Energy Management Strategy of Microgrids based on Benders Decomposition Method

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Abstract— **This paper discusses an optimal energy management system for microgrids, taking into account distribution power flow and dynamic loads, in presence of storage units and all associated constraints, aiming to reduce microgrid costs under two grid-connected and islanded modes. Getting the unit commitment, the microgrid energy management problem is introduced as a mixed integer nonlinear problem (MINLP). Since solving MINLP problems is complex and time consuming, a linearization technique is applied for simplification of the problem as a mixed integer linear programming (MILP) problem. Then, the Benders decomposition method is used to reach an efficient and accurate answer. The model proposed is implemented on a 14-bus microgrid including conventional and renewable distributed resources, storage units, and dynamic loads. The results indicated fair and fast performance of the proposed model.**

Keywords—Benders decomposition; energy management; microgrid; unit commitment;

I. Introduction

The growing increase in energy consumption today, environmental issues, increased use of distributed energy sources and storage units has highlighted the concept of microgrids [1]. Microgrids can work in either grid-connected or islanded modes [2]. Introducing an energy management system, taking into account the optimal performance of all power generation resources available in the microgrid and aiming to better manage the demand side in order to control power flow under different operational conditions is very important to achieve the microgrid potentials, namely high reliability, improved power quality, low cost of energy supply, and decreased generation of greenhouse gases [3]. Microgrid optimal management problems include economic dispatch (ED), unit commitment (UC), and demand side energy management (DSM) problems [4]. Very few studies have discussed distribution power flow in microgrid energy management problem. without considering distribution power flow and its associated constraints, the methods proposed have no application in the real word. In [5], an online energy management strategy (EMS) by considering power distribution network and the associated constraints is proposed for real-time operation of microgrids. In [6], a distributed energy management strategy for the optimal operation of microgrids in order to preserve the privacy of the distributed energy resources (DERs) and the loads, with consideration of the distribution network and the associated constraints is proposed. Although small fossil-fuel based generators in microgrids generally have fast start-up, shut-down, and ramp characteristic, it is desirable to limit them due to increased cost and frequency of maintenance [7]. Hence, In [7], [8], UC based EMS models for renewable based microgrids are proposed. Adding distribution power flow and operational constraints to microgrid equations results in nonlinearity and complexity of the energy management problem, and its calculations would become time consuming [6], [7].

Due to some constraints applied on the microgrid energy management problem, namely the constraints related to fossil fuel production units, storage units, as well as the constraints on AC distribution power flow, solving this problem with solvers of GAMS software would be very difficult and time consuming. Furthermore, the possibility of reaching a local optimal answer in meta-heuristic methods is high [7]. Different methods based on decomposing the problem into simpler sub-problems, namely Benders method, have been used in the literature to reach a suitable answer on solving distribution and transmission network optimization efficiently and reach an accurate and precise answer [9]–[11]. In [12], distributed and real time algorithm based on dual decomposition method is introduced to solve the energy management problem in a radial microgrid. In [13], the economic dispatch of a multi-carrier energy system is discussed by linearization and Benders decomposition algorithm in a transmission network.

In this study, an optimal energy management system for microgrids is discussed, taking into account distribution power flow, unit commitment, the adjustable loads, and all associated constraints. Then, in order to decrease the complexity of the MINLP problem obtained and to reach an absolute answer, distribution power flow and other nonlinear constraints are linearized. In the next step, Benders method is used to reach an accurate and efficient answer. This paper is organized as follows: section 2 presents the formulation of energy management problem, mathematical equations of all components of the microgrid, and operational equations. Sections 3, 4, and 5 discuss equation linearization, Benders method and structure of the problem, and analyzing the results, respectively. Finally, the conclusion is presented in section 6.

II. Problem Formulation

This microgrid has energy storage unit B, conventional and renewable distributed generation systems G, and different electrical loads L. The main purpose of the problem is to reduce the production cost and interruptible and defferable load costs, and also to reduce power losses in a day-ahead approach. Distribution power flow equation and the operational constraints of microgrid resources are considered to make the optimization problem applicable in real microgrids.

