Quantified Risk and Uncertainty Analysis

Bayesian belief networks provide a powerful means for analysing uncertainty in terms of accident risk, and aid key decision making

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THE legal requirement in the UK for the duty holder of a chemical process plant to demonstrate that risk is as low as reasonably practicable (ALARP) means that quantified risk assessments (QRAs) must be accurate and robust and that identified risks are adequately mitigated. The overall risk assessment process normally applied to chemical plants is illustrated in Figure 1. Bayesian belief networks (BBN)[1,2] is an emerging technique which can be used to determine the likelihood of an event in support of the QRA process. It is a statistical method involving estimating the probability distribution for a given hypothesis. The most interesting features which distinguish this QRA technique from all the others are:

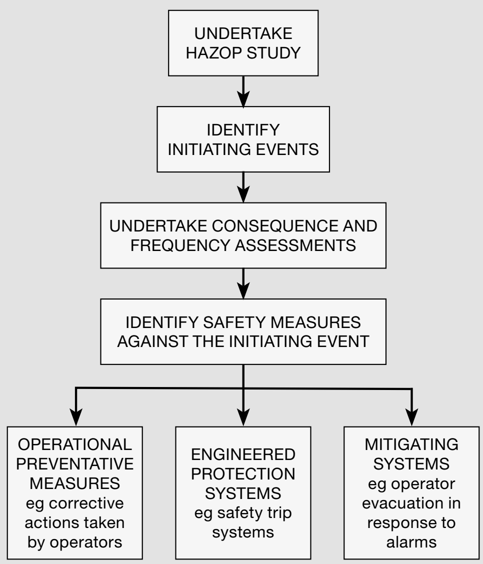


FIGURE 1: QUANTIFIED RISK ASSESSMENT PROCESS (QRA)

- it can analyse complex systems of any given number of variables and their dependability within a single analysis;
- it can analyse parameters over a range of probability values for any given set of conditions, providing a better understanding in terms of sensitivity analysis;
- it engages expert judgement and learning from previous events to update the probability distribution, thus improving QRA accuracy; and
- it is not just restricted to fault analysis and can be used to support plant operational decision making using a quantified approach.

The successful search for the Air France Flight 447 (2009) wreckage using Bayesian inference[3] is perhaps one of the best examples to illustrate the power and accuracy of BBN. Principally, BBNs were used to combine evidence gathered from the previous failed searches for the flight and expert judgement, to update the probability distribution. This led to precisely locating the wreckage in the Atlantic.

In this article I will illustrate the principles behind the Bayesian technique and how it actually provides the benefits listed above. Through a plant case study, I will also demonstrate how BBNs can be used to undertake other forms of uncertainty analysis such as plant operability issues.

BAYESIAN BELIEF NETWORKS – THE CONCEPT

Bayesian inference essentially uses a statistical hypothesis commonly referred to as the Bayes Theorem, which is expressed mathematically as follows.

$$P(X|Y) = \frac{P(Y|X)\,P(X)}{P(Y)}$$

where:

- P(X) and P(Y) are the probabilities of observing events X

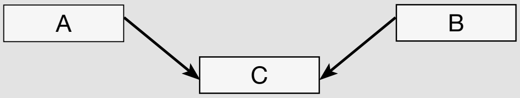


FIGURE 2: BAYESIAN NETWORK SHOWING RELATIONSHIP BETWEEN PARENT AND CHILD NODES

and Y which are independent of one another;
- P(X) is referred to as the prior probability, ie before any information is available about the event;
- P(X|Y), is a conditional probability which represents the likelihood of observing event X given that Y is true; and
- P(Y|X) represents the probability that event Y occurs, given that X is true.

We can apply the above equation to a typical hydrogen safety issue concerning the likelihood of a hydrogen explosion in a sealed process pipe. Suppose the probability of an explosion occurring in the pipe is P(X) and the probability that hydrogen is present in a pipe is P(Y). In this case Bayes Theorem can be used to update the likelihood of an explosion event occurring in the light of new evidence, as illustrated by the following example.

Consider a plant consisting of 1,000 sealed pipes in which one explosion incident has previously been observed. Then the prior probability P(X) = 1/1,000, or 0.1%. Now let's assume that upon a sampling of the pipes, hydrogen was detected in 5% of the cases, so P(Y) = 0.05. If we also assume that the probability that hydrogen would have been detected given that an explosion event occurred, P(Y|X) = 1, then applying Bayes Theorem tells us that if hydrogen is detected in a given pipe, the probability of an explosion rises from 0.1% to 2% (ie P(X |Y) = (1 x 0.001)/0.05 = 2%).

