Dynamic Phasors to Enable Distributed Real-Time Simulation

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***Abstract*—Distributed real-time simulation allows the sharing of simulator equipment and components connected in Hardware-In-the-Loop experiments. In this paper, we analyze the challenges of geographically distributed real-time power system simulation and how dynamic phasors could be applied to improve the accuracy of the simulation results for large time steps. The time step is of particular interest since the communication delay between simulators interconnected through wide area network is much larger than the simulation time step typically used in electromagnetic transient real-time simulations. However, commercially available real-time simulators use either the electromagnetic transient or classic complex phasor representation. Results in the dynamic phasor and electromagnetic transient domain are compared to quantify the advantage of dynamic phasor simulations in practice. The test platform for this evaluation is a power system simulator which is currently under development.**

***Index Terms*—Power system simulation, Distributed computing, Real-time systems, Computational modeling, Power system modeling**

I. Introduction

Deployment of new power system components such as power electronics devices and measurement units, advanced communication infrastructure as well as new control and management algorithms is rapidly increasing. Therefore, it is important to study the interactions among these components to ensure the stability and controllability of future grids.

Real-time simulation is becoming increasingly popular as a means to test and validate physical components and algorithms in a controlled and realistic environment. The synchronization of simulation time with wall clock time allows the exchange of physical inputs and outputs between externally connected devices and the real-time simulator. When a device is attached to a simulation, so-called Hardware-In-the-Loop (HIL), the behavior of components in the system can be safely tested also during emergency operations, and the integration of the device into the system can be easily validated.

However, the real-time simulators and the devices-under-test might be geographically distributed or the capabilities of the local real-time simulator might not be sufficient for the given simulation scenario. Then, the model which is to be simulated can be partitioned for a distributed simulation. Typically, geographically distributed simulators are connected via Wide Area Networks (WAN). In addition to the challenge of partitioning the model and creating interfaces for parallel execution, the communication delay can be even larger than the simulation time step which is typically used in power system electromagnetic transient (EMT) simulations. One way to overcome this are interface algorithms that compensate for the delay [1].

In this paper, we pursue another strategy: increasing the simulation time step to enable the exchange of signals for each simulation step in real-time. The minimal frequency that has to be represented in power system simulations is the system frequency of $50\,\mathrm{Hz}$ or $60\,\mathrm{Hz}$ depending on the region. Hence, it would be advantageous to integrate the frequency implicitly as it is done in traditional complex phasor calculation but without being fixed to this frequency. Dynamic phasors meet both requirements [2]. Since commercially available real-time simulators do not provide dynamic phasor models, the advantage of dynamic phasor over EMT for distributed real-time simulations is not quantified, yet. That is why this paper presents a simulation study on EMT and dynamic phasor simulations for varying time steps. The simulator used in the study is currently developed at ACS, RWTH Aachen University.

The paper is structured as follows: Section II introduces the techniques and definitions which are used throughout the paper. Sections III and IV analyze the challenges of distributed real-time simulation in depth and define the scope of the solution presented here. The simulation study and results are given in Section V.

II. State of the Art

A. Real-Time Simulation

Simulations can be classified according to the relation between simulation time and wall clock time and the time flow mechanism, fixed / variable time stepped or event-driven simulation [3]. Typically, power system real-time simulators progress in time with fixed time steps since more sophisticated variable time step integration methods are not suitable for fast calculations as needed for real-time execution and small time steps. In case of nonlinear events such as transistor switching, the time step synchronization can give rise to jitter: events are considered only at the

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beginning of the next step and not when they are actually happening. Particular techniques and solving methods are therefore needed to avoid numerical instability [4]. Real time simulation of power systems is intended to accurately reproduce the dynamic behavior of a physical system, for example, during very fast transient situations like system outage management or fault location [5]. Protection and control system development and testing, distributed generation modeling, especially with renewable energy resource (RES) integration, and microgrid control are some of the application fields in which real-time simulation is widely utilized [6].

B. Distributed and Parallelized Simulation

Parallel and distributed simulation may refer to sharing the computational load among several processing units which can belong to one computer, or multiple computers interconnected in a lab, or several computers located geographically distant.

