**Mechanical Models for Local Buckling of Metal Sandwich Panels**

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**Abstract**

Modern design methods for sandwich panels must attempt to maximise the potential of such systems for weight reduction thus achieving highly optimised structural components. A successful design method for large-scale sandwich panels requires the consideration of every possible failure mode. An accurate prediction of the various failure modes is not only necessary but it should also utilise a simple approach that is suitable for practical application. To fulfil these requirements, a mechanics-based approach is proposed in this paper to assess local buckling phenomena in sandwich panels with metal cores. This approach employs a rotational spring analogy for evaluating the geometric stiffness in plated structures, which is considered with realistic assumed modes for plate buckling leading to accurate predictions of local buckling. In developing this approach for sandwich panels with metal cores, such as those with a rectangular honeycomb structure, due account is taken of the stiffness of adjacent co-planar and orthogonal plates and its influence on local buckling. In this respect, design-oriented models are proposed for core shear buckling, intercellular buckling of the faceplates and buckling of slotted cores under compressive patch loading. Finally, the proposed design-oriented models are verified against detailed nonlinear finite element analysis, highlighting the accuracy of buckling predictions.

**Keywords**

Buildings, structures & design; Mathematical modelling; Slabs & Plates;

**List of notation**

 is the plate dimension in the x-direction

 is the plate dimension in the y-direction

 is the buckling coefficient

 is the compressive stress resultant in x-direction

 is the compressive stress resultant in y-direction

 is the planar shear stress resultant

 is the critical buckling load

 is the critical buckling stress

 is the SDOF unknown coefficient

 is the vector of discrete freedoms

 is the vector of deformation modes *a*

 is the elastic modulus of the plate

 is the thickness of the plate

 is the Poisson’s ratio

 is the stiffness of the equivalent rotational springs

 is the geometric stiffness matrix

 is the material stiffness matrix

 is the stiffness of the edge rotational spring

 is the deformation matrix

 is the deformation mode

 are the rotations in the x-direction

 are the rotations in the y-direction

 is the stress ratio 

 is the edge rotation of the plate in the x-direction

 is the edge rotation of the plate in the y-direction

**1. Introduction**

Sandwich systems are recognised for achieving enhanced structural performance at minimal weight and are therefore becoming attractive for applications where saving weight is a fundamental objective. Maximising the potential for weight savings requires an accurate design method to be established, which should also be simple for practical application. Achieving an accurate and practical design methodology for any structural component requires a comprehensive understanding of the various failure modes. Plate buckling constitutes an integral part of such design methodology for rectangular honeycomb core sandwich panels, as these are typically fabricated from thin plates. The elastic local buckling load of plates can be predicted in several ways, ranging from simple analytical expressions based on idealised support conditions to complete geometric and material nonlinear analysis accounting for initial imperfections, thus achieving different degrees of accuracy obtained with methods of different sophistication and computational demand.

Consider a thin, perfectly flat, elastic rectangular plate subject to in-plane compressive and shear loads. Plate buckling occurs at specific levels of loading which cause the initial flat configuration of equilibrium to become unstable. Under small disturbances, the plate adopts a nontrivial equilibrium state defined by a deflection pattern, which induces bending deformations on the plate. The bending of thin plates is effectively described by Kirchhoff’s plate theory, where the transverse shear strain is neglected across the thickness of the plate. Figure 1 presents a rectangular plate, with planar dimensions  and  under general planar loading:

The determination of the critical buckling load can be achieved by transforming the stability problem into an eigenvalue problem, considering proportional planar loads that are scaled by a load factor . Solving the governing differential equation of plate buckling in such a way results in several non-trivial solutions, or eigenvalues, in which the smallest is the critical buckling load . In plate buckling, the critical buckling load  and respective critical buckling stress  are generally written as:



1.

where  is the elastic modulus of the plate,  is plate thickness,  is Poisson’s ratio,  is the plate bending stiffness,  is a dimension of the plate and  is the buckling coefficient. The definition of the buckling coefficient, hence the corresponding buckling load, depends on the aspect ratio of the plate, the stress state and the support conditions.