A. Power production systems

The production systems available in the microgrid under study contain conventional and sustainable generators. The conventional systems are fossil-feuled units and sustainable systems are solar cells and wind generators.

1) Sustainable generation units

The produced power of the sustainable units is non-adjustable reling on the amount of wind and photovoltaic resources available within the entire day. The prediction approach of sustainable resources should be employed to specify their generated power. In this study, the production of the sustainable units determined as the known data. No cost was specified for sustainable units.

2) Fossil-fueled generators

The conventional units have variable production amounts with maximum and minimum limitations which are given in [14]. The objective function of fossil-feuled units is defined as:

$$costP(P_g) = \sum_t [(a_0\, P(t)_g^2 + b_0 P(t)_g + c_0 v_g(t))\Delta t(t) + C(t)_{sup.g} + C(t)_{sdn.g} \tag{1}$$

where $costP(P_g)$ is the power generation cost of each generator, and a_0, b_0, and c_0 are objective function constants. $C(t)_{sup.g}$ and $C(t)_{sdn.g}$ are turn-on and turn-off costs, respectively.

B. Battery model and loads model

The formulation of battery as well as interruptible and defferable loads is given in [14]. A cost function is expressed to prevent battery damage due to charging and discharging operations, which is modeled as follows.

$$C(B) = \sum_t \left\lceil \frac{SOCmax - SOC(t)}{100} \right\rceil * X \tag{2}$$

where X is the cost of battery penalty, which is a fraction of the cost of electricity supplied by the main grid or. $C(B)$ represents the cost of charge and discharge operations. State of charge ($SOC(t)$) is a variable representing the energy stored in the battery and $SOCmax$ is its maximum limit.

The cost function of interruptible loads is correlated with the load removed, and is expressed by the following equation.

$$costint(P_l) = \sum_{t.P_l} \alpha_l \left(P_l(t) - P_l^f(t)\right) \tag{3}$$

where $costint(P_l)$ represents the penalty due to consumers dissatisfaction with load interruption, $P_l^f(t)$ represents the value of the forecasted load, and the coefficient α_l has a fixed value. Equation (3) is equal to 0 except when there is load removal. The cost function for deferrable loads is defined as equation (14).

$$costshift(P_l) = \alpha_l \left(Emin_l - \sum_t P_l(t)\Delta t \right) \tag{4}$$

where $costshift(P_l)$ is cost imposed due to not generation of the energy required by the deferrable load, and the coefficient α_l has a fixed value.

C. Distribution system power flow equations

Generally, the configuration of the distribution system is radial. The set N, specified by $i = 0.1.\dots.n$, is equivalent to the busses of a distribution system, and the line E is defined between two buses i and j. Bus 0 is considered as a constant voltage bus. Impedance, current, and complex power along the branch between busses i and j are expressed as $z_{ij} = r_{ij} + ix_{ij}$, $I_{ij}(t)$, and $S_{ij}(t) = P_{ij}(t) + iQ_{ij}(t)$. In each buss, $V_i(t)$ is the complex value of the voltage. Each bus can be linked to a group of loads or production and storage units. The net power value for each bus can be defined as follows.

$$S_i(t) = S_{li}(t) + S_{bi}(t) - S_{gi}(t) \tag{5}$$

where S_{li}, S_{bi}, and S_{gi} are the total amount of consumed power, stored energy, and generated complex power at bus i, respectively. Given the radial network structure, the below expressions for voltage, impedance, current, and flow in lines are valid in all lines at all time steps.

$$V_i(t) - V_j(t) = z_{ij} I_{ij}(t) \tag{6}$$

$$S_{ij}(t) = V_i(t) I_{ij}^*(t) \tag{7}$$

$$S_{ij}(t) - z_{ij}\left|I_{ij}(t)\right|^2 - \sum_{k:(j.k)\in\varepsilon} S_{jk}(t) = S_j(t) \tag{8}$$