In case of offline or as-fast-as-possible simulations where simulation time is not tied to wall clock time, the motivation for distributed computation is the acceleration of the simulation process by utilizing many processing units in parallel. In contrast, this paper is focused on parallel and distributed simulation of power systems in real-time. Again, one objective is to increase the overall computational power of the simulator. Besides, there are more advantages of distributed real-time simulation [7], [8]. The following list summarizes the most important ones:

- Available hardware and software in different real-time simulation laboratories can be shared among participants in order to enhance computation power and facilitate remote Software-In-the-Loop (SIL) and (Power-)Hardware-In-the-Loop
- Larger scales of systems could be simulated by assigning different parts of the model to several laboratories
- Expertise and knowledge in different energy fields and use cases could be shared by applying the same case study concurrently without the need to move researchers and equipment
- Confidential data does not need to be shared as each laboratory can be responsible for simulating its own part of the model locally, solely exchanging interface variables with other interconnected systems, imitating the real world where regional or national power grids are interconnected through tie-lines
- Several algorithms to control, manage, or regulate systems can be tested in laboratories where no realistic models of the environment (e.g. power grid model) are available

C. Dynamic Phasors

Dynamic Phasors are known as a powerful and efficient analytic tool used in simulations, which is based on the concept of time varying Fourier coefficients. As the size and number of new components of power systems are rapidly increasing, especially by introducing power electronics devices, there is a need for simulation methods which can address and describe the dynamic changes of grid states without demanding the same computational cost as EMT simulations. In this way, larger grids under several conditions can be simulated and studied.

Application of dynamic phasors in power systems is ubiquitous. It was firstly used for studying and simulating general converter technologies. Nowadays, one of the main goals is to simulate and integrate new Distributed Energy Resources (DER) and HVDC converter technologies. The aim is to construct an efficient model for the dynamics of switching gates phenomena with a high level of detail [2], [9], [10], [11].

Fault analysis and unbalanced conditions are other important research topics in which dynamic phasors allow larger models and more efficient simulations. Asymmetrical faults are studied in [12], [13], [14], [15]. In the last two articles the behavior of AC machines like Doubly-Fed Induction Machine (DFIG) wind turbines or synchronous generators are evaluated using dynamic phasors. In [16] an unbalanced distribution system consisting of a single-phase PV system, a three-phase induction machine and a three-phase power factor correction capacitor is simulated using dynamic phasors, trying to achieve a comprehensive modeling approach. Results show great similarity with time-domain simulations.

In [17] an effort to generalize the dynamic study with dynamic phasors is made by modeling and validating a multiple synchronous generator test grid. The goal was twofold: application of dynamic phasors for multi-source, multi-frequency systems and modeling of systems with time-varying frequencies. In [18] a frequency matched linear numerical integration technique is used to improve efficiency and accuracy of the dynamic phasor simulation.

Although dynamic phasors allow a significant saving in terms of computational cost, the idea to apply it to real-time simulation is still fairly novel. Commercial simulators like RTDS and OPAL-RT offer analytic modeling tools being able to perform EMT simulations e.g. eMEGAsim developed by OPAL-RT. OPAL-RT introduced also simulation tools in the traditional complex phasor domain called ePHASORsim which is limited to system fundamental frequency. As mentioned in Section I, dynamic phasors might allow larger systems to be simulated in real-time with larger time-steps (i.e. milliseconds instead of microseconds) while catching the dynamic behavior of a system with frequency deviation. In this paper, this capability is highlighted and proposed to be applied in distributed real-time simulations where communication latency and the size of the model lead to serious limitations when using conventional EMT solvers. The following section analyzes these limitations and challenges.

III. Challenges of distributed real-time simulation

Geographically distributed simulation borrows problems from parallel simulation. In both cases the model has to be partitioned and it has to be assured that the parallel simulation results do not differ from the results of a sequential simulation. Furthermore, real-time requirements decrease the number of suitable simulation techniques [19].