Throughout the years, the prediction of buckling of several plated structures has been mathematically realised for idealised support conditions and loadings. For support conditions that cannot be idealised either by simple or fixed supports, the calculation of the buckling coefficient becomes more complex, where a methodology based on the energy method can be advantageous. For the application of the methodology proposed in this paper, the previous knowledge on the buckling of rectangular plates is thoroughly reviewed, mainly focusing on simply supported and clamped plates, under uniaxial and biaxial compressive stress as well as under shear stress. Navier derived the stability equation for rectangular plates under lateral load and formed the basis for much of the work subsequently undertaken on the stability of plated structures. By considering that the deflection can be written as a double Fourier series, Bryan (1891) obtained the general exact solution for the stability equation applied to the buckling behaviour of rectangular simply supported plates under biaxial compressive stress. However, the buckling of clamped plates under biaxial compressive stress proved to be remarkably more complex. By applying edge moments to the simply supported case and therefore enforcing zero slope at the edges of the plate, Levy (1942) presented a solution for a clamped plate under uniaxial compressive stress. Timoshenko and Gere (1961) assumed a shape function which accurately predicts the buckling mode of a square clamped plate subjected to constant biaxial compressive stress, subsequently leading to the critical buckling capacity of such plates. Wong and Bettess (1979) established a numerical procedure capable of accurately predicting the elastic buckling load of rectangular clamped plates subjected to general biaxial compressive stress state, which however proved to be rather unsuitable for practical application since it required an advanced mathematical formulation to overcome the lack of computational power at the time and does not provide an intuitive framework for practitioners for buckling assessment of structures. On the other hand, Zenkert (1995) proposed a deformation mode for the global buckling for clamped isotropic sandwich panels subjected to uniaxial compression, and achieved an accurate prediction of the buckling capacity of such structures. Timoshenko and Gere (1961) presented the exact solution for the buckling of simply supported rectangular plates subjected to the action of constant shear forces along the edges by using a double trigonometric series.

The application of sandwich panels for structural purposes drove the scientific research on the topic, from which design-oriented methods were developed to assess its local buckling capacity. Kolsters and Zenkert (2006a; 2006b) established a simplified methodology for local intercellular buckling assessment of I-core sandwich panels, under stress parallel and normal to the core strips, including the influence of a continuous foam core using a Pasternak foundation model. Experimental validation of the developed approach was further achieved by means of edgewise compressive tests, verified by finite element analysis (Kolsters and Zenkert 2009). Jimenez and Triantafyllidis (2013) determined the local critical buckling loads under compression and transverse shear based on unit-cell calculations, considering both hexagonal and rectangular honeycomb cores and considering the orthogonal strips to be fully connected.

A mechanics-based approach is proposed in this paper to predict the buckling of plates under complex support conditions, which are of practical importance for the establishment of a design methodology for rectangular honeycomb core sandwich panels. The local buckling performance of such panels can intrinsically be determined by three failure modes: Intercellular buckling of the faceplates subjected to biaxial compressive stress state, considering the effect of the discrete rectangular honeycomb core; Buckling of the plates of the core under pure shear, considering rotational springs along their edges; Compressive buckling of rectangular honeycomb cores, where the orthogonal strips are slotted together even though they are not posteriorly welded. The establishment of a mechanics-based design approach enables the critical buckling load associated with these different failure modes to be evaluated and to be subsequently implemented in the design and optimisation of all-metal sandwich panels.

**2. Rotational Spring Analogy**

A “rotational spring analogy” (RSA) (Izzuddin 2007) is employed in this paper for the assessment of buckling, as it enables the effective and accurate formulation of the geometric stiffness for various structural forms. In this approach, the geometric stiffness is accurately represented by a set of equivalent rotational springs, and is thus easily recovered using first-order kinematics relating the rotations of the equivalent rotational spring to the system degrees-of-freedom (DOF). This analogy describes buckling phenomena using common notions from linear structural analysis, rendering it a highly intuitive approach for practical application (Izzuddin 2006).

