The complex interplay between COVID-19 and economic activity

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Abstract

We introduce a dynamical system to model the complex interaction between COVID-19 and economic activity. The model introduces some novelties not accounted by SIR-like models. The equilibrium of the system is an unstable focus, with fluctuations having increasing size and periodicity. Numerical simulations of the model produce waves which reproduce the pandemic dynamics. In observing the stylized facts linking economics and pandemic and stating related reasonable assumptions, we obtain a Lotka-Volterra co-dynamics. This outcome is confirmed by extensive simulations. The outcomes obtained qualitatively replicate some important stylized facts deepening the knowledge about the role of some parameters in their origin and eventually in their shaping.

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1 Introduction

Over the recent months scientists, researchers and policy makers have devoted considerable efforts to study and face complex and unprecedented problems raised by the COVID-19 outbreak. Among the many, one unsolved issue is the contrasting interplay between pandemic and economy. The trade-off between halting the virus spread and halting economy has been largely debated tacking the issue directly and/or indirectly. Among the former we find articles dealing with optimal macroeconomic policies (Lee, 2021; Faria-e-Castro, 2021; Jiwei et al, 2020), distributional consequences of macroeconomic policies (Graham and Ozbilgin, 2021), real business cycles and epidemiology models (Eichenbaum et al, 2021; Glover et al, 2020) and the literature therein, just to cite some of the most representative works of this copious strand of the literature. Among the latter, we find studies focused on the heterogeneous effects produced by shelter-in-place policies (Cerqueti et at, 2021; Dizioli and Pinheiro, 2021; Gallic et al., 2021), the economic determinants of timing and intensity of the policy reaction to pandemic (Ferraresi et al, 2020), and the cost of strict policy measures, such as the lockdown. In turn, this cost has been declined in terms of job losses (Friedson et al., 2021), changes to consumers behavior (Goolsbee and Syverson,

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2021), households income and wealth (Coibion et al., 2020), fairness and cooperation (Buso et al., 2020), uncertainty and expectations (Pellegrino et al., 2021), externalities (Rothert, 2021) and GDP (Ilzetzki and Moll, 2020).

This paper moves from this premise and provides a model for the formal description of the interplay between economic activity and COVID-19 diffusion. Until few years ago, epidemiology and economics have long operated in distinct silos (Murray, 2020). The modeling of the interactions between the spread of a pandemic and economic activity is a recent topic, emerged as a consequence of the COVID-19 pandemic.[1] Most of these models are compartmental models, originated from the work of Kermack and McKendrick (1927), where economic variables have been inserted into a SIR (Susceptible-Infected-Removed) model or into one of its numerous augmented versions (SEIR models, with the addition of Exposed; SEIRD models, with also Deceased, and so on).

We move from some empirical examples to show that some features of the interplay between economy and COVID-19 resemble the classical dynamics obtainable by a Lotka-Volterra type model. For this reason, we present a new theoretical model from which several interesting results emerge. First, our model leads to a waves-shaped dynamics for the pandemics. This is very close to the empirical evidence of the sequence of waves, defined as an increasing number of infected individuals followed by a decrease of such a number (for a formal definition of waves, see Zhang et al., 2021).

Second, we find an unstable focus as an equilibrium of the dynamical system, with fluctuations having increasing size and periodicity. Such an outcome can be interpreted in a very intuitive way by stating that the waves are endogenous to the system. Once received the initial input – i.e. once detected the first cases of infection – waves become endogenous to economic activity. This finding is consistent with the real world, classical SIR-like models are unable to capture it and it is also particularly appropriate in our context. Indeed, waves are an unpleasant feature of the pandemic. Periods of "high" pandemic alternate with periods of relative low virus diffusion and contained consequences. In this respect, we also point out that the proposed model excludes the presence of recovered and immunized elements – hence leading to an infinite number of waves. Therefore, we do not have a herd equilibrium of the pandemic, which is associated to the absence of waves. It is worth noticing that the herd immunity scenario cannot be observed from the data on COVID. Indeed, recovered people move directly to the compartment of susceptible, since vaccines do not provide a complete immunization against the disease and its variants. This makes our model particularly close to the empirical evidence.

Third, the dynamics of the two variables obtained from stylized facts and reasonable assumptions make our model consistent with a Lotka-Volterra prey-predator model. This outcome is confirmed also from the simulations. With respect to the existing literature this is a novelty because while some models assume the Lotka-Volterra co-dynamics (Younes and Hasan, 2020; Mohammed et al, 2021) our analysis attains it as a result of the preliminary analysis of the considered economic-pandemic context.

Fourth, from an empirical viewpoint we show that economic activity and new cases seem to be strictly related in the initial phase of the pandemic with waves of increasing amplitude. This is attributable to the fact that the virus can still affect a quite large number of susceptible individuals in a preliminary phase when no restrictions were into place and in absence of vaccines. The model captures this fact. As time goes by, the introduction of

[1]The most relevant exception is probably Goenka et al. (2014) who, before COVID-19, analyze a neoclassical growth model under a SIS epidemic environment.

vaccines changes the whole story, so that governments do not need any stronger intervention to limit the spreading of the virus; it follows that the co-evolution of economics and epidemics intrinsically changes. We give full account for these aspects by showing that a structural break occurred in the real world time series. Furthermore, one unpleasant characteristic of COVID-19 outbreak is that a recovered individual can be affected again. In terms of stylized facts, this feature works as if recovered do not exist. On this particular stylized fact, our model can be of great usefulness to describe the inception of the pandemic. Interestingly, the outcomes obtained qualitatively replicate some important stylized facts deepening knowledge about the role of some parameters in their origin and eventually in their shaping.
The remainder of the paper proceeds as follows. Section 2 presents a motivating example showing evidence for the US case and refers to the Appendix for further evidence about other countries. Section 3 outlines the dynamical system model capable of accounting for the empirical features emerged in Secion 2. Section 4 provides a study of the map, Section 5 elaborates on the economic implications of the results and finally Section 6 concludes.

2 Motivating examples and arguments

Figure 1 reports the time plot of COVID-19 cases (source: World Health Organization, WHO) and GDP (source: OECD Weekly Tracker of Economic Activity) on a weekly basis for the US from 05jan2020 to 06mar2022.[2] The picture shows a number of interesting features.

[2]Data on COVID-19 have been downloaded from https://covid19.who.int/ and those on weekly GDP from https://www.oecd.org/economy/weekly-tracker-of-gdp-growth/