In case of geographically large distances between the simulators the time needed for information exchange can have a huge impact on the simulation. In fact, a communication delay of more than $10\,\mathrm{ms}$ might cause large errors and even instability [1]. One cause for this is the sampling requirement imposed if an AC $50\,\mathrm{Hz}$ or $60\,\mathrm{Hz}$ system is simulated in EMT. According to the sampling theorem, the minimum sampling frequency is twice the maximum frequency expected in the system. This combined with the large Round-Trip Time (RTT) expected in geographically distributed simulations, complicates the synchronization among the simulators. Simulations using traditional complex phasors do not impose the strong sampling requirement since the system frequency is implicitly included but this frequency is fixed. Therefore, this approach does not support frequency control or stability studies, for example, on transmission level.

The authors of [8] realized an integrated real-time co-simulation laboratory by applying a communication platform as a simulator-to-simulator interface proposed in [1] in order to enable remote and online monitoring of an interconnected transmission-distribution system. Based on that novel approach, each simulator carried out simulations in time domain, while the time-varying Fourier coefficients of the quantities in the interconnection node, i.e. decoupling point, are exchanged. EMT values could not be exchanged for every simulation step due to the communication RRT. Following transformed quantities could be exchanged: Traditional complex phasor or dynamic phasors of the fundamental and harmonic components. The problem with the former solution is the following. For transient analysis, frequency deviation in one side cannot be captured on the other side to perform a distributed real-time simulation with the same results as a local real-time simulation. With the latter solution, dynamic phasor exchange, simulations do not imply fixed system frequency. Still, the local EMT simulation is computational less efficient compared to phasor simulations. Besides, this approach requires the extraction of phasor information from the EMT signals. Therefore, transparency of the interface is not given since the interface algorithm may alter the exchanged signals. So far, the interface algorithm can extract magnitude and phase for several harmonic components. The frequency is assumed to be the nominal system frequency with a DC link connecting the two systems.

The following section describes the use case of our dynamic phasor solver which is supposed to avoid these limitations.

IV. Use case

Recently, real-time control of Distributed Energy Resources (DER), especially from renewable energy resources with intermittent behavior, is attracting more attention due to increasing penetration of such energy resources in grids. Renewable energy sources aim to reduce CO2 emissions, but as a side effect, power system stability and quality of electric power supply are jeopardized as they do not provide inertia by default due to power converters being used as interfaces to the grid.

A H2020 European project named RE-SERVE[1] was just initiated to address such challenges and investigate control strategies for DERs using a pan-European simulation network. In order to validate the performance of new control algorithms for large grid scenarios, integration of geographically distributed simulation facilities is anticipated in RE-SERVE.

The pan-European real-time simulation infrastructure will be implemented by interconnecting laboratory facilities of four universities, that are depicted in Figure 1, to test frequency and voltage control strategies, providing support to energy stakeholders and regulators in their decision making. Performing the simulations requires large computational power and efficient simulation solvers to enable large-scale network studies and validation of frequency and voltage control algorithms. Geographically distributed simulation would bring together available hardware facilities located in different laboratories.

As discussed in the challenges section, applying an alternative solver seems inevitable. Our dynamic phasor solver as an open source code together with the novel pan-European real-time simulation platform would enable an integrated European virtual simulation environment to simulate larger systems in real-time.

By integrating simulation facilities in different laboratories, the network data and system components do not need to be shared, and all susceptible data can be kept confidential. Only boundary quantities are exchanged via WAN interfaces. This approach would encourage system operators including DSOs and TSOs to perform simulations in collaboration with other laboratories without sharing confidential data. While keeping the main fundamental dynamics of the system, the proposed solver allows to increase simulation time-steps to enable large-scale power system simulation. Commercial EMT real-time simulators such as RTDS and OPAL-RT could be connected to this simulation to investigate specific parts of the grids with a small time step. This would require an interface algorithm between the EMT and the dynamic phasor part such as the one described in [8] but the interface between two dynamic