For continuous systems, the equivalent rotational springs, which are associated with the internal stresses of the structure, are uniformly distributed over the element domain with a rotational stiffness per unit volume equal to the internal normal stress. A compressive stress would be associated with a negative rotational spring that contributes to destabilise the structure, while a tensile stress would have an opposite stabilising effect. In the case of plate buckling under loading that induces biaxial normal stresses, the geometric stiffness is obtained from two sets of orthogonal equivalent rotational springs distributed over the plate area. Considering problems defined by a single degree-of-freedom (SDOF), the RSA enables the determination of the Rayleigh quotient (Bažant and Cedolin 1991), which has the characteristic of producing the exact critical buckling load when the exact buckling mode  is employed. For a plate sustaining general planar stress resultants , and and discretised using a SDOF unknown coefficient , which is associated to a generalized displacement, the geometric stiffness  is obtained as follows (Izzuddin 2007):



2.

where  and  are the dimensions of the plate presented in Figure 1,  represents the stiffness of the equivalent rotational springs distributed over the area of the plate and  is a 2×1 matrix expressing the first-order kinematic relationship between rotations and the SDOF unknown coefficient :



3a.



3b.

where  and  represent the rotations of the plate in the x- and y-directions which are respectively obtained as the derivatives of the deformation mode  with respect to the x- and y-axes.

The material stiffness, on the other hand, is obtained using the deformation matrix , which expresses the relationship between curvatures and the SDOF unknown coefficient . Assuming linear elastic isotropic material:



4.

Buckling can then be considered to occur when the geometric stiffness becomes sufficiently negative to equal the positive material stiffness leading to a structure in neutral equilibrium. The buckling load can therefore be obtained by equating the sum of the geometric and material stiffness to zero.

For problems that are not adequately represented with a single deformation mode, a multiple degree-of-freedom (MDOF) formulation is required. The application of the RSA to continuous MDOF problems is formulated similarly to the Rayleigh-Ritz variational method (Bažant and Cedolin 1991), where the buckling mode of the plate is estimated as a linear combination of several predefined modes:



5.

where  is a vector of assumed modes, and  is a vector of unknown coefficients.

Both geometric and material stiffness matrices will therefore be  matrices, determined by (2), (3a) and (4), where  is now a  matrix expressing the first-order kinematic relationship between rotations and the unknown coefficients:



6.

Buckling occurs when the tangent stiffness matrix, which consists of the contributions from the geometric and material stiffness matrices, becomes singular, leading to the evaluation of the buckling load factor as the solution of the corresponding eigenvalue problem.

**3. Buckling failure modes**

As sandwich panels are structural components fabricated from thin plates, local plate buckling represents one of the main sources of failure that should be considered in the design and assessment of such components. Three failure modes associated with plate buckling and characteristic of rectangular honeycomb core sandwich panels are considered hereafter: i) intercellular buckling of faceplates, ii) shear buckling of the core, and iii) compressive buckling of rectangular slotted cores.

The detailed response of a rectangular honeycomb core sandwich panel under flexural loads can be obtained using high fidelity nonlinear finite element analysis (Nordas *et al.* 2018), with a typical deformed configuration illustrated in Figure 2. The behaviour of sandwich panels is fundamentally defined by the sandwich effect: the core resists shear loads while stabilising the top and bottom faceplates which act together to form a stress couple resisting the resulting bending moment. This implies that the faceplate stiffened by the core might buckle under biaxial compressive stress, with a ‘rippling’ intercellular buckling mode exemplified by the deformations shown in Figure 2.

On the other hand, this paper also investigates the shear buckling of the plates of the core as it is identified as a typical buckling failure mode due to high shear stresses, usually close to the supports or under heavy concentrated patch loads.

Finally, the third characteristic buckling phenomenon that is investigated in this paper relates to one of the manufacturing processes for rectangular honeycomb cores, where the core is fabricated from continuous strips that are partially cut over the depth and slotted together to form the core geometry (Wadley 2006) as shown in Figure 3. Clearly, if the honeycomb core is only attached to the faceplates, with the individual core strips slotted but not completely attached to each other, the buckling capacity under compressive transverse loading can be significantly affected, and therefore an effective buckling assessment method is required to deal with such a case.

The above proposed developments are presented in the following sections, where verification is undertaken against the results of nonlinear finite element analysis using ADAPTIC (Izzuddin 1991), where a 9-noded shell element is employed (Izzuddin and Liang 2016, 2017). A linear elastic material model is used, characterised by an elastic modulus of 210 GPa and Poisson’s ratio of 0.3.

Small imperfections are introduced in the models to avoid numerical problems arising from bifurcation points and to ensure the response follows a unique equilibrium path. An estimation of the buckling load of the perfect structure, as presented onwards, is obtained by using a suitable bifurcation load recovery method such as a Southwell plot (Mandal and Calladine 2002).