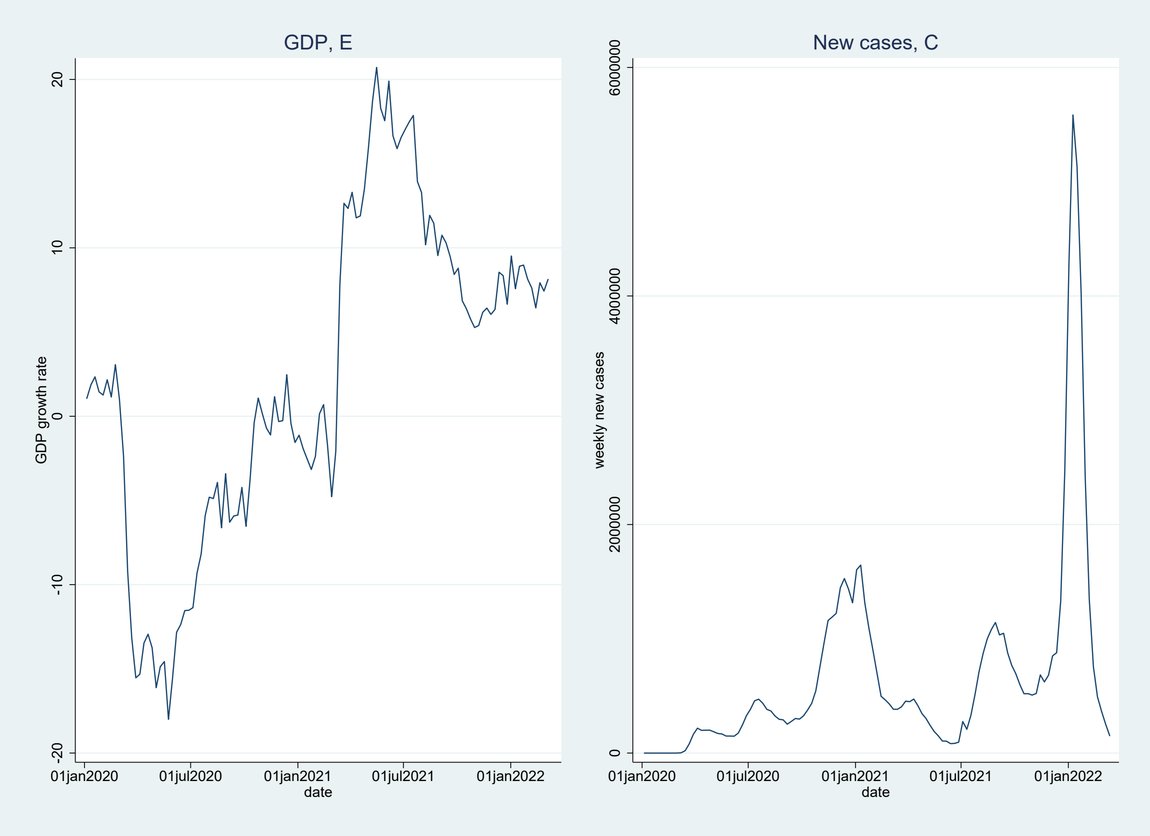

Figure 1: Time series plot of GDP and COVID-19 new cases. The figure reports the time series plot of the US annual GDP growth rate (left panel) and COVID-19 new cases (right panel). Both variables are observed on a weekly basis from 05jan2020 to 06mar2022

First of all the plot of COVID-19 new cases presents waves. Moreover, such waves are associated with two different scenarios so that, ideally, the plot can be broken down into two parts defined by the introduction of vaccines that engender a structural break at the beginning of 2021. Before January 2021, the evolution of new cases is characterized by waves of increasing amplitude. Afterwards, new cases decrease until new variants of the virus emerge. Vaccines are not able to avoid the spread of the new variants, and a second series of waves of increasing amplitude starts in the second half of 2021. Loosely speaking, the time-plot of new cases is the plot of two different viruses, but during the second pandemic a lower amount of infected people is hospitalized (even lower in intensive care units) because vaccines are effective in avoiding severe symptoms. The structural break in the number of new cases automatically affects the relationship with economic activity. Roughly speaking, before the introduction of vaccines for each peak in the new cases time series there is a correspondence with a peak in the GDP growth rate series. The latter seem to anticipate the peaks in the former. Differently, in the second stage, i.e. after the introduction of vaccines, GDP and new cases seem to be unrelated. Indeed, simple statistics reveal that until 31jan2021 the two variables moved together with a correlation coefficient of 0.36 statistically significant at 1%, after that date the correlation drops to -0.17 and becomes not statistically significant at conventional levels. This structural break in the co-movement is essentially due to the fact that once infections became less dangerous restrictions to economy became less stringent, avoiding, for instance the extreme policy

of shelter-in-place order. Therefore, the first phase of the pandemics, the one in which the relationship between the two phenomena is stricter, offers a stylized fact which can be studied and formally modelled. Summing up, the stylized facts (SF) emerging from the example are:
SF1: the evolution of the pandemics is waves-shaped;
SF2: the evolution over time of new cases is a series of waves of increasing amplitude;
SF3: peaks of new cases are almost always anticipated by peaks of GDP per capita.
In other words, when the virus becomes less effective, restrictions can be lessened or even removed. In the short run, restrictions removal has a positive effect on economic activity, so that it peaks, but it lays the foundations for a revival of new cases and GDP peaks anticipate the ones of new cases. As a consequence, new cases lead to new restrictions, causing a drop in economic activity, and keep on doing. It is noteworthy that this process just described resembles very closely what happens in a prey-predator model. This motives us to adapt a prey-predator model to capture the co-evolution of epidemics and economics, instead of using a classical SIR model.
The empirical evidence shown for the US is not different form the one obtainable for other countries. To this aim, we have provided other evidence for Germany, Italy, Japan, Canada and France to further corroborate the theoretical model with empirical instances (see Figure .1 − .5 in the Appendix). In order to account for these interesting features we present a bi-dimensional discrete-time dynamical system modelling the interconnected evolution of a daily measure of the economic activity and the daily number of infected. In doing so, we are in line with the epidemic models based on dynamical systems, by including also the interaction of the economic variables of the environment considered. An overview of these models can be found in Atkenson (2020) that provides obviously just a photography of the state of the art a few months after the emergence of COVID-19. Many works have been done recently. For instance, Droste and Stock (2021) move from a SEIRD model assuming an index of economic activity to be positivily related to the deaths rate caused by the epidemic and the larger the economic activity the larger the coefficient of transmission of the epidemic. They show that their model can fit US data. Eichenbaum et al. (2021) develop an extension of the classical SIR model to test the effects of containment policies when people cut back on consumption and work to reduce the chances of being infected. Further extensions of the SIR model developed to test the effects of containment policies, where epidemic and economic variables are interconnected can be found in Acemoglu et al. (2020), Alvarez et al. (2021), Jones et al. (2021), Zhong et al. (2021), among the others.

3 The model

This section provides a formalization of the complex interaction between COVID-19 and economy. Since the two entities are mutually affected and strictly entangled a dynamic system is the most appropriate tool. We can think of the variable capturing COVID-19 as the number of infected individuals in period t, i.e. the so called new cases in period t, represented by C_t. The evolution of economy can be represented by an index capturing the intensity of economic activity, E_t, such as GDP. It is worth noticing that even though GDP is released quarterly by national institutes of statistics, this does not imply that the phenomenon does not exist on a more frequent basis, e.g. weekly or daily, it is just a matter of measurement. The current frequency of GDP releases does not alter anyway the generality of our analysis and does not prevent GDP from being released on a more frequent basis in the future, should more timely and efficient information systems become available. By definition, both C_t and E_t range in $[0, +\infty)$, for any time period $t = 0, 1, 2, \ldots$. In studying the interplay between E and C some specific instances must be discussed.

(a) If $C = 0$, E growth is proportional to the value reached in the previous period. In other words, the only factor hindering economic growth is COVID-19 and whenever the latter is zero economy is assumed to grow exponentially. This assumption is made in order to concentrate the study only on the relationship between the two variables, abstracting away from any other factor that may generate economic fluctuations, such as economic recessions or fiscal and monetary policies.

(b) If $E = 0$, C decreases proportionally to the value reached in the previous period. The only factor feeding COVID-19 spread is economic activity, whenever the latter is zero infections are assumed to decay at a (negative) exponential rate.