Based on [15], employing the equations (16)-(18) and converting them to real parameters, the expressions of power flow for all time horizons become:

$$p_j(t) = P_{ij}(t) - r_{ij} l_{ij}(t) - \sum_{k:(j.k)\in\varepsilon} P_{jk}(t) \tag{9}$$

$$q_j(t) = Q_{ij}(t) - x_{ij} l_{ij}(t) - \sum_{k:(j.k)\in\varepsilon} Q_{jk}(t) \tag{10}$$

$$v_j(t) = v_i(t) - 2\left(r_{ij}P_{ij}(t) + x_{ij}Q_{ij}(t)\right) + (r_{ij}^2 + x_{ij}^2) l_{ij}(t) \tag{11}$$

$$l_{ij}(t) = \frac{P_{ij}(t)^2 + Q_{ij}(t)^2}{v_i(t)} \tag{12}$$

$$l_{ij}(t) = \left|I_{ij}(t)\right|^2 \tag{13}$$

$$v_i(t) = |V_i(t)|^2 \tag{14}$$

D. Operational limitations of the conventional generators

The operational constraints related to optimal performance of the fossil-feuled units are defined according to [16]. Only one binary variable $v_g(t)$ is used in these equations. Using only one binary variable, $v_g(t)$ in the constraints decreasing the time of the problem.

III. Equation Linearization

The main objectives of energy management in the microgrid under study are:

- Minimizing the cost of distributed power generation units, energy storage unit, and energy transferred to the network
- Minimizing the consumers' dissatisfaction in demand side management
- Minimizing power loss
- Minimizing production costs

Solving MINLP problems, such as the problem of this study is hard with a high computational burden, and none of the GAMS solvers give a correct answer for them. The meta-heuristic methods also may give local optimal answers in these kinds of problems. So, this problem is practically unsolvable, even in small networks. In this study, the equation linearization is done, especially distribution power flow linearization without removing the losses, to reach an absolute answer for the MINLP problem defined.

A. Linearization of distribution power flow equations

The reason behind nonlinearity of distribution power flow equations is the constraint on the current. The nonlinearity in losses results in nonlinear distribution power flow equations and loss equation. Due to the constraint on voltage (the change in voltage is considered to be 0.05), linearizing the current equation results in linear distribution power flow and loss equations. Voltage changes in the denominator of equation (12) is very little, so a constant value can be considered for voltage [17]. With this assumption, equation (12) becomes a second order one and its linearization would result in a linear equation for current to be applied in the distribution power flow expression. To linearize the equation (12), $P_{ij}(t)$ and $Q_{ij}(t)$ are converted to small linear segments using Piecewise linear method. The linear equations for current and loss are shown below.

$$I_{ij}^2(t) = \sum_{s\in SP} SP_l(s) \times TPF_l(s.t) + \sum_{s\in SQ} SQ_l(s) \times TQF_l(s.t) \tag{15}$$

$$P_{ij}(t) \le \sum_{s\in SP} TPF_l(s.t) \tag{16}$$

$$Q_{ij}(t) \le \sum_{s\in SQ} TQF_l(s.t) \tag{17}$$

$$TPF_l(s.t) \ge 0 \tag{18}$$

$$TQF_l(s.t) \ge 0 \tag{19}$$

where s is the number of segments, and SP and SQ are the number of intervals considered for $P_{ij}(t)$ and $Q_{ij}(t)$, respectively. Also, $l \in (i.j).(j.i)$ is the line between busses i and j. $TPF_l(s.t)$ and $TQF_l(s.t)$ represent the horizontal axis intervals for active and reactive power of the lines, respectively.

IV. Introducing Benders Decomposition Method

Benders decomposition method is one of the most desirable methods for solving MILP problems in terms of optimal answer and time [18]. Using this algorithm, the problem will be decomposed to a master problem (MP) and one more sub-problems (SP). The master problem is usually an integer problem, and the sub-problems are linear. Solving the master problem determined the binary variables indicating the status of units' operation, and the sub-problems use this information to satisfy the remaining constraints. In the first iteration, primary values are considered for the integer variables and the lower bound (LB) of the objective function would be achieved. Solving the sub-problems determines the upper bound (UB) of objective function, and the master problem will be solved again based on the constraints added. In each iteration, upper and lower bounds of the objective function are updated, and the termination condition of the algorithm is reaching a negligible difference between these two bounds. Benders decomposition method is based on mathematics and has less computation time and better convergence that meta-heuristic algorithms (e.g. genetic algorithm) due to decomposing one problem into a number of simpler problems. Fig. 1 shows all steps of Benders decomposition method [19].