The hydrogen explosion hypothesis above applies Bayes Theorem to a relatively simple analysis with only two events. However, for a large number of events and interactions, the Bayesian algorithm for the hypothesis would be almost impossible to compute manually. Hence commercial software applications such as *Netica*[4] have been developed, based on Bayes Theorem. The software enables modelling of a hypothesis of any given number of variables, in the form of a graph network, commonly referred to as the Bayesian belief network (BBN).

SO HOW DO BAYESIAN BELIEF NETWORKS ACTUALLY WORK?

The construction of Bayesian networks consists of the following key steps:

1. Identifying variables for the problem in question and causal relations between the variables in the form of a directed acyclic graph. A directed acyclic graph is a group of random variables or nodes. If there is a causal dependability between two nodes, the corresponding two nodes are connected by an arc or a link. In Figure 2 the arcs from nodes A and B to a node C indicates random variables A and B (often termed as parent nodes) cause random variable C (child node).
2. Identifying a conditional probability distribution for each node. Here, conditional probability tables (CPTs) are constructed for each node. A CPT for a child node identifies probabilities of the node accepting each of its values which are conditional on values of the parent node. The nodes can be either 'discrete' and expressed as end states such as 'yes' or 'no' or they can be 'continuous' in the form of a range or a distribution.
3. Verification of CPT data through either discussions with experts or experimental evidence.

Suppose the risk analyst has predicted that the likelihood of a detonation or a deflagration in a pipe (child node) is influenced by the parent nodes: pipe diameter, flammability of the hydrogen in air gas mixture, and if an ignition source is present in the pipe. This hypothesis is illustrated in Figure 3 using *Netica* software[4] which models all of the parent

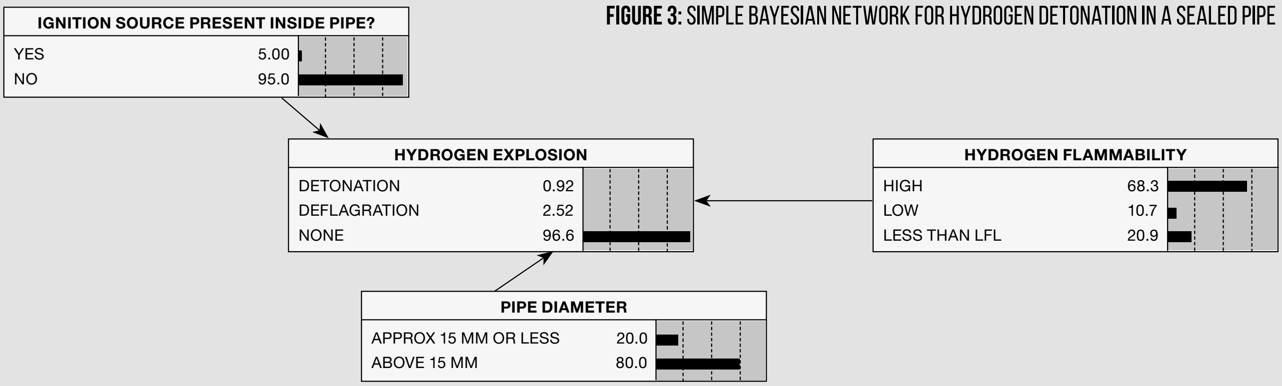


FIGURE 3: SIMPLE BAYESIAN NETWORK FOR HYDROGEN DETONATION IN A SEALED PIPE

nodes as discrete nodes with varying probabilities for each state. For illustration purposes, Figure 3 calculates that even with a high probability (68.3% chance) of high hydrogen flammability, but only a 5% chance that an ignition source is present, there is still only a 0.9% probability that a detonation will occur.

CASE STUDY – APPLYING BBNS TO PROCESS PLANT OPERABILITY ISSUES

The following case study will demonstrate how the technique can be applied to resolve a plant operability issue and aid operational decision making. It looks at the uncertainty associated with mixing process sludge (magnesium hydroxide) and the likelihood of sludge of acceptable product quality being easily retrieved from the mixing vessel.

THE HYPOTHESIS

Chemical plant A provides the process for treatment and encapsulation of magnesium hydroxide sludge retrieved from underwater storage of metallic magnesium in plant B.

The waste encapsulation process in plant A primarily involves transferring the sludge from plant B donor plants via a skip to a mixing vessel in plant A and mixing the sludge with cement grout. Subsequently the grouted contents of the mixing vessel are tipped into a waste encapsulation drum for curing. A key plant operability requirement is that the grouted product within the mixing vessel should be sufficiently mobile to enable it to be transferred or tipped into the waste encapsulation drum.