(c) If $C > 0$, in presence of non null economic activity, a higher interaction between E_t and C_t, captured by C_tE_t, leads to a decrease in E_{t+1}. Pandemic wrecks economy and such a deterioration is supposed to be proportional to the amount of C_t and to the level of E_t. This interplay can be affected by stringency measures adopted by national countries, such as the popular lockdown policy.

(d) If $E > 0$, in presence of pandemic, a higher interaction between E and C, captured by C_tE_t, leads to an increase in C. Economic activity eases virus spread, which is supposed to grow proportionally to the amount of C_t and the level of E_t. As before, this interplay can be affected by more or less severe lockdown policies.

The stylized facts described in bullet (a) through (d) can be formalized as follows:

$$\begin{cases} E_{t+1} = [(1+a)E_t - f(C_t, C_{t-1}) \cdot E_tC_t]^+, \\ \\ C_{t+1} = [(1-b)C_t + g(C_t, C_{t-1}) \cdot E_tC_t]^+. \end{cases} \tag{1}$$

where $a, b \in [0, +\infty)$, $[\bullet]^+ = \max\{\bullet, 0\}$, f and g are two functions appropriately defined to take into account the government's intervention affecting the interplay between the two variables as described in point (c) and (d) before.
Specifically, the parameter a represents the growth rate of economy in the absence of pandemics, while b is the natural decay rate of the virus. By definition, a might capture business cycle and the economic status of the regional reality under scrutiny.
The f and g functions play the role of modifying the patterns of economic activity and pandemic, respectively, in accordance with specific government's interventions. In particular, f captures the policies aimed at reducing social interactions among individuals having strong economic implications. Such a function is a quantitative term modeling the effects of the stop to the economic activities – like e.g., entertainment activities or restaurants – and the shelter-in-place order on the dynamics of economics. Differently, the g function translates the effects of non-pharmaceutical interventions on the spread of the disease – like e.g, the use of face masks. The interaction between E and C leads to an indirect effect of non-pharmaceutical interventions on the economy and generates a feedback from the restrictions to the economic activities to the evolution of the pandemic.

The next subsection is devoted to the definition and discussion of the details related to such complex functions.

3.1 The effects of lockdown policies: definition of f and g functions

Intervention policies alter the natural dynamics between E and C aiming at increasing the former and hindering the latter. Their intensity as well as their effectiveness depend on many factors that in our stylized representation are summarized by the level of E. Therefore, we consitently define the functions f and g as follows.

$$f(E_t, C_t, C_{t-1}) = \begin{cases} 0 & \text{if } C_t = 0; \\ \gamma & \text{if } 0 \leq \frac{C_t}{C_{t-1}} < 1 - \lambda_1 \exp\{-\delta_1 E_t\} \text{ and } C_t \neq 0; \\ \psi & \text{if } 1 - \lambda_1 \exp\{-\delta_1 E_t\} \leq \frac{C_t}{C_{t-1}} < 1 \text{ and } C_t \neq 0; \\ \theta & \text{if } \frac{C_t}{C_{t-1}} \geq 1 \text{ and } C_t \neq 0. \end{cases} \tag{2}$$

and

$$g(E_t, C_t, C_{t-1}) = \begin{cases} 0 & \text{if } C_t = 0; \\ \alpha\left(1 - \exp\left\{-\rho_1 \cdot \frac{C_t}{C_{t-1}}\right\}\right) & \text{if } 0 \leq \frac{C_t}{C_{t-1}} < 1 - \lambda_2 \exp\{-\delta_2 E_t\} \text{ and } C_t \neq 0; \\ \alpha\left(1 - \exp\left\{-\rho_2 \cdot \frac{C_t}{C_{t-1}}\right\}\right) & \text{if } 1 - \lambda_2 \exp\{-\delta_2 E_t\} \leq \frac{C_t}{C_{t-1}} < 1 \text{ and } C_t \neq 0; \\ \beta & \text{if } \frac{C_t}{C_{t-1}} \geq 1 \text{ and } C_t \neq 0. \end{cases} \tag{3}$$

where $\alpha, \beta, \gamma, \psi, \theta, \rho_1, \rho_2, \delta_1, \delta_2 > 0$ and $\lambda_1, \lambda_2 \in (0, 1)$. The first line of the two curly brackets represents the trivial case of no pandemic or absence of economic activity, for the f and g functions, respectively. As for the other lines we move from the consideration that an intervention policy, roughly speaking referred to as lockdown, is effective if it leads to $C_t < C_{t-1}$. Accordingly, the cases of C_t/C_{t-1} less than or greater than one are kept separately. In turn, the first case, i.e. $C_t/C_{t-1} < 1$ is broken down into two sub-cases, that is: very effective or fairly effective lockdown. These two sub-cases are distinguished according to a critical value of the ratio C_t/C_{t-1} and to this aim we introduce a threshold, $1 - \lambda \exp\{-\delta E_t\}$. This threshold, no negative and strictly less than one, takes into account an intertemporal discount rate, δ, and an effectiveness parameter, λ which shapes the stringency of the intervention. The more stringent the intervention, i.e. the higher λ, the lower the threshold and the more effective the policy will be, as measured by a lower C_t/C_{t-1} ratio. The same argument is used both in f and g but in order to keep the discussion as general as possible we do not constrain the parameters δ and λ to be the same across functions. This, can be formalized by the following two explicit functional forms of the threshold(s):

$$
\begin{cases}
\text{Regime of very effective lockdown} & \text{if } 0 \le \frac{C_t}{C_{t-1}} < 1 - \lambda_2 \exp\{-\delta_2 E_t\}; \\
\text{Regime of fairly effective lockdown} & \text{if } 1 - \lambda_2 \exp\{-\delta_2 E_t\} \le \frac{C_t}{C_{t-1}} < 1; \\
\text{Regime of ineffective lockdown} & \text{if } \frac{C_t}{C_{t-1}} \ge 1.
\end{cases} \tag{4}
$$

In general, $\lambda_1 \neq \lambda_2$ and $\delta_1 \neq \delta_2$, this allows to model separately the effects of the policy on the evolution of C_{t+1} and E_{t+1}. The g function in (3) is slightly more complicated than f in (2) because the constant term α is multiplied by a function of the COVID growth rate, $\left(1 - \exp\left\{-\rho_i \cdot \frac{C_t}{C_{t-1}}\right\}\right)$ for $i = 1, 2$ in the first two regimes. This complication allows a smooth transition between effective regimes, i.e. from very effective to fairly effective policy and viceversa. Under these regimes, *ceteris paribus*, an increase in C_t does not translate into a proportional increase of C_{t+1} by α, but the proportionality factor is smoothed by $\left(1 - \exp\left\{-\rho_i \cdot \frac{C_t}{C_{t-1}}\right\}\right)$ leading to a smooth transition between regimes. Differently, under the ineffective regime smoothness does not appear and the proportionality factor between C_t and C_{t+1} becomes β, third line of (3). The interpretation of the parameter ρ_i $i = 1, 2$ is akin to the popular R_0 index, measuring the capability of the virus to be transmitted from one individual to another. In other words, ρ_i can be considered as capturing the speed with which the virus spreads around. It clearly appears now that f and g are increasing with respect to the ratio C_t/C_{t-1} and to this aim, we assume that $\alpha < \beta$ and $\rho_1 < \rho_2$ for the function f and $\gamma < \psi < \theta$ for the function g. Eventually, we discuss the specific shape of the functions f and g. Both of them are piecewise-defined functions with discontinuities, in order to capture the effects of intervention policies and the possibility of crossing states, namely the possibility to move from one regime to another, according to the dynamics in (4). In particular, the relationship between intervention policies and a decrease of E proceeds in a constant stepwise form, this is to model the fact that intervention policies are not unique but may take on different intensities from reducing social interactions to stopping entire sectors of economic activity, so that the proportional factor takes on different values in the three regimes.