A. Master Problem

The objective function and all constraints in the master problem have only binary variables. Solving this problem determines on and off conditions of the units. The objective function of the master problem is defined below [20]:

$$min\, Z_{master} = \alpha \tag{20}$$

$$\alpha \ge \sum_{g\epsilon G.t\in T} c_0 v_g(t) + C(t)_{supg} + C(t)_{sdng} \tag{21}$$

$$\alpha \ge \sum\nolimits_{g\epsilon G.t\in T} c_0 v_g(t) + C(t)_{supg} + C(t)_{sdng} + \bar{Z}_{subproblem} + {\pi^p}_{g.iter}(t)(v_g(t) - \bar{v}_{g.iter-1}(t)) \tag{22}$$

$$0 \ge \bar{Z}_{feasibility\ check-subproblem} + {\pi^r}_{g.iter}(t)\left(v_g(t) - \bar{v}_{g.ter-1}(t)\right) \tag{23}$$

Equation (22) is Benders cut and equation (23) is Benders feasibility cut, generated by Benders sub-problem and Benders feasibility sub-problem, respectively. ${\pi^r}_{g.iter}$ and ${\pi^p}_{g.iter}$ are dual variables in Benders sub-problem. $\bar{Z}_{subproblem}$ is the optimal answer of Benders sub-problem, and $\bar{Z}_{feasibility\ check-subproblem}$ is the optimal answer of Benders feasibility sub-problem. $\bar{v}_{g.iter-1}$ is a binary variable obtained

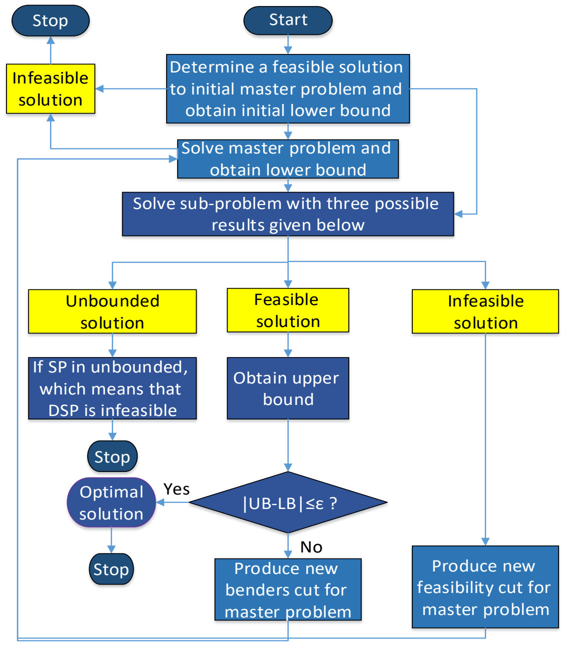

Fig.1. proposed microgird optimal scheduling model

from the master problem in previous iteration. Benders feasibility cut will be added to the master problem by the feasibility sub-problem in case of not reaching the feasible answer in the sub-problem. Benders cut will also be added to the master problem in case the upper and lower bounds of the objective function are not equal in an iteration. Solving the master problem, LB at each iteration would be calculated as:

$$LB(iter) = \bar{Z}_{master}(iter) \tag{24}$$

B. Benders decomposition sub-problem

After calculating the binary variables of the master problem, these variables will be assumed constant in the sub-problems, so that the optimal answer of the sub-problem objective function would be calculated, satisfying its constraints, by simplifying the problem as a LP. The sub-problem objective function is defined as equation (25). According to equation (26), the binary variable obtained from the master problem will be applied in Benders decomposition sub-problem. The UB value in each iteration is calculated using equation (27).