IDENTIFYING BBN VARIABLES AND THEIR DEPENDABILITY

This particular analysis assumed the following variables would affect sludge mobility:

- **Water/solids ratio**, which is a function of free water, sludge internal water content quantity of dry cement and organics grout plus the sludge dry solids content.
- **The dry sludge mass** – this is affected by the amount of total material present in the skip and quantity of water associated with the sludge.
- **Fluidity**, which is dependent on the dry sludge and dry powder grout (DPG) mass, the water/solids ratio and sludge stickiness.
- **Product quality**, ie mix too hard, too runny or acceptable, which is a function of the parent node fluidity and mixing parameters including mixing time and speed.

THE BBN RESULTS

For the above variables and their dependability, a Bayesian belief network was constructed using *Netica*, as shown in Figure 4. Three of the parent nodes in the network are observational, requiring input of the CPT data by the user. These are skip fill volume, active water mass fraction, and dry powder grout mass. The conditional probabilities for all the remaining nodes were calculated by the network using equations based on the relationships outlined in the previous bullet points. The network shows that at the selected values for the three input parent nodes there is a 78% chance that the sludge will come out from the mixing vessel.

THE MAIN CHALLENGES REVEALED DURING CONSTRUCTION OF THE NETWORK WERE THAT THE ANALYSIS IS DEPENDENT ON ACCURATE PREDICTION OF KEY VARIABLES AND THEIR DEPENDABILITY AS WELL AS EXPERT JUDGEMENT OF THE CPT DATA

The maximum and minimum ranges for the input parent node values used in Figure 4 are hypothetical values to illustrate the sensitivity associated with the key variables. As for any modelling work, the BBN output required validation. To test the validity, the values for the input parent nodes in Figure 4 were changed to replicate previous experimentally trialled values. With the modified values for the input parent nodes, the updated water/solids calculated ratio by the BBN compared well with the actual trialled water/solids ratio.

LEARNING FROM THE BBN CASE STUDY

The comparison of the BBN output with the experimental data clearly showed that the BBN model is reasonably accurate, which can be used to analyse real plant conditions. Effectively, this would suggest that the network could be used by plant operators to determine the effect of change in plant parameters on the mixing vessel performance and mix quality, without the need for additional trials, which can be costly and time consuming. That said, the main challenges revealed during construction of the network were that the analysis is dependent on accurate prediction of key variables and their dependability as well as expert judgement of the CPT data. Using expert judgement may be acceptable for plant operability type uncertainties. However when the BBN analysis is being used to support a key safety case, in particular when the perceived risk is considered to be high – eg due to hydrogen explosions – then for ALARP purposes the CPT data would require a high level of confidence, for example through experimental evidence.

IS THE BBN TECHNIQUE TENABLE?

One may argue that there are other QRA techniques such as fault tree analysis (FTA)[5], so why should we choose to use Bayesian networks? The main benefit of BBNs is that, unlike FTA, they enable parameters of uncertainty to be modelled over a range or a distribution, thus providing a better understanding in terms of sensitivity of key variables. Additionally,