3.2 The final map

System (1) is a two-dimensional dynamic system made up of two second order difference equations. To obtain a system consisting of only first order difference equations we introduce an auxiliary lagged variable, defined as:

$$
Z_{t+1} = C_t
$$

Accordingly, we can write the final three-dimensional map as follows:

$$
\begin{cases}
E_{t+1} = [(1 + a)E_t - f(C_t, Z_t) \cdot E_t C_t]^+, \\
C_{t+1} = [(1 - b)C_t + g(C_t, Z_t) \cdot E_t C_t]^+. \\
Z_{t+1} = C_t
\end{cases} \tag{5}
$$

with:

$$f(C_t, Z_t) = \begin{cases} \gamma & \text{if } 0 \leq \frac{C_t}{Z_t} < 1 - \lambda_1 \exp\{-\delta_1 E_t\} \text{ and } C_t \neq 0; \\ \psi & \text{if } 1 - \lambda_1 \exp\{-\delta_1 E_t\} \leq \frac{C_t}{Z_t} < 1 \text{ and } C_t \neq 0; \\ \theta & \text{if } \frac{C_t}{Z_t} \geq 1 \text{ and } C_t \neq 0; \\ 0 & \text{if } C_t = 0. \end{cases} \tag{6}$$

and

$$g(C_t, Z_t) = \begin{cases} \alpha\left(1 - \exp\left\{-\rho_1 \cdot \frac{C_t}{Z_t}\right\}\right) & \text{if } 0 \leq \frac{C_t}{Z_t} < 1 - \lambda_2 \exp\{-\delta_2 E_t\} \text{ and } C_t \neq 0; \\ \alpha\left(1 - \exp\left\{-\rho_2 \cdot \frac{C_t}{Z_t}\right\}\right) & \text{if } 1 - \lambda_2 \exp\{-\delta_2 E_t\} \leq \frac{C_t}{Z_t} < 1 \text{ and } C_t \neq 0; \\ \beta & \text{if } \frac{C_t}{Z_t} \geq 1 \text{ and } C_t \neq 0; \\ 0 & \text{if } C_t = 0. \end{cases} \tag{7}$$

4 Study of the map

Let us start with considering the null equilibrium, where all the variables take value 0. Even though an analytical analysis of this point does not lead to straightforward conclusions, numerical simulations show that it is not possible find convergence to that point.

Besides, from the third equation of (5) we know that in equilibrium $C_t = Z_t$. As a consequence, at the equilibrium $\frac{C_t}{Z_t} = 1$, so $f(C_t, Z_t) = \theta$ and $g(C_t, Z_t) = \beta$. Under this circumstance the equilibrium values $\bar{E}$ and $\bar{C}$ solve the following system of equations:

$$\begin{cases} E = (1 + a)E - \theta EC \\ C = (1 - b)C + \beta EC \end{cases}$$

Thereby we obtain the positive valued equilibrium Q where $\bar{E} = \frac{b}{\beta}$ and $\bar{C} = \bar{Z} = \frac{a}{\theta}$.

4.1 Stability of the equilibrium

The Jacobian matrix of (5) calculated at the equilibrium condition $\frac{C_t}{Z_t} = 1$ is the following:

$$J : \begin{bmatrix} 1 + a - \theta C & -\theta E & 0 \\ \beta C & 1 - b + \beta E & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

which calcuated in Q is:

$$J(Q) : \begin{bmatrix} 1 & -\theta \frac{b}{\beta} & 0 \\ \beta \frac{a}{\theta} & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

where one eigenvalue is 0 and that other two come from the submatrix:

$$J_s(Q) : \begin{bmatrix} 1 & -\theta \frac{b}{\beta} \\ \beta \frac{a}{\theta} & 1 \end{bmatrix}$$

$a = 0.05$	$b = 0.025$	$\rho_1 = 0.15$	$\rho_2 = 0.22$	$\alpha = 0.04$
$\beta = 0.07$	$\psi = 0.05$	$\theta = 0.07$	$\lambda_1 = 0.3$	$\lambda_2 = 0.5$
	$\delta_1 = 0.6$	$\delta_2 = 0.35$	$\gamma = 0.04$	

Table 1: Benchmark Scenario.

whose trace is equal to 2 and the determinant is $1+ab$. It follows that, the discriminant of the characteristic polynomial is:

$$\Delta = -4ab < 0$$

and the equilibrium is a focus, so that the eigenvalues are complex and conjugated. The Real part of the eigenvalues is equal to 1, while the Imaginary part is $\pm\sqrt{ab}$.

In order to determine the local stability properties of Q we must check:

$$\mathrm{Re}^2 + \mathrm{Im}^2 = 1 + ab$$

which is clearly larger than one. It follows that the equilibrium is an unstable focus.

4.2 Dynamics and the role of the parameters

In order to understand the typical outcome produced by the dynamical system, let us consider the following pictures.

Figure 2 represents the phase plane (C, E), while Figure 3 plots the simulated time series generated by 2,000 iterations (with C in black and E in blue). The set of parameters used to generate the series are reported in Table 1.

The parameterset in Table 1 are obtained through a trial-and-error mechanism. However, as we will see below, the main analytical result about the nature of the equilibrium (unstable focus) is independent of any feasible combination of the parameters. That is, its qualitative content does not change according to the value of the parameters one chooses.

The pictures show a typical dynamics around an unstable focus, where the variables oscillate in a sort of spiral with fluctuations of increasing amplitude (Figure 2).

Another interesting aspect consists in the asymmetry of the waves. epidemic waves have a very sharp increase and a slow decrease, while economic waves have a slow increase and a crash (Figure 3), in addition E reaches its peaks before C. The role of the various parameters in this kind of dynamics can be numerically investigated. Indeed, they can be gathered into three groups according to the effects they produce on the fluctuations when increasing their value:

- Less frequent fluctuations but with higher peaks (LF);
- More frequent fluctuations with lower peaks (MF);
- No influence (NO)

The numerical simulation gives us the possibility to classify the parameters according to the type of fluctuation they generate. The results are reported in Table 2.

As an example, we report in Figure 4 what we obtain with a value of ψ equal to 0.1355 (less frequent, higher peaks), while Figure 5 shows what one obtains with $\alpha = 0.088$ (more frequent, lower peaks).