**3.1. Intercellular buckling**

At a local level, the phenomenon of intercellular buckling in rectangular honeycomb core sandwich panels can be represented as the buckling of a rectangular plate, corresponding to part of the faceplate, with sides equal to the cell sizes in the two directions. Since this rectangular plate is supported by the core plates, the contribution of these plates should be considered in the buckling assessment. The thickness of the core plates, hence their contribution to rotational restraint along the edges, influences the buckling phenomena of the faceplate. The RSA is used hereafter to investigate this influence, so as to achieve a simplified yet accurate methodology for the prediction of intercellular buckling in rectangular honeycomb core sandwich panels.

The deformation mode that can be used to obtain the exact buckling load of simply supported rectangular plates under general biaxial stress states is defined by the following equation (Bryan 1891):



7.

Taking  to be the stress ratio , the buckling coefficient  is determined as:



8.

which should be minimised by an appropriate selection of  and for each value of  and aspect ratio .

Timoshenko and Gere (1961) established the deformation mode which accurately predicts the buckling load of approximately square clamped plates subject to constant biaxial compressive stress state:



9.

By testing this shape function with the RSA as a SDOF and comparing to the results obtained by Wong and Bettess (1979), it is concluded that a reasonably accurate prediction is achieved for rectangular plates under equal biaxial compressive stress states, hence = 1.

In order to account for the influence of the rotational stiffness of the core plates, the RSA is used to formulate a 2-DOF system, where the buckling mode is considered as a linear combination of the aforementioned approximation modes for simply supported and clamped plates:



10a.



10b.



10c.

Constant rotational springs are initially considered along the edges of the rectangular plate to assess their influence on the critical buckling load. The contribution is introduced into the material stiffness matrix, as follows:



11a.



11b.

where  is the constant stiffness of the distributed edge rotational springs, and and represent the rotations of the plate evaluated over the edges. The contribution of the rotational springs along the edges is therefore only added to the material stiffness of the first mode , corresponding to the simply supported condition, since the second mode has zero rotations along the edges.

With this formulation, the determination of the buckling coefficient, which reflects the interaction between the two assumed modes, is postulated as an eigenvalue problem of rank 2. Finite element analysis is used to verify the analytical predictions arising from the proposed simplified methodology considering a rectangular plate model, supported by rotational springs along the edges, where small imperfections (1/2000 of the shorter plate length) are introduced using a half sine wave in each direction, similar to the buckling mode assumed for the simply supported plates.

The predictions of the proposed analytical method are presented in Figure 4 for three different aspect ratios , where a favourable comparison is achieved against the results of finite element analysis for a varying rotational stiffness along the edge. The results present the variation of the dimensionless buckling coefficient against the stiffness of the rotational edge springs. These results are associated with a 4 mm plate, where the shorter side  is equal to 200 mm and  is determined by the considered aspect ratio. These plates are subjected to biaxial compression  considering that the deformed shape assumed for the clamped case is adequate only under this scenario. Theoretical values for the buckling coefficient are also included for simply supported (SSSS) and fixed supported (FFFF) conditions.

A qualitative description of these results can be made with reference to three different regions on each graph on Figure 4. Relatively flexible edge springs do not influence the buckling of the plate, where the buckling mode is therefore defined by corresponding to the simply supported case, rendering the respective buckling coefficient. On the other hand, relatively stiff edge springs restrain the rotations along the edges of the plate by adding a dominant contribution to the 2×2 material stiffness matrix corresponding to the first mode, , thus effectively restraining this mode and shifting buckling towards the second mode. For intermediate values of rotational spring stiffness, an interaction between the two modes is clear, reflected by the critical buckling load following a smooth transition from the buckling loads of simply supported to clamped plates.

The above results demonstrate that applicability of a 2-DOF idealisation with the RSA to offer a simplified yet accurate buckling assessment of a rectangular plate subject to rotational edge restraint. However, for intercellular buckling of a faceplate in a sandwich panel with rectangular honeycomb core, the rotational restraint provided by the core plates cannot be considered as constant along the faceplate edges, hence a modification of the above simplified buckling model is required. A more realistic approach is therefore proposed, which is verified against a single cell finite element model, as illustrated in Figure 5.