$$\min Z_{subproblem} = \sum_{t.g}[(a_0 + b_0 P(t)_g]\Delta t + \sum_{b\in B} C_b(P_{bat}) + \sum_{l\epsilon L} costint(P_l) + costshift(P_l) + \sum_{t\epsilon T} Costgrid(t.P_0(t)) + \sum_{t\epsilon T}\sum_{(i.j)\epsilon\varepsilon} r_{ij} l_{ij}^2 \tag{25}$$

$$v_g(t) = \bar{v}_{g.iter1}(t) \tag{26}$$

$$UB(iter) = \bar{Z}_{subproblem} + \sum_{g\epsilon G.t\epsilon T} c_0 \bar{v}_{g.iter1}(t) + + C(t)_{supg.\bar{v}_{g.iter1}(t)+C(t)_{sdng.\bar{v}_{g.iter1}}(t)} \tag{27}$$

It has to be noted that $\pi^p{}_{g.iter}(t)$ is the dual variable of equation (26), applied in equation (22).

C. Benders decomposition feasibility sub-problem

One of the methods solving the problem of infeasibility in Benders decomposition sub-problem is adding positive variables to some of constraints on the sub-problem (constraints related to distribution power flow and line voltage) in order to create a feasibility condition in the sub-problem leading to solving the new sub-problem and creating a feasibility cut for the master problem. Adding feasibility cut as a new constraint in the master problem results in calculation of the answers of the master problem in the next iteration and prevention from infeasibility of the answer in the sub-problem. The objective function and constraints of the feasibility sub-problem are [21]:

$$\min Z_{feasibility\ check-subproblem} = \sum_t S_{1jk}(t) + S_{2jk}(t) + S_{3jk}(t) \tag{28}$$

$$p_j(t) + S_{1jk}(t) = P_{ij}(t) - r_{ij} l_{ij}(t) - \sum_{k:(j.k)\epsilon\varepsilon} P_{jk}(t) \tag{29}$$

$$q_j(t) + S_{2jk}(t) = Q_{ij}(t) - x_{ij} l_{ij}(t) - \sum_{k:(j.k)\epsilon\varepsilon} Q_{jk}(t) \tag{30}$$

$$v_j(t) + S_{3jk}(t) = v_i(t) - 2\left(r_{ij}P_{ij}(t) + x_{ij}Q_{ij}(t)\right) + (r_{ij}^2 + x_{ij}^2) l_{ij}(t) \tag{31}$$

$$v_g(t) = \bar{v}_{g.iter1}(t) \tag{32}$$

where $S_{1jk}(t)$, $S_{2jk}(t)$, and $S_{3jk}(t)$ are the extra positive variables added to the distribution power flow and line voltage constraints. It has to be noted that $\pi^r{}_{g.iter}(t)$ is the dual variable of the constraint (32), imposed on the equation (23). Obtaining the feasibility answer in the sub-problem at each iteration, the UB and LB values will be compared. If they have negligible difference, the Benders iterations stops and the problem reaches optimal answer.

V. Simulation and Result Analysis

The energy management problem of this study is implemented on a radial 14-bus microgrid with 10kV nominal voltage depicted in Fig. 2. The microgrid is connected to the upstream transformer through bus No. 1. This microgrid includes diesel generators, renewable resources, battery, and dynamic loads. The minimum and maximum voltage was considered to be 9.5 and 10.5, respectively. The maximum production and consumption power for each bus is specified in Fig. 2. Simulation time is considered to be 24 hours, equal to $T \triangleq \{0.1. \dots .23\}$. This microgrid has two diesel generators and a storage resource. The information about cost function of interruptible and deferrable loads is described in the following. It is assumed that only the load associated with bus No. 5 is deferrable, and complete load transfer is only feasible in this bus. Furthermore, the required energy for the deferrable load must be delivered in time intervals t=12 to t=23. The maximum and minimum energy of the deferrable load, denoted by $Emax_l$ and $Emin_l$, are 11.4 MW.h and 7.6 MW.h, respectively. The