[1] http://www.re-serve.eu/

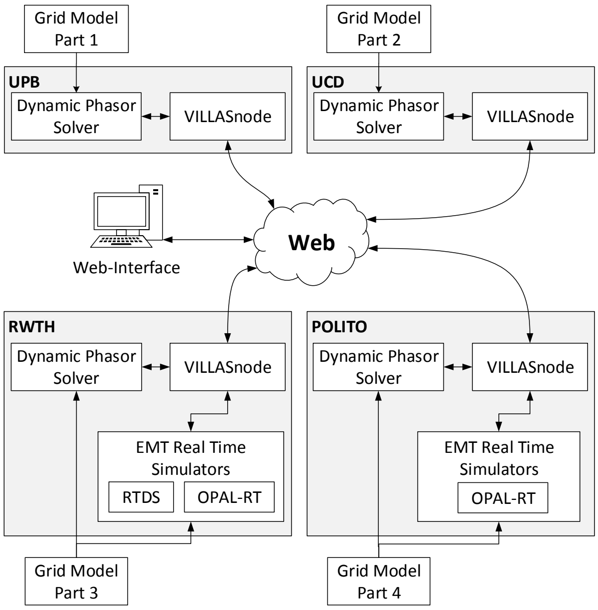

Fig. 1. Distributed real-time simulation infrastructure integrating laboratories of RWTH Aachen University, Politecnico di Torino (POLITO), University Politehnica of Bucharest (UPB), University College Dublin (UCD)

phasor simulators is facilitated a lot since the extraction of the phasor information is not required and the time step can be increased. Then, the communication delay is easier to compensate for.

V. Dynamic phasors in real-time simulation

As mentioned in Section III, the exchange of time-domain values among simulators imposes strong requirements on the sampling rate. Therefore, previous work already introduced dynamic phasors as a means of exchanging data in the frequency domain rather than the time domain [20]. However, this requires the extraction of the dynamic phasors from the time domain signal for every simulation step. Instead, we propose to simulate the entire system in dynamic phasors to be able to increase the simulation time step and to avoid the conversion from the time domain to the frequency domain.

A. Development of Dynamic Phasors

Dynamic phasors were initially developed for power electronics analysis [2]. Later, the concept was extended to power systems analysis [21]. The use of dynamic phasors for power system simulation is described in [22].

This section covers the general approach of dynamic phasors for power system simulation while pointing out the main features that are interesting for distributed real-time simulation. Using Dynamic phasors, it is possible to treat an AC signal as a DC signal without losing its dynamic properties as it is the case when using classic complex phasors in power system analysis. Instead of fixing the frequency, the signal is shifted by the system frequency, e.g. $50\,\mathrm{Hz}$. Besides, one time domain variable can be approximated by several dynamic phasors of different harmonics, each of these shifted by their center frequency. However, this shift only decreases the maximum frequency of the simulated signals if all frequencies of interest lie in a small band around these center frequencies. The fundamental frequency of power systems is normally varying in a region close to the nominal system frequency. Hence, the bandpass limitation is fulfilled. The shift in the frequency domain is visualized in Figure 2.

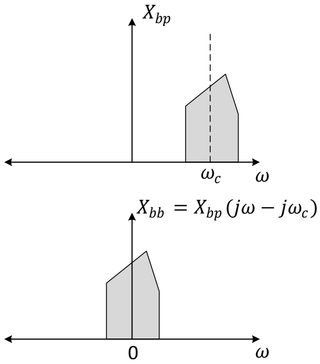

Fig. 2. Shift of band limited signal in the frequency domain

The bandpass signal X_{bp} centered around ω_c is real valued and can be represented in the right half plane of the frequency spectrum. The shifted signal, which is sometimes termed baseband signal, X_{bb} features a smaller maximum frequency. According to the sampling theorem, the baseband signal requires a smaller sampling rate to be represented correctly. This property is very important in the application of real-time simulation since the round trip time (RTT) between two simulators in different locations can be very significant. In case of pan-European simulations the RTT has been found to be several tens of ms [20], whereas links between Europe and the US can exhibit RTT of well over $100\,\mathrm{ms}$ [1]. Therefore, the default time step of $50\,\mu\mathrm{s}$, used by many commercial real-time simulators, does not allow a data exchange between the simulators for every simulation step without compensation for the communication delay.