As the plates are connected to form a cell, a deformation mode for the side plates that respects continuity of displacements and rotations in the system is required. This deformation mode can then be used to obtain the contribution of the side plates to the material stiffness, using standard discretisation principles, instead of utilising rotational springs along the edges of the faceplate. The assumed approximation mode is a polynomial function in the z-direction that is fixed against rotation at the bottom end, multiplied by a function that guarantees continuity of rotation between the top and side plates alongside the shared edge.



12a.



12b.

where  is the height of the cell,  is the flexural stiffness of the side plate, and  and  represent the rotation of the top plate evaluated over the edges. It is important to highlight this approximation mode violates the kinematic constraint of rotations between adjacent side plates, though it has been established that this incompatibility has a negligible effect on the buckling prediction of the system.

Results for the single cell model are presented in Figure 6, including an excellent comparison of buckling coefficients obtained from the analytical model and finite element analysis for a varying thickness of the side core plates. The theoretical values for simple and fixed supports are also presented. These results are associated with 200×200 mm2 top plate, 4 mm thick and a height of 200 mm. Theoretical values for the buckling coefficient are also included for simple supported (SSSS) and fixed supported (FFFF) conditions.

As a further refinement, account should be taken of the fact that intercellular buckling does not occur in a single plate cell but involves the sympathetic buckling of a group of cells, as observed in the high fidelity model of a sandwich panel in Figure 2. Therefore, the main difference compared to the single cell model is that the rotational restraint offered by the core plates is shared between adjacent rectangular faceplate cells, hence the contribution of the last two terms in (12b) should be halved. In order to verify the revised simplified model, a detailed finite element model for a sandwich panel substructure consisting of 3×3 cells (Figure 7) is considered under a biaxial deformation state in the faceplate which corresponds to . The influence of the core plate thickness is considered for three different  aspect ratios, with 200 mm and a thickness of 4 mm for the faceplate, where an excellent comparison is demonstrated in Figure 8 between the simplified model and FE results. Clearly, as the thickness of the core plates is reduced, the buckling coefficient approaches the theoretical solution for the simply supported (SSSS) case, while for large core plate thickness it approaches that of the clamped (FFFF) case; this is also demonstrated in Figure 7 with the finite element model, where two respective core plate thicknesses of 0.5 mm and 10 mm are considered. Importantly, the results in Figure 8 demonstrate that the simplified 2-DOF buckling model utilising the RSA is capable of capturing the influence of intermediate core plate thickness on the buckling coefficient very accurately.

Consider a sandwich panel under general loading that leads to different bending moments in the x- and y-directions, leading to different biaxial compressive stress states defined by  and . As a final refinement, consideration is given here to intercellular buckling under a general biaxial stress state, with arbitrary *=*. The accuracy of simplified approach using the RSA dependent on the selection of the approximation modes, where it has been demonstrated in the previous 2-DOF model for = 1 that the consideration of the two modes associated with the simply supported and clamped edge conditions, respectively, is important. Since the previously defined mode for clamped rectangular plates (9) cannot be employed to accurately predict the buckling load under general biaxial compressive stress, the following more general buckling mode, based on a double Fourier series, is considered:



13.

which readily satisfies the boundary conditions for fixed plates. By testing this shape function against the results obtained by Wong and Bettess (1979), it is concluded that, considering enough terms of the series, an accurate prediction can be achieved for rectangular plates of any aspect ratio under general biaxial compressive stress. Moreover, a single term, which can be determined by the appropriate selection of  and  that minimizes the buckling coefficient, can be used to provide an estimate of the critical buckling load with a maximum error of 15%.

Results are presented in Figure 9(i) for a stress ratio = 0.5 and aspect ratio = 1.5, where a very good comparison of buckling coefficients is obtained from the simplified RSA model and the 3×3 cell finite element model for a varying thickness of the core plates. For this comparison, a 300×200 mm2, 4 mm thick faceplate is considered, while the side plates have a height of 200 mm. The analytical prediction is obtained with 9 Fourier terms from (13), 3 in each direction, and, alternatively, with the single term which minimizes the bucking coefficient, for which . In Figure 9(ii), the results for a stress ratio of = 0.2 and aspect ratio of = 2.5 are presented, where again the simplified model achieves a very good comparison with the finite element results. For this comparison, a 500×200 mm2, 4 mm thick faceplate is considered, while the side plates have a height of 200 mm. As discussed before, the governing simply supported mode is obtained, which in this case is represented by (7) with , since it is the one that minimises the buckling load. The clamped mode is once again considered via 9 Fourier terms from (13), and, alternatively, with only two terms, specifically and 