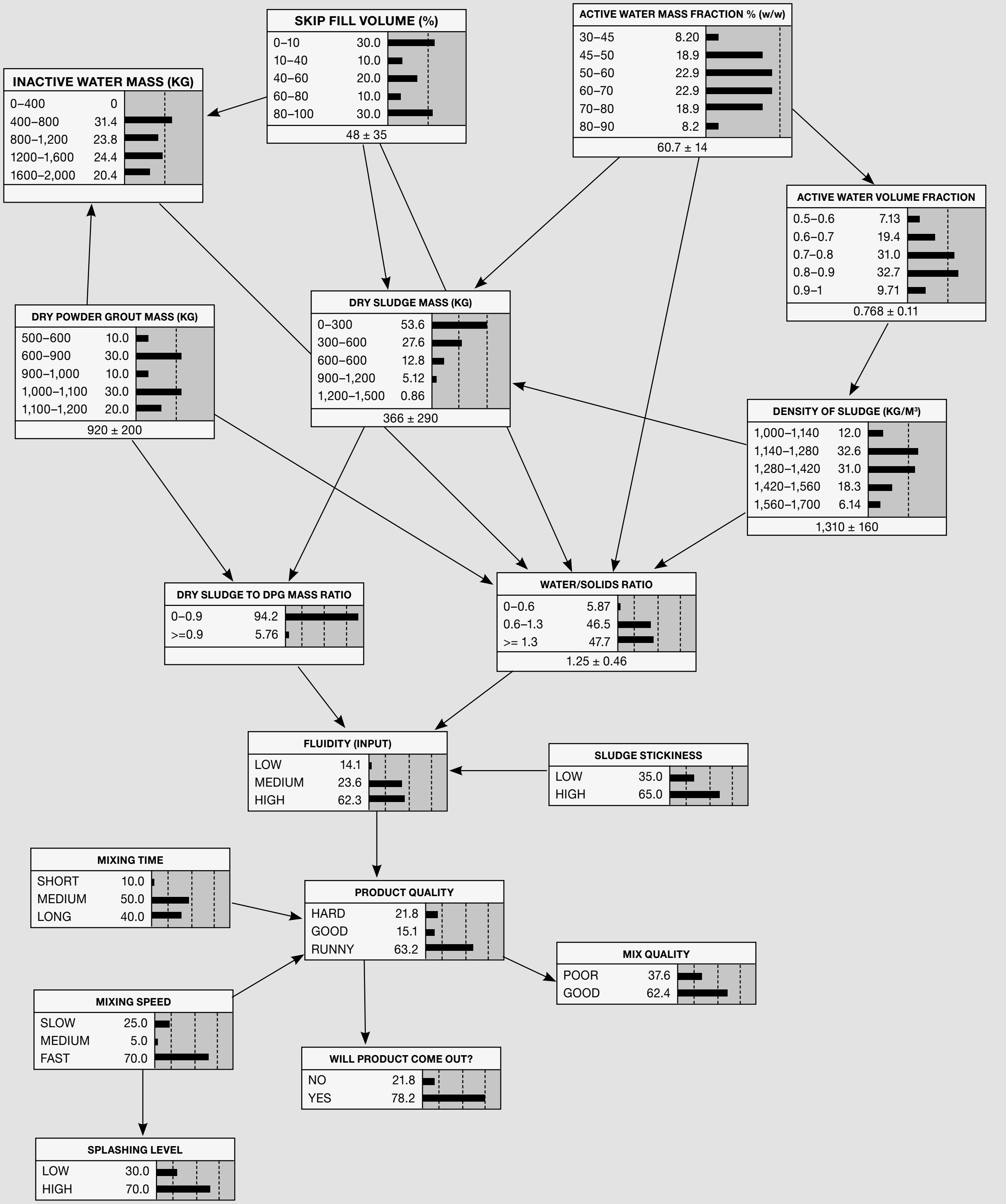
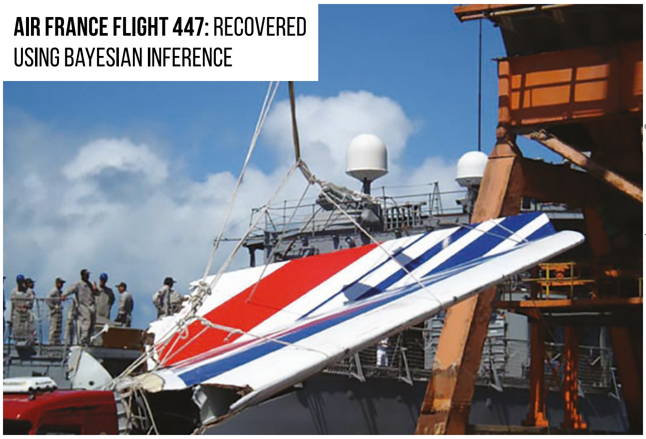


FIGURE 4: BAYESIAN BELIEF NETWORK ANALYSIS OF UNCERTAINTY ASSOCIATED WITH MIXING MAGNESIUM HYDROXIDE SLUDGE



AIR FRANCE FLIGHT 447: RECOVERED USING BAYESIAN INFERENCE

Roberto Maltchik/Agencia Brasil/Wikipedia

as explained earlier, in the search for Air France Flight 447, this technique can update the probability of known variables through evidence, and improves accuracy of the analysis.

THE MOST UNIQUE FEATURE IS THAT BBNS ARE NOT JUST RESTRICTED TO FAULT ANALYSIS... THEY ARE ALREADY BEING USED IN THE OUTSIDE WORLD TO AID KEY DECISION MAKING

Interestingly, the most unique feature is that BBNs are not just restricted to fault analysis. As illustrated in the case study, it can use Bayes Theorem to quantify the likelihood of an uncertainty to aid key operational decision making. Most often the process industry is also required to carry out optioneering studies in order to arrive at the best practicable design or environmental option, and usually methods such as value engineering (VE) studies are used. VE studies use a qualitative approach, whereas the Bayesian technique uses a more structured and quantified method for deriving the best option.

BBNs are already being used in the outside world to aid key decision making. For example the medical profession uses this technique for patient diagnosis and prognosis. If important decisions are being made which would affect human life, then surely there is enough confidence that the technique can be used to for QRA in the process industry. ■

FURTHER READING

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4. *Netica*, Norsys, www.norsys.com
5. Andrews, J, *Fault Tree Analysis Proceedings of the 16th International System Safety Conference*, 1988, available at www.fault-tree.net