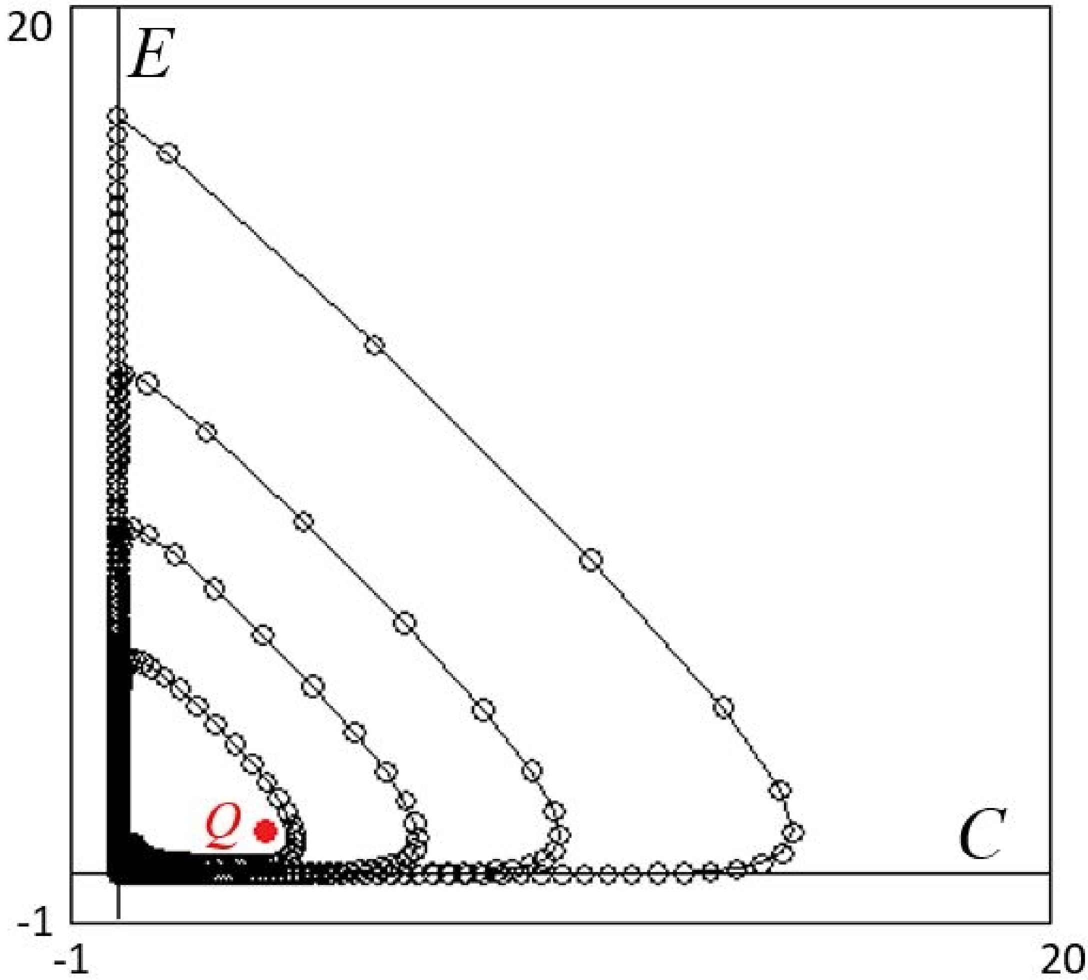

Figure 2: Phase plane (C, E). The figure reports the points of a typical trajectory moving away, oscillating, from the unstable focus Q. Consecutive points are linked with a segment.

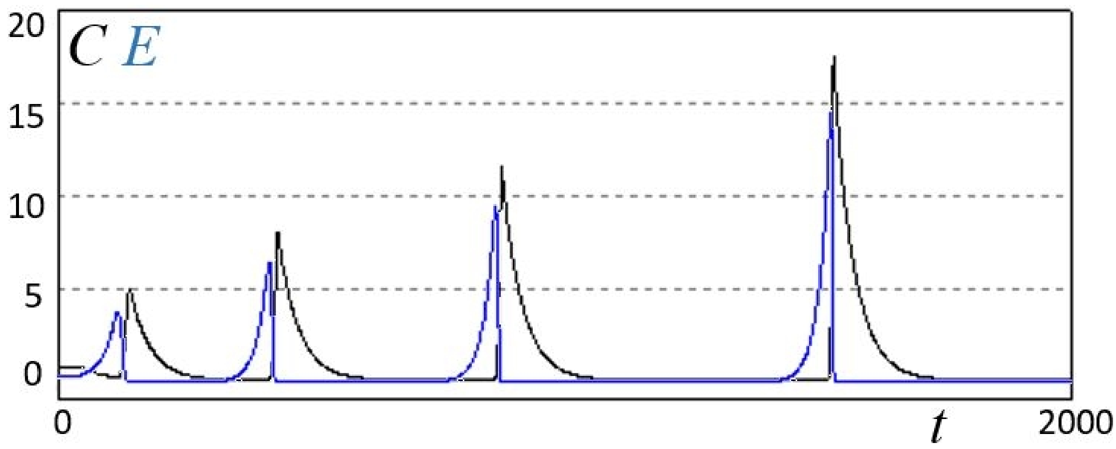

Figure 3: Timeplots. The figure reports the time series of C (panel (a)) and E (panel (b)) corresponding to our benchmark scenario in Table 1.

LF	MF	NO
(b, β, ψ)	$(a, \rho_2, \alpha, \theta)$	$(\gamma, \rho_1, \lambda_1, \lambda_2, \delta_1, \delta_2)$

Table 2: The role of parameters and fluctuations.

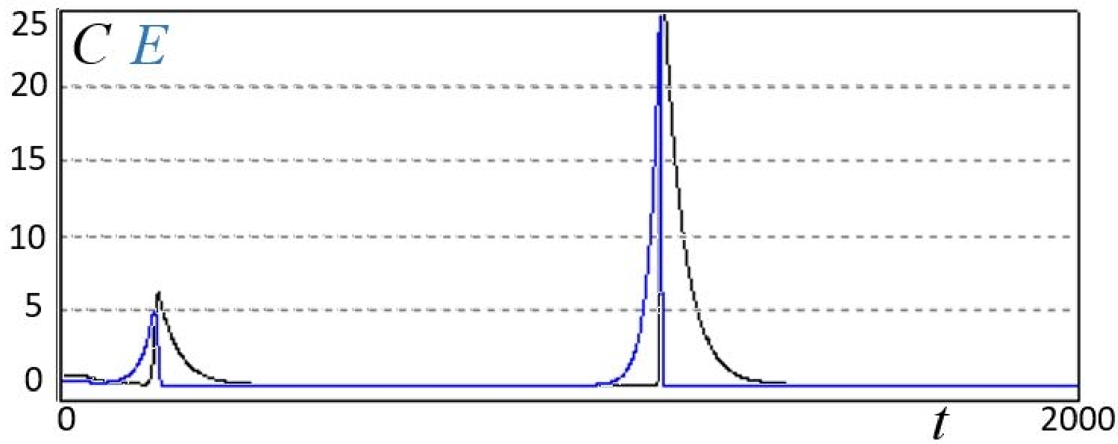

Figure 4: Timeplots. The figure plots the time series of C (panel (a)) and E (panel (b)) corresponding to our benchmark scenario in Table 1 with the exception of the parameter ψ which has been increased to 0.1355.

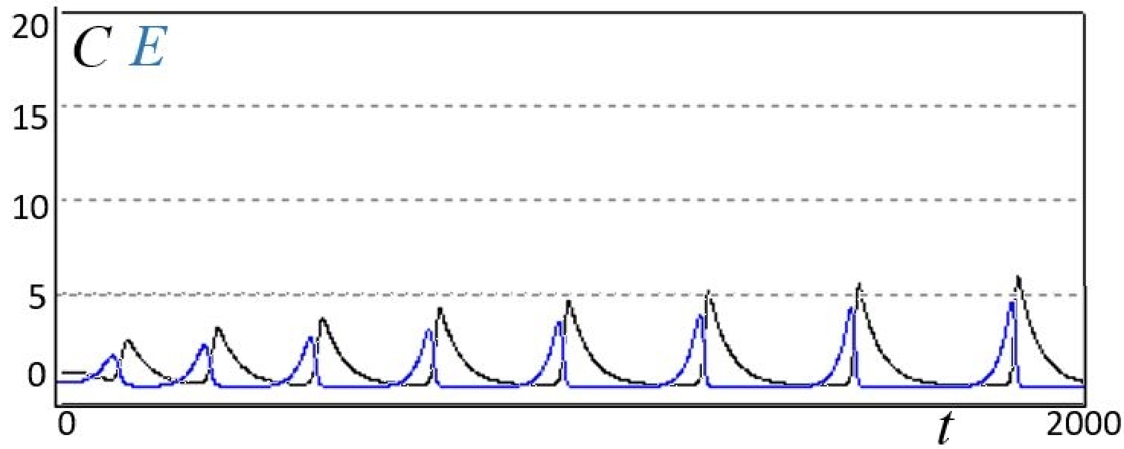

Figure 5: Timeplots. The figure plots the time series of C (panel (a)) and E (panel (b)) corresponding to our benchmark scenario in Table 1 with the exception of the parameter α which has been increased to 0.08.