**3.2. Shear buckling**

As previously mentioned, a common failure mode in rectangular honeycomb sandwich panels relates to shear buckling of the core plate, depending on the slenderness of this plate. This failure mode can occur around patch loads, due to punching shear, or along a supported edge of the panel due to high reactions. A simplified RSA model, based on assumed modes, is hereafter used to assess the shear buckling capacity of the core plate, allowing further for the rotational stiffness along the top and bottom edges to represent faceplates.

For both the simple and fixed supports, double Fourier series are used to assess the critical buckling load. Both series are respectively presented below:



14a.



14b.

Approximate best-fit solutions have been established for the simple  and clamped  support cases, which are given for by:



15a.



15b.

A linear combination of the two series is used to obtain the buckling load for a plate supported by rotational springs along its edges and subjected to pure shear, similar to the previous case of biaxial compression. A favourable comparison of the results from the analytical model and finite element analysis applied to square plates under constant shear with uniform edge rotational springs is presented in Figure 10. The analytical prediction is obtained with 4 Fourier terms in each direction for each series, where taking advantage of orthogonality conditions for trigonometric functions significantly reduces the computational cost of the simplified model. The numerical results are obtained for a 300×300 mm2 plate and for a 600×300 mm2 plate, both 4 mm thick.

**3.3. Compressive buckling of slotted core**

As mentioned before, the manufacturability of the core for rectangular honeycomb core sandwich panels can significantly be improved by avoiding the welding between orthogonal slotted strips, where a typical configuration is shown in Figure 3. However, this lack of connection influences the buckling behaviour of the core plate when subjected to significant out-of-plane compressive stresses arising from heavy patch loads. The RSA is therefore used to provide a simplified assessment of the elastic buckling capacity of the core plate when the orthogonal strip are slotted without further direct mechanical connection (e.g. through welding). Considering the simplest case of a square core assembled from orthogonal plates with the same thickness, the buckling capacity could be estimated by analysing a single rectangular plate which is the repetitive unit of such cores.

A representative plate is shown in Figure 11, with is subjected to uniform compressive loading in the vertical direction, , and which is restrained from out-of-plane displacements at over the two vertical edges along half of the height to simulate the restraint offered by the orthogonal strips. The edges along the top and bottom edges are assumed to be clamped, considering that the bottom plate is generally under tensile stress and that the compressive load would typically be applied through a thick base plate that prevents the rotation of the top faceplate.

For the idealised plate model, the buckling coefficient depends only on the plate aspect ratio . For a wide plate with a large aspect ratio , which corresponds to a sandwich panel with a large core cell size relative to the height, as illustrated in Figure 12(i), the buckling mode resembles that of a long clamped plate of height , for which the buckling coefficient would be = 4.0. For a narrow plate with a small aspect ratio , which corresponds to a sandwich panel with a small cell size relative to the height, as illustrated in Figure 12(ii), the buckling mode resembles that of a clamped plate of height , leading for which the buckling coefficient would be = 16.0.

Essentially, the effect of the side supports, hence the slotted configuration, on the buckling mode of the idealised plate is more prominent for narrow plates. The nonlinear response of narrow, intermediate and wide plates, idealising the slotted configuration, is depicted in Figure 13(i), where the load factor  is normalised according to (1) to correspond to the buckling coefficient. The post-buckling deformed shape and the distribution of curvatures in the vertical y-direction are respectively presented in Figure 13(ii) and 13(iii) for various aspect ratios.

Figure 13 demonstrates that the theoretical buckling coefficients for wide and narrow plates serve as lower and upper bounds for the various aspect ratios. By analysing the results, it is also possible to anticipate that the buckling coefficient, as a function of the aspect ratio , is defined by a continuous transition from the narrow case to the wide case, similar to what is seen in the previous sections under different loading and boundary conditions.

As before, the RSA is used to provide a simplified buckling assessment of the idealised plate model representing the slotted core. To estimate the buckling coefficient for wide plates, the following mode of a rectangular plate clamped on all four edges is assumed:



16.