In the following, the general dynamic phasor approach is explained which is the basis of the simulation example in the next subsection. First of all, the time domain signal x is approximated with a Fourier series representation.

$$x(\tau) = \sum_k X_k(t) e^{jkw_s(\tau)} \tag{1}$$

where $\tau \in (t-T, t]$. The k^{th} coefficient is determined by

$$X_k(t) = \langle x \rangle_k(t) = \frac{1}{T}\int_{t-T}^{T} x(\tau) e^{-jkw_s(\tau)} d\tau \tag{2}$$

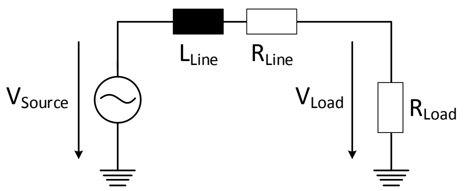

Fig. 3. Example circuit for the comparison of dynamic phasor and EMT simulations

where ω_s is the fundamental system frequency and $k\omega_s$ are its harmonics. Deriving equation (2) leads to

$$\frac{d}{dt}\langle x\rangle_k(t) = \left\langle \frac{d}{dt}x \right\rangle_k(t) - jk\omega_s\langle x\rangle_k(t) \tag{3}$$

Accordingly, a state space model of the general form

$$\frac{d}{dt}x(t) = f(x(t), u(t)) \tag{4}$$

would be transformed to

$$\frac{d}{dt}\langle x\rangle_k(t) = \langle f(x(t), u(t))\rangle_k - jk\omega_s\langle x\rangle_k(t) \tag{5}$$

Applying (5) to the equation of an inductance

$$\frac{d}{dt}i(t) = \frac{1}{L}\cdot v(t) \tag{6}$$

results in the following equation for the fundamental dynamic phasor:

$$\frac{d}{dt}\langle i\rangle_1(t) = \frac{1}{L}\cdot\langle v\rangle_1(t) - j\omega_s\langle i\rangle_1(t) \tag{7}$$

In the next section, a simple circuit, that includes an inductance modeled according to (6) and (7), is simulated using the EMT and dynamic phasor approach for different time steps.

B. Simulation Study for Different Time-Steps

To support the theoretical advantage of dynamic phasor over EMT simulations, we present the simulation results for a simple circuit as depicted in 3. The circuit consists of an AC voltage source of $V_{Source} = 1\,\text{kV}$ peak voltage with a resistance of $R_{Source} = 1\,\Omega$, an RX-series element of $R_{Line} = 1\,\Omega$ and $L_{Line} = 100\,\text{mH}$ and a load resistance of $R_{Load} = 100\,\Omega$. Internally, the voltage source is transformed to its Norton equivalent.

The simulation scenario is as follows. At $0.2\,\text{s}$, the load resistance is decreased to $50\,\Omega$ and at $0.4\,\text{s}$ the frequency of the AC voltage source is decreased from $50\,\text{Hz}$ to $45\,\text{Hz}$. This scenario is simulated for different time steps between $50\,\mu\text{s}$ and $45\,\text{ms}$ using our own simulator that is based on the resistive companion method. In the following, we compare the voltage V_{Load} across the load resistance.

As can be seen in Figure 4 and Table I, the results are almost identical for time steps of $50\,\mu\text{s}$. Figure 4 shows the EMT results, the absolute value of the fundamental

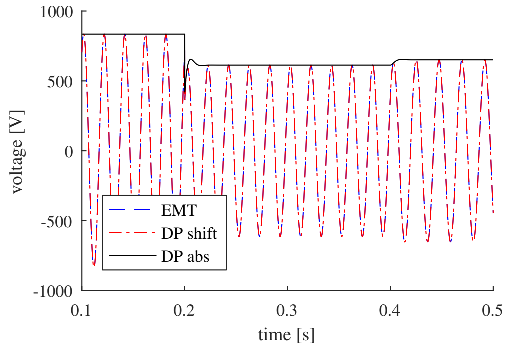

Fig. 4. Comparison of dynamic phasors and EMT simulation for time steps of $50\,\mu\text{s}$