Very similar results are obtained by perturbing the other parameters belonging to each group.[3]

[3]The results are omitted for sake of room but evidence can be provided to the interested reader upon request.

5 Economic implications of the results

The simulation gives us the possibility to interpret the dynamics traced by the system from a socio-economic point of view as well as the possibility to calibrate it in order to match the actual observed dynamics. The following interesting insights emerge from the simulation.

First, the unstable null equilibrium may be regarded as representative of the fact that it is not a feasible configuration, in the sense that no economy and no COVID is clearly inconceivable. Even more interesting, if one considers as the initial point of the economy the one in which COVID showed up for the first time, i.e. without loss of generality if one normalizes to zero the level of the economy at time $t = 0$, there are no endogenous forces leading back the system to its original value. This gloomy scenario seems (unfortunately) quite real. There are few but significant cases of countries which decided not to introduce restrictions leaving the system to work freely, waiting the variables to go back to its original position: notably the UK and Sweden. In March 2020 the prime minister of the former declared to pursue heard immunity, not introducing restrictions of any type. After few weeks the prime minister himself got sick and the government changed dramatically its strategy introducing travel restrictions, quarantines and closures of entire economic sectors. These limitations have been repeatedly enforced over the year 2020. Similarly, Sweden never adopted a true lockdown measure but at the end of December 2020 the king officially declared that the Swedish strategy against COVID had to be considered as a failure. Successively, the country has introduced restrictions, even tough not as strict as in many other countries.

Second, as we move from the initial point, peaks become higher and higher. This feature is consistent with the fact that the second and the third wave of pandemic are even tougher than the first one.

Third, the fact that the peaks of economy preempt the ones of COVID captures well the fact that when economic activity restarts human interactions invariably increase and the risk of contagion increases, therefore the COVID peak, in such a situation, is the direct consequence of an increased economic activity.

Fourth, the simulated dynamics in (3) is the result of contrasting forces. On the one side, there are forces pushing towards less frequent and stronger waves of pandemic, LF. On the other side, there are forces pushing the system towards exactly the opposite direction, MF. Daily chronicle shows us that the first effect occurs when the lockdown policy is stringent because it delays the wave but once the lockdown finishes the wave is stronger than ever. This has occurred, for instance in Italy and Germany with the second COVID wave occurred in Autumn 2020 after a complete lockdown. Few months before, over the summer, the economic recovery had been substantial, with an GDP increase of 16.1% and 8.2% on a quarterly basis in the third quarter in Italy and Germany, respectively. Concerning the effect induced by LF, it seems to capture well the effect generated by constant and less stringent measures, such as those introduced in 2021 in Sweden, aiming at avoiding agglomerations of people but not intended to stop the economic activity.

Given the strong halt that strict measures have imposed to economy, governments throughout the world now favour less stringent measures. Under this circumstance the simulations of the model predict a shorter time between one wave and the other but with lower virulence, as long as lower economic rebounds.

6 Concluding remarks and further research

The theoretical model and its simulated results present interesting features capable of mimicking the actual fluctuations of economy and COVID. Notably, the waves of COVID are endogenous to the economic system. Periods of "high" pandemic alternate with periods of relative low virus diffusion and contained consequences. This feature is not accounted by SIR-like models and we deem it as important given the adherence with the real world. In addition, the dynamics of the two variables simulated from our model are consistent with the Lotka-Volterra prey-predator model, hence being consistent with stylized facts and reasonable assumptions employed for properly formulating the two-dimensional map. This result can be considered as an ex-post validation of the papers in the literature that assumes such a dynamic, without deriving it from scratch. The results obtained can be useful to tailor effective public health strategies to curb the spread of COVID-19 and its relationships with economic activity. Notably, the model seems to be a useful tool to envisage the effects of the policies put into place, on both economic activity and the outbreak spread. As far as further research is concerned, an important step ahead is represented by the inclusion of recovered and immunized elements in the model, in order to envisage the pandemic dynamics in presence of effective vaccines.

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Appendix

This appendix reports the time plot of COVID-19 cases (source: World Health Organization, WHO) and GDP (source: OECD Weekly Tracker of Economic Activity) on a weekly basis for Germany, Italy, Japan, Canada and France from 05jan2020 to 06mar2022. These pictures show a profile similar to the one for the US, as in Figure 1 extending the validity of the motivating example and the stylized facts captured by the model. Data on COVID-19 have been downloaded from https://covid19.who.int/ and those on weekly GDP from https://www.oecd.org/economy/weekly-tracker-of-gdp-growth/