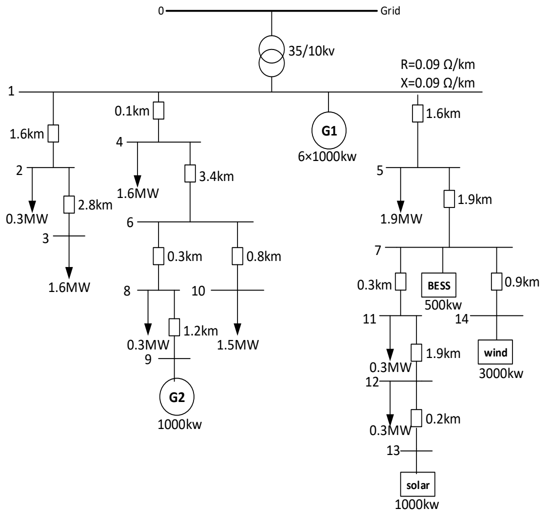

Fig. 2. Configuration of the studied microgrid

values of α_l in the cost function of interruptible and deferrable loads are assumed to be 1000 and 100, respectively.

A. Results of analysis in islanded mode

Fig. 3 shows the iterations of Benders decomposition algorithm to reach the optimal answer of the problem in islanded mode. Benders algorithm is converged after 27 iterations.

The upper bound of the problem has not changed until 22nd iteration due to infeasibility of the sub-problem's answer. The sub-problem has reached a feasible solution after 22nd iteration, and the upper bound of Benders decomposition algorithm has approached optimal answer by an irregular pattern. On the other hand, the lower bound of Benders decomposition algorithm has increasingly approached the optimal solution at each iteration. The total cost obtained by Benders decomposition algorithm is 10,863 dollars. Fig. 4 shows the day-head output program of the optimization problem. The productivity of fossil fuel generators is low due to the operation constraints and operation costs. It is also decreased significantly during increased production of the renewable resources. During the time intervals 3-5, the system operator used wind generator energy for supplying the microgrid load in order to decrease the costs. Fig. 5 shows a comparison between interruptible load and the predicted load. It can be observed that the load is significantly removed to reduce the total cost. Fig. 6 shows the deferrable loads. The overload is transferred from hours 12-16 to 18-22 to reduce the costs.

B. Results of analysis in grid-connected mode

Fig. 7 shows the iteration of Benders algorithm to reach the optimal answer of the problem in grid-connected mode. After 18 iterations, the Benders decomposition algorithm has reached the optimal answer of 9,114 dollars. This cost is significantly decreased compared to the islanded mode due to the possibility of supplying electricity from the main network. The upper bound of Benders decomposition algorithm has approached the optimal answer by an irregular pattern, while the lower bound of Benders decomposition algorithm has approached the optimal answer increasingly at each iteration. Fig. 8 shows the day-head output program of the microgrid in grid-connected mode. Decreased participation of the fossil fuel generators in supplying the network load is clear in this figure. Due to the high cost of generators and lower electricity cost of the main network, the electricity is supplied from the main network in most of the time. Fig. 9 shows the electricity supply and injection to the main grid. It can be seen that the electricity is mainly supplied from the main grid due to lower cost of electricity in the main network compared to the cost of fossil fuel generators, or renewable distributed resources. Only during hours 16-19, in order to increase profitability and decrease the microgrid costs, the electricity is injected to the main network due to increased cost of the electricity of the main network. Fig. 10 shows the interrupted loads in grid-connected mode, which is decreased compared to the islanded mode. No load interruption occurred at early times due to power generation of renewable generators and the electricity supplied from the network.