TABLE I
MEAN SQUARED ERRORS FOR V_{Load}

Timestep	EMT	EMT interp.	DP	DP interp.
50 µs	0	-	0.3973E−4	-
1 ms	105.6141	138.9220	97.0038	110.7485
5 ms	7.9126E+3	2.1133E+4	2.0867E+3	588.3851
10 ms	2.5305E+5	2.2442E+5	5.7648E+3	914.9802
15 ms	3.8408E+5	4.5783E+5	1.2080E+4	2.0503E+3
20 ms	2.4815E+5	1.0170E+6	1.6898E+4	3.1879E+3
25 ms	1.5840E+5	5.6950E+5	1.7379E+4	3.9371E+3
30 ms	1.4264E+5	3.1919E+5	2.6352E+4	3.2719E+3
35 ms	3.6699E+5	4.7725E+5	2.9534E+4	3.4567E+3
40 ms	2.6983E+5	0.9913E+6	2.8822E+4	3.6437E+3

dynamic phasor and the time domain signal of the fundamental dynamic phasors after it is shifted back by $50\,\text{Hz}$ in the frequency domain. The shift and transformation into the time domain is accomplished by taking the real part of the signal after applying equation (1). Furthermore, Table I depicts the mean squared error for the signals after linear interpolation. It is important to point out that the interpolation of the dynamic phasors is applied for real and imaginary part separately and before shifting the signal back to $50\,\text{Hz}$. Comparing the $20\,\text{ms}$ time step results presented in Figure 5 and 6, it can be seen that the dynamic phasor simulation is very accurate even for large time steps. Without interpolation, the fundamental sinusoidal is not represented correctly by the dynamic phasor values since the number of data points is too small.

From Table I, it can be concluded that the error is growing much slower for dynamic phasor simulations. Tens of ms seem to be feasible time steps for dynamic phasor simulations if the system transients are not too fast. Therefore, the dynamic phasor approach could enable distributed simulations without having to compensate for the communication delay in some cases, for example, distributed simulation among participants in Europe.

VI. Conclusion

In this paper, the requirements and challenges of distributed real-time simulation are deduced from use cases

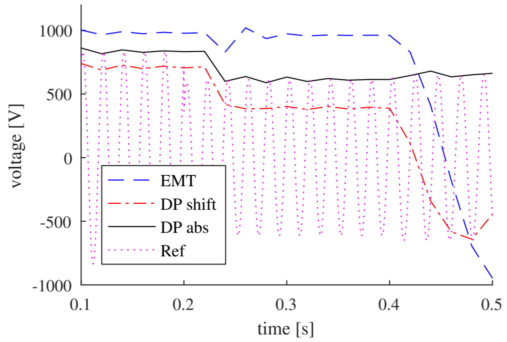

Fig. 5. Comparison of dynamic phasors and EMT simulation for time steps of 20 ms

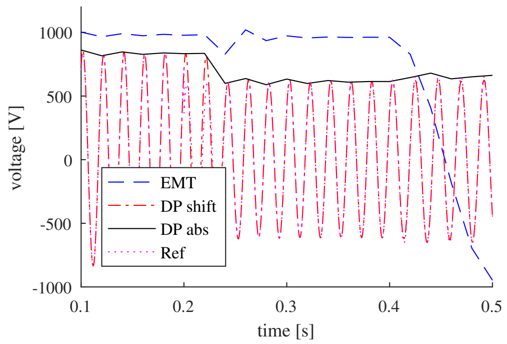

Fig. 6. Comparison of dynamic phasors and EMT simulation for time steps of 20 ms with interpolation

such as the pan-European real-time simulation of the power system. Distributed real-time simulation involving frequency stability and control are found to be very important. Dynamic phasors are proposed as solution to solve one of the main problems of distributed real-time simulation, the large round trip time between the simulators which determines the minimum time step if the simulators are to exchange data for every step. Furthermore, we present a study that shows the advantage of using dynamic phasors for distributed real-time simulation. The focus of this study is on the simulation time step. The larger the time step, the lesser the impact of the communication delay between two geographically distributed simulators on the real-time data exchange.

Currently, the real-time simulation capability is added to the dynamic phasor solver presented in Section V. As next step, we plan to investigate its capabilities regarding parallel power system simulation.

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