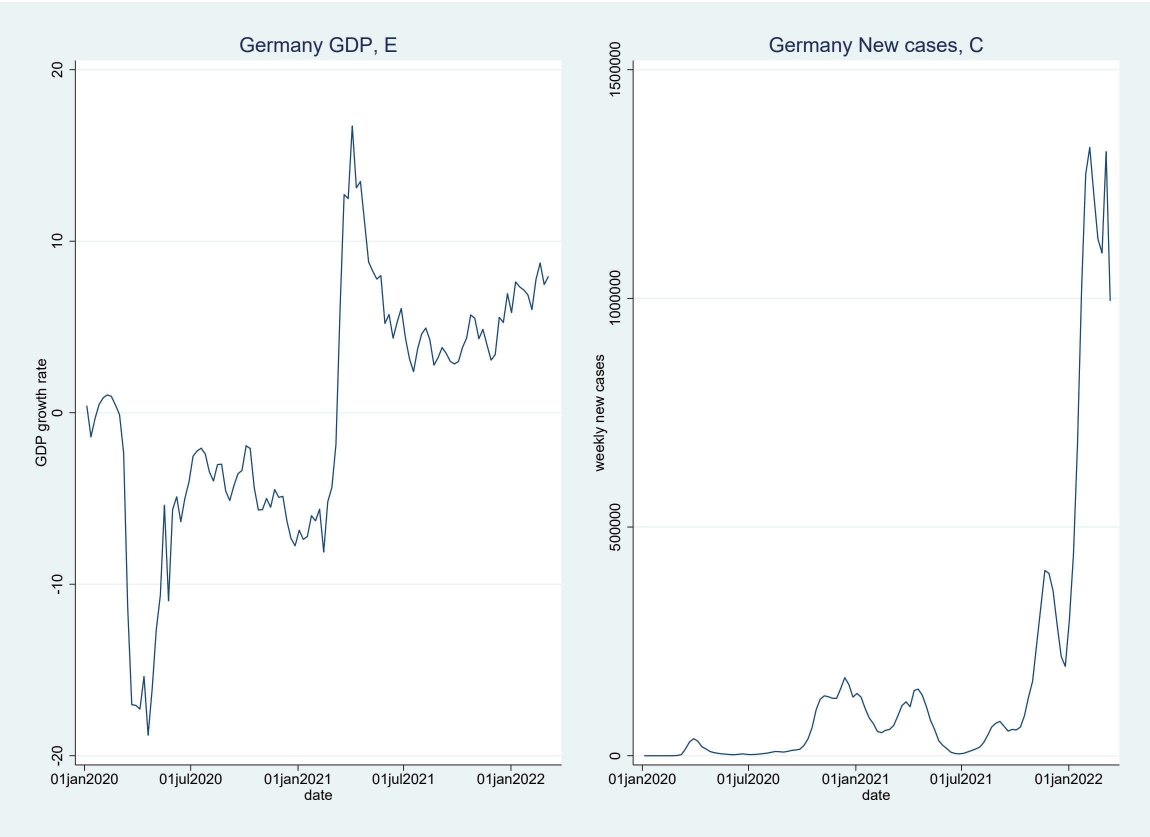

Figure .1: Time series plot of GDP and COVID-19 new cases. The figure reports the time series plot of annual GDP growth rate (left-hand-side) and COVID-19 new cases (right-hand-side) for Germany. Both variables are observed on a weekly basis from 05jan2020 to 06mar2022

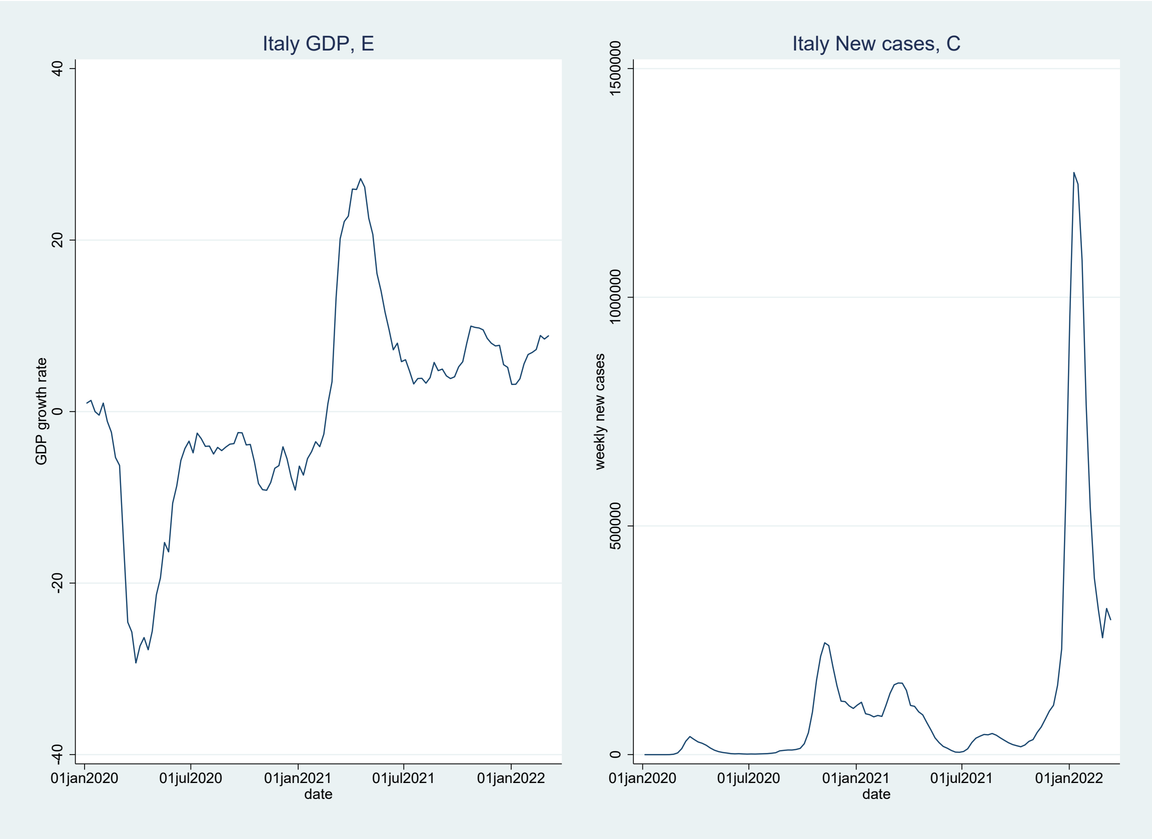

Figure .2: Time series plot of GDP and COVID-19 new cases. The figure reports the time series plot of annual GDP growth rate (left-hand-side) and COVID-19 new cases (right-hand-side) for Italy. Both variables are observed on a weekly basis from 05jan2020 to 06mar2022

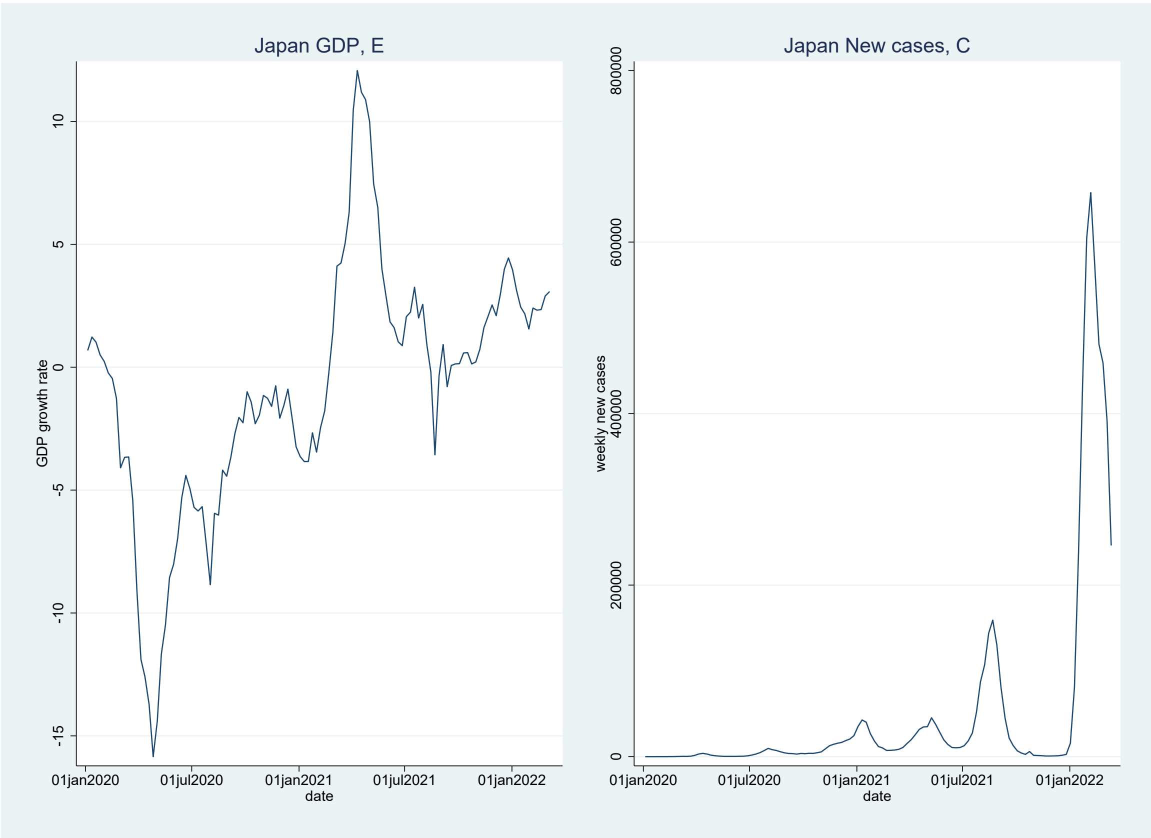

Figure .3: Time series plot of GDP and COVID-19 new cases. The figure reports the time series plot of annual GDP growth rate (left-hand-side) and COVID-19 new cases (right-hand-side) for Japan. Both variables are observed on a weekly basis from 05jan2020 to 06mar2022

