IX International Conference on Computational Plasticity
COMPLAS IX
E. Oñate and D. R. J. Owen (Eds)

NUMERICAL MODELLING OF FORMATION AND PROPAGATION OF DRYING CRACKS IN SOILS

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Key words: drying cracks; state surfaces; fracture mechanics; remeshing; boundary value problems.

Summary: In this paper a general framework is presented using a Finite Element Model (FEM) combining Continuum Mechanics, Fracture Mechanics and Unsaturated Soil Mechanics, together with re-meshing techniques, to simulate and solve problems of cracking due to drying in soils in a consistent manner. The formulation includes the unsaturated soil equilibrium equation and the mass balance equation of continuum mechanics and then by means of fracture mechanics criteria the initiation and propagation of the cracking mechanism is simulated. The re-meshing technique permits to model the time evolution of cracking in form of discrete fissures and update the mesh at each time step. The change of geometry and boundary conditions is also considered when the cracks initiate or propagate.

1 INTRODUCTION

Present models available in the literature dealing with drying cracks in soils do not include all aspects of the problem in an integral manner. Usually they only solve partial aspects separately like occurrence and morphology of cracks, depth and spacing of cracks, consolidation and desiccation, shrinking-swelling, etc. In this paper a general framework is presented using a FEM (Galerkin method) combining Continuum Mechanics, Fracture Mechanics and Unsaturated Soils Mechanics, together with re-meshing techniques, to simulate and solve problems of cracking due to drying in soils in a consistent manner. Continuum Mechanics contributes with two balance equations: Equilibrium and Water Balance Equation. Unsaturated Soils Mechanics appears with the State Surface concept, Darcy's Law and Retention Curve. Concerning Fracture Mechanics we use Linear Elastic Fracture Mechanics (LEFM), not taking into account the plastic zone at the crack tip.

Two state variables are adopted in this model: net stress and suction. The model is hydro-mechanically coupled: for the mechanics part, a non-linear elasticity based model is chosen, while for the hydraulics problem, Darcy's law, including unsaturated flow, is used.

The theoretical formulation has been made in the most general manner thinking in solving 3D problems. Implementation results are left for subsequent work because the main purpose of the current research is to establish a solid basis for the finite element code that will be a tool capable to approach complex phenomena such as drying cracks in soils.

2 HYDRO-MECHANICAL MODEL

Two state stress variables are adopted in this model: net stress $\boldsymbol{\sigma}^*$ and suction s.

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma} - p^a \mathbf{1} \tag{1}$$

$$s = p^a - p \tag{2}$$

With the assumption that atmospheric pressure p^a is constant and equal to zero, the state variables become the total stress $\boldsymbol{\sigma}$ and the negative pore water pressure p.

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma} \qquad \text{and} \qquad s = -p \tag{3}$$

Under the assumptions of small-strain theory, isothermal equilibrium and negligible inertial forces, we obtain the following balance equations. First, the linear momentum balance equation for a two phase medium, where ρ is the density and $\mathbf{g}$ is the gravity vector.

$$div(\boldsymbol{\sigma}) + \rho \mathbf{g} = 0 \tag{4}$$

Second, the mass balance equation for water, where ρ^w is the water density, $\mathbf{q}$ is Darcy's velocity, n the porosity and S_r the degree of saturation.

$$div(\rho^w \mathbf{q}) + \frac{\partial}{\partial t}(\rho^w n S_r) = 0 \tag{5}$$

We can summarize the coupled problem through the next system of differential equations:

$$\begin{cases} \mathbf{K}\dfrac{\partial \overline{\mathbf{u}}}{\partial t} + \mathbf{Q}\dfrac{\partial \overline{\mathbf{p}}}{\partial t} - \mathbf{f}^u = \mathbf{0} \\ \mathbf{P}\dfrac{\partial \overline{\mathbf{u}}}{\partial t} + \mathbf{S}\dfrac{\partial \overline{\mathbf{p}}}{\partial t} + \mathbf{H}\overline{\mathbf{p}} - \mathbf{f}^p = \mathbf{0} \end{cases} \tag{6}$$

Where $\overline{\mathbf{u}}$ and $\overline{\mathbf{p}}$ are respectively nodal displacements and nodal pore pressure. $\mathbf{K}$, $\mathbf{Q}$, $\mathbf{P}$, $\mathbf{S}$ and $\mathbf{H}$ are matrices, and $\mathbf{f}^u$ and $\mathbf{f}^p$ vectors, that result from the FEM approach.

