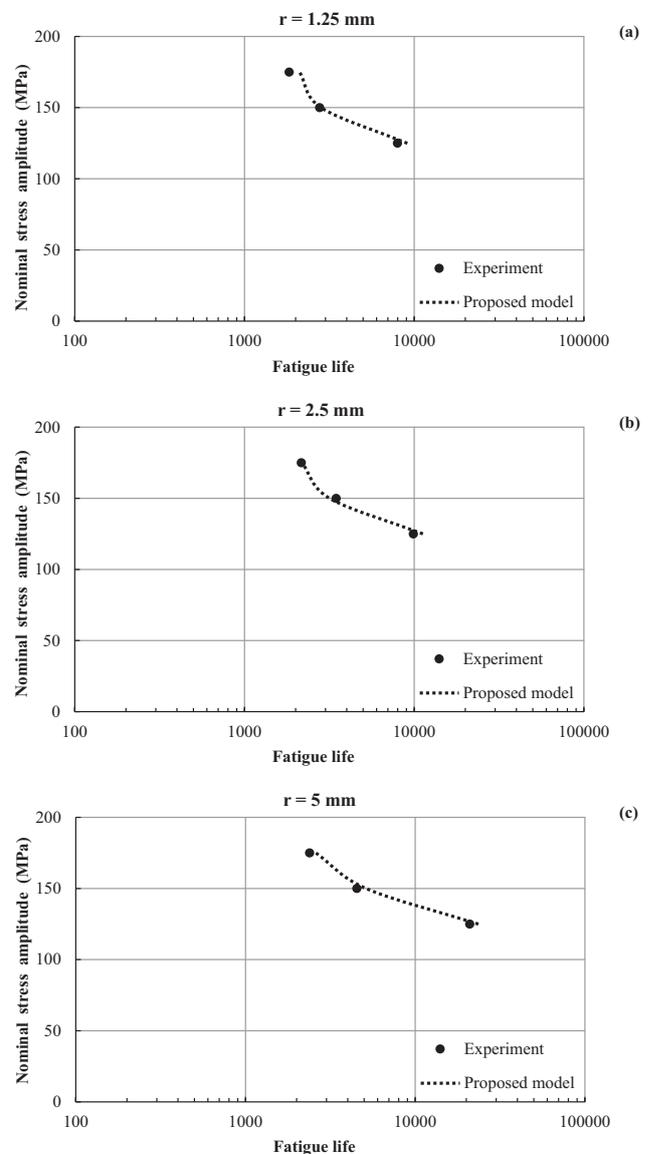


Fig. 6. Comparison between the estimated fatigue lives through the proposed model and the experimental data provided by Veerababu et al. [27] under different stress amplitudes for (a) 1.25 mm, (b) 2.5 mm, and (c) 5 mm notch root radii



equation (i.e. Eq. (12)) enables the estimation of the fatigue life of U-notched specimens made of modified 9Cr–1Mo steel under 550 °C, with a minimum relative error compared to all the previously used analytical methods. For instance, the relative error between the experimental data and the calculated fatigue lives using Eq. (12) are 1.97, –8.67, and 13.54%, under  $\pm 149.95$  MPa, for 1.25, 2.5, and 5 mm notch root radius, respectively. Therefore, the suggested mathematical model for modified 9Cr–1Mo notched steel provides better fatigue life predictions than those predicted by the other analytical methods under different stress amplitudes and notch root radii.

## 4. CONCLUSIONS

The elastic-plastic local strains, as well as the fatigue lives, were estimated for circumferentially notched cylindrical specimens made of modified 9Cr–1Mo steel with a notch-root radius of 1.25, 2.5, and 5 mm, and subjected to uniaxial nominal stress amplitudes of  $\pm 124.95$ ,  $\pm 149.95$ , and  $\pm 174.95$  MPa, under 550 °C temperature loading conditions. The analytical approximations used in this study to calculate the strains at the notch-tips were: Neuber's rule [8], modified version of Neuber's rule [9], Equivalent Strain Energy Density method [10], and linear rule [11]. The fatigue lives were predicted through the strain-life equation, based on the local strains obtained analytically.

Compared to the local strains obtained from the Finite Element Analysis, it was found that those calculated from Neuber's rule [8] are non-conservative for all notch-root radius and under all applied stress amplitudes, except at higher notch root radius, and higher nominal stress amplitudes, i.e., at 5 mm and under  $\pm 174.95$  MPa, respectively. The modified Neuber's rule [9] resulted in a non-conservative estimation of the local strains, only at higher notch root radius, namely 5 mm and under lower applied stress amplitudes, i.e.,  $\pm 124.95$  and  $\pm 149.95$  MPa. Otherwise, it gave conservative results. Besides, the ESED method [10] slightly overestimated the local strains under lower applied stress amplitude, for all notch root radius. However, at higher nominal stress amplitudes, the method underestimated the local strain values. The obtained local strains using the linear rule [11] were conservative for all notch root radius and under all applied stress amplitudes.

The fatigue lives obtained through the strain-life equation were compared to the experimental data provided by Veerababu et al. [27]. Thus, it was found that the local strains obtained from the ESED method [10] led to a conservative fatigue life estimation at 1.25 mm notch root radius, under all applied stress amplitudes and for 2.5 and 5 mm notch root radius, under smaller applied stress amplitudes, i.e.,  $\pm 124.95$  MPa. Otherwise, it gave non-conservative fatigue life results. Besides, the Neuber's rule [8] underestimated the fatigue life at 1.25 as well as at 2.5 mm notch root radius, under all the applied stress amplitudes. However, it overestimated the fatigue lives for a notch root

radius of 5 mm and under  $\pm 149.95$ ,  $\pm 174.95$  MPa. The modified Neuber's rule [9] resulted in a conservative fatigue life under all nominal stress amplitudes for 1.25 mm notch root radius, and at lower nominal stress amplitudes, i.e.,  $\pm 124.95$  MPa, at 2.5 and 5 mm notch root radius, otherwise, it overestimated the fatigue life. For all notch root radius, the linear rule [11] underestimated the fatigue lives only when the applied stress amplitude was  $\pm 124.95$  MPa, otherwise, it was found to provide non-conservative fatigue life results.

A simple mathematical equation was suggested based on the regression analysis. It was found that the calculated fatigue lives correlate well with the experimental data for circumferentially notched cylindrical specimens made of modified 9Cr–1Mo steel, and subjected to stress-controlled loading mode, under 550 °C temperature condition.

## ACKNOWLEDGMENT

The first author would like to acknowledge the promoter of scientific and technical research in Morocco; CNRST, for the Ph.D. excellence grant.

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