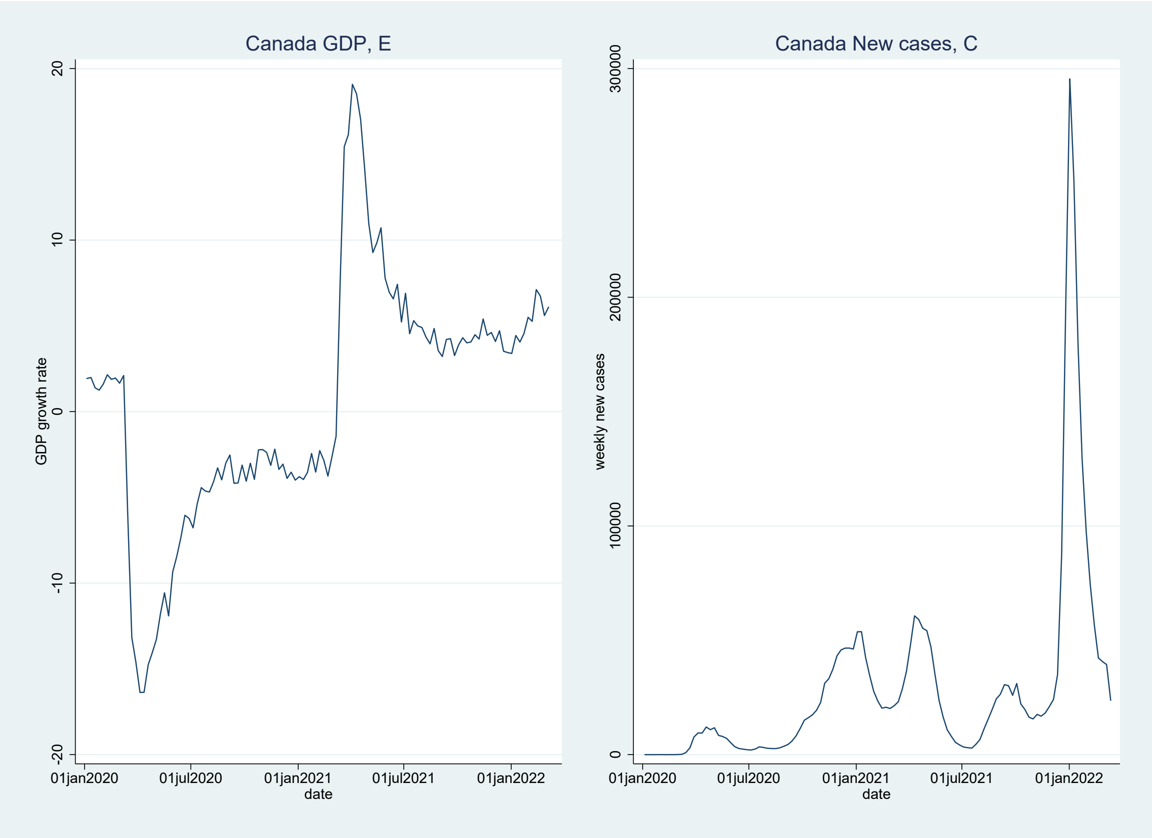

Figure .4: Time series plot of GDP and COVID-19 new cases. The figure reports the time series plot of annual GDP growth rate (left-hand-side) and COVID-19 new cases (right-hand-side) for Canada. Both variables are observed on a weekly basis from 05jan2020 to 06mar2022

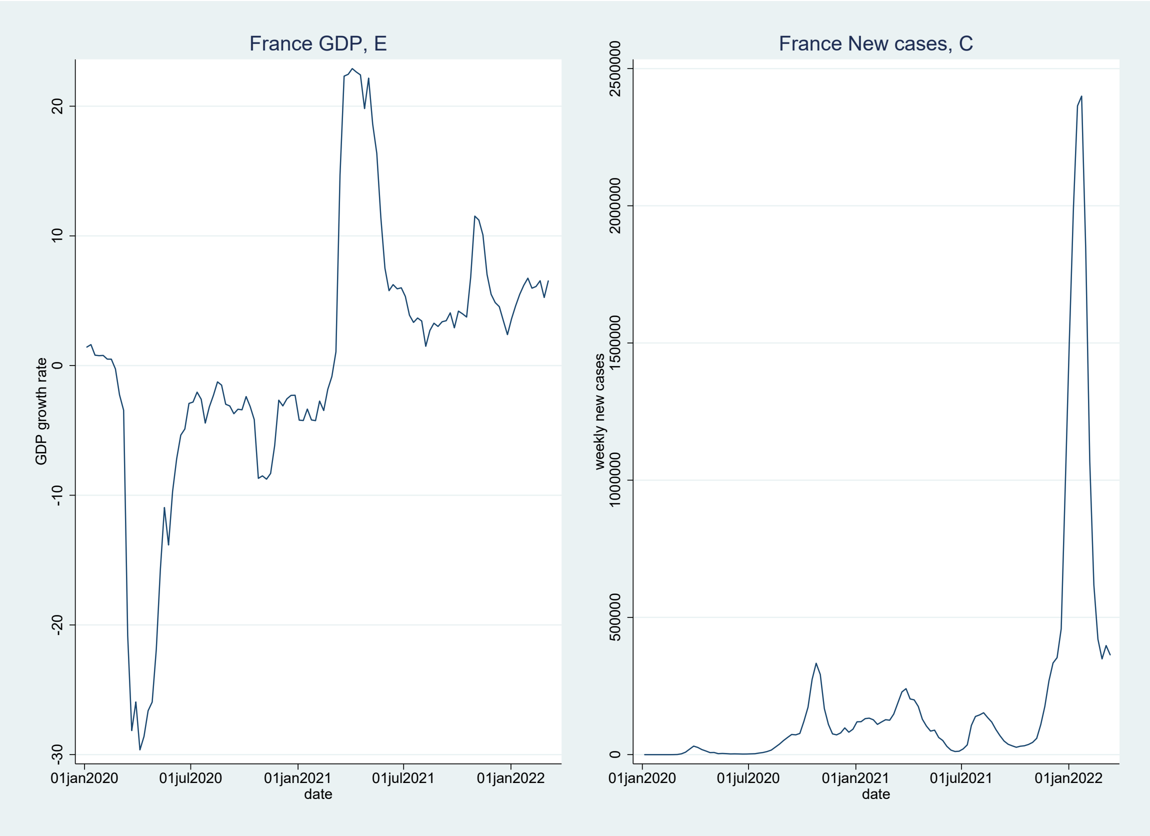

Figure .5: Time series plot of GDP and COVID-19 new cases. The figure reports the time series plot of annual GDP growth rate (left-hand-side) and COVID-19 new cases (right-hand-side) for France. Both variables are observed on a weekly basis from 05jan2020 to 06mar2022