For the narrow plate a single mode is used over half the plate, also considering clamped support conditions, as follows:



17.

To account for only half the plate, the integral defining both the material and geometric stiffness matrices is computed separately for two the halves of the plate, separated at half the height. Figure 14 displays the results of combining these two modes.

It can be observed from the comparison in Figure 14 that the first mode can accurately predict the buckling load of wide plates. There is, however, an evident discrepancy for both the narrow and intermediate plates, which is related to the fact that neither of the considered modes accounts for rotations about the horizontal line located at mid-height of the plate, which is characteristic of the deformed shape observed in the FEM, as presented in Figure 13(ii) for = 1.0.

To resolve this discrepancy, a much richer set of modes is required, where a total of 8 modes have been found to provide good accuracy to within 4%: 2 applied to only the top half of the plate domain, denoted by , and 6 applied over the full plate domain, denoted by , as given by:



18a.





18b.

Figure 15 presents the results of combining the 8 modes using the RSA methodology, where a much improved comparison with the FE results can be observed.

**4. Conclusions**

A simplified mechanics-based methodology is proposed in this paper for assessing the elastic local buckling of the main plate components in rectangular honeycomb core sandwich panels, which is both accurate and practical. The proposed approach, utilising a rotational spring analogy for establishing the geometric stiffness, considers the important phenomena of intercellular buckling of the faceplate, shear buckling of the core plate, and compressive buckling of slotted cores.

The importance of selecting representative buckling modes for the considered phenomena is highlighted and investigated. In applying the RSA with assumed modes, the addition of modes towards a richer representation of the actual lowest buckling mode tends to improve the prediction of the buckling load; however, this addition increases the complexity of the methodology and the computational effort, hence there is a trade-off between computational efficiency and accuracy that should be considered in a simplified approach. For intercellular buckling of the faceplates, it is shown that very good accuracy can be obtained for different cell aspect ratios using only two modes, provided the equal biaxial stresses are considered in the two orthogonal directions. A more involved set of modes associated with clamped edge supports is presented for non-equal biaxial stresses, although accurate results can be achieved by an appropriate selection of a reduced set of the modes. For the core plate subject to shear buckling under arbitrary rotational support conditions, good accuracy is generally obtained with 4 Fourier terms for the modes associated with simple and clamped supports in each of the two directions. Finally, for the slotted core plate subject to compressive loading, very good accuracy is demonstrated with 8 modes, though reasonable approximation may also be obtained with only 2 modes, particular for a relatively wide plate between two successive slots.

Considering the demonstrable practicality of the rotational spring analogy, the proposed methodology offers a simplified approach for plate buckling assessment, which can form an important component of a practical design method for rectangular honeycomb core sandwich panels.

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**Figure captions**

Figure 1. Thin rectangular plate under general planar loading

Figure 2. High fidelity response of a rectangular honeycomb core sandwich panel (Nordas *et al.* 2018)

Figure 3. Square honeycomb core built from slotted strips

Figure 4. Buckling coefficient *k* for variable spring rotational stiffness under constant biaxial compression: i) = 1.0, ii) = 1.5 and iii) = 5.0

Figure 5. Finite element model of single cell model

Figure 6. Buckling coefficient  for variable thickness of side plates: single cell model

Figure 7. Post-buckling deformed shape of 3×3 cell model: i) Thin core plates and ii) Thick core plates

Figure 8. Buckling coefficient  for variable thickness of side plates – 3×3 cells model: i) = 1.0, ii) = 1.5 and iii) = 5.0

Figure 9. Buckling coefficient  for variable thickness of side plates – 3×3 cells model: i) = 1.5 and = 0.5, and ii) = 2.5 and = 0.2

Figure 10. Buckling coefficient  for variable spring rotational stiffness under constant shear i) = 1.0 and ii) = 2.0

Figure 11. Thin slotted rectangular plate under uniform compressive loading

Figure 12. Idealised post-buckling deformed shape of slotted cores: i) large cell size relative to the height, and ii) small cell size relative to the height

Figure 13. Buckling of idealised slotted plates: i) nonlinear response, ii) post buckling deformation, and iii) curvatures in the y-direction for final deformed configuration

Figure 14. Buckling coefficient  of slotted core plate for variable aspect ratio: 2-DOF model

Figure 15. Buckling coefficient  for variable aspect ratio: 8 modes