The algebraic system of equations that results from this formulation is highly nonlinear and non-symmetric, in general. For this reason we need to use iterative strategies to solve it. The first equation in (6) is the mechanical part and the second one is the hydraulic part.

3 UNSATURATED SOIL MECHANICS CONCEPTS

For hydro-mechanical formulation we need to employ two constitutive models. First we apply a mechanical constitutive model based on the concept of state surfaces. Next a hydraulic constitutive model, including unsaturated flow (Darcy's law) is used. We write both constitutive models as follow:

$$\varepsilon_v = -\frac{e}{1+e_0} = a_1 \ln(\boldsymbol{\sigma} + a_4) + a_2 \ln\left(\frac{p + p_{ref}}{p_{ref}}\right) + a_3 \left[\ln(\boldsymbol{\sigma} + a_4)\ln\left(\frac{p + p_{ref}}{p_{ref}}\right)\right] \tag{7}$$

$$\mathbf{q} = -\mathbf{K}(S_r)\cdot(\nabla p - \rho^w \mathbf{g}) \tag{8}$$

Where ε_v is the volumetric strain, e and e_0 are the current and initial void ratios, a_1, a_2, a_3, a_4 are state surface constants and p_{ref} is a reference pressure.

Furthermore, we need the relation between suction and degree of saturation. In our case we chose the Van Genuchten equation (Van Genuchten 1980 [8]):

$$S_r = \left[1 + \left(\frac{p}{P_0 f_n}\right)^{\frac{1}{1-\lambda}}\right]^{-\lambda} \tag{9}$$

In (9) S_r is the degree of saturation, λ is a material parameter, P_0 is the air entry value at the reference porosity n_0 and f_n is a function of porosity and material parameter η.

4 MODELING FRACTURE BEHAVIOUR

To model fracture behaviour we need to compute the stress tensor at each node through our hydro-mechanic model, and we use it to calculate the maximum principal stress. If the maximum principal stresses exceed the threshold stress of the material, we mark the corresponding node "n" as a possible candidate for fracture initiation.

Once a set of candidate nodes are identified, we extend the fracture to these nodes and the finite element mesh is adjusted. We use the same procedure for both incorporation of the onset of a new fracture and the propagation of an existing fracture. The input procedure is the location of the fracture, specified by fracture node "n" and the corresponding nodal stress tensor $\boldsymbol{\sigma}$. The fracture plane "p" is defined by the eigenvector of $\boldsymbol{\sigma}$ which corresponds to the maximum principal stress.

5 IMPLEMENTATION STRATEGIES

The model will be able to reproduce laboratory tests on rectangular and circular specimens of different thickness. Real soil is not homogeneous and at each point we can find different physical properties. This fact will be captured by imposing at each node small random differences in its properties.

Finally we present a flow chart of drying cracks in soils (Figure 1). The three main program's blocks are represented in light blue colour. These are: Discretization and re-meshing routines, Hydro-mechanics model and unsaturated soil mechanics model, and Fracture Mechanics Model for crack initiation and propagation. Yellow colour indicates the two most important decision structures.

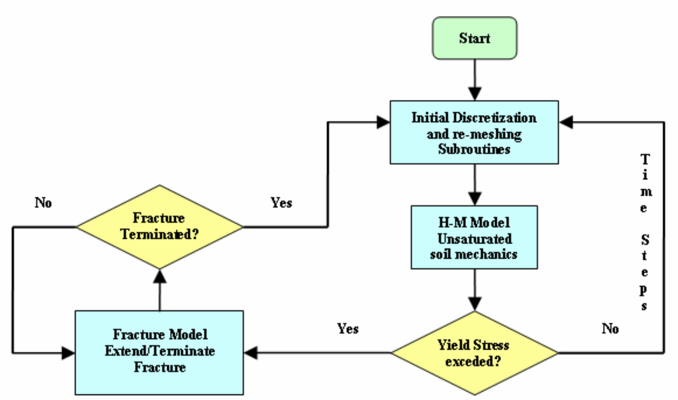

Figure 1: Program Flow Chart of Drying Cracks in Soils.

6 CONCLUSIONS

A complete formulation to solve cracking in drying soils problems has been presented. Meshing techniques, hydro-mechanical model for unsaturated soils and Linear Elastic Fracture Mechanics will permit to reproduce laboratory tests of soils under desiccation. Preliminary results indicate that the Finite Element Method with Unsaturated soils mechanics concepts and Re-meshing Techniques, Continuum Mechanics and Fracture Mechanics are viable tools for modelling desiccation cracks processes in soils.

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