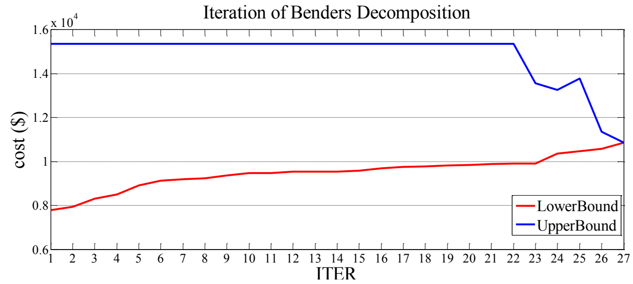

Fig. 3. Number of iteration in islanded mode

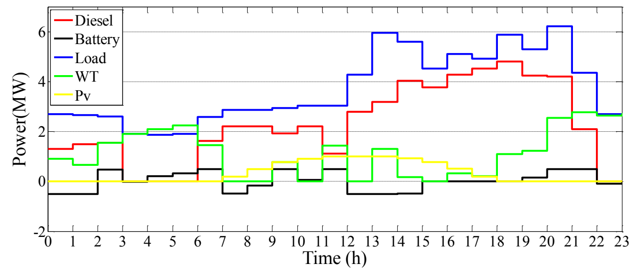

Fig. 4. Day-ahead output program of the isolated mode

VI. CONCLUSION

Optimal production of a microgrid in both islanded and grid-connected modes are discussed, and the microgrid performance is investigated during 24 hours. The constraints on fossil fuel generators, energy storage systems, AC distribution power flow, and demand-side management are discussed. Since solving MINLP problems by GAMS solvers is very time consuming and unpractical, and the possibility of reaching a local solution by meta-heuristic methods is also very high, problem simplification is used to reach an absolute solution in the MINLP problem. Due to nonlinearity of the constraints on AC distribution power flow, it has been transformed to a second order constraint, and piecewise linearization method has been apllied. Other nonlinearities on this problem have been simplified using this method, and then the problem has been converted to an MILP master problem and a number of LP sub-problem by Benders decomposition method and implemented in GAMS software to reach an efficient accurate answer. The results proved fair, accurate, and quick performance of this method..

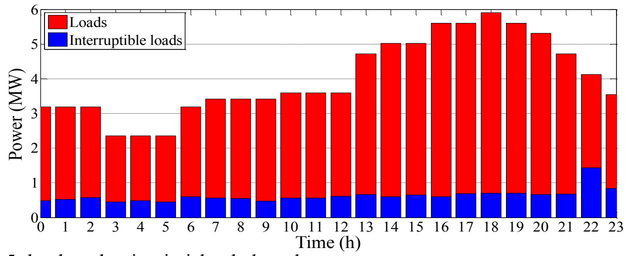

Fig. 5. loads reduction in islanded mode

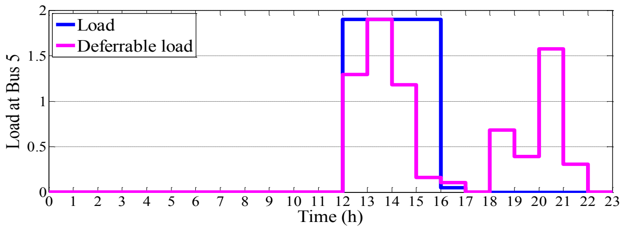

Fig. 6. deferrable load in Bus 5 in islanded mode

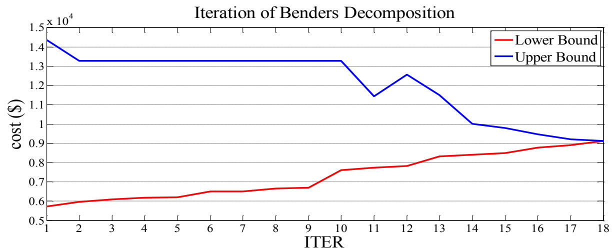

Fig. 7. number of iteration in grid-connected mode

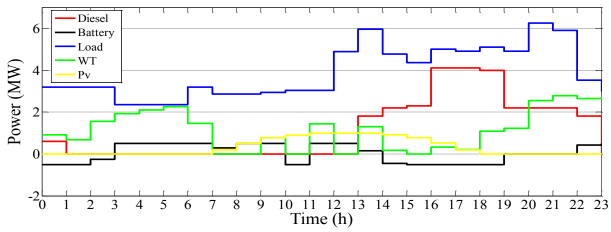

Fig. 8. Day-ahead output program of the grid-connected mode

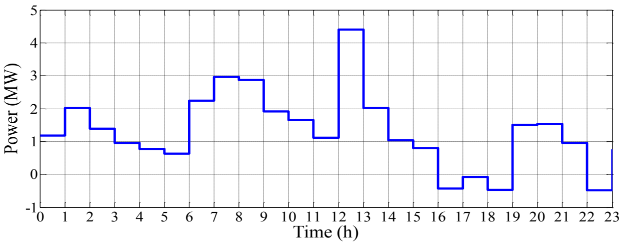

Fig. 9. Transferred power to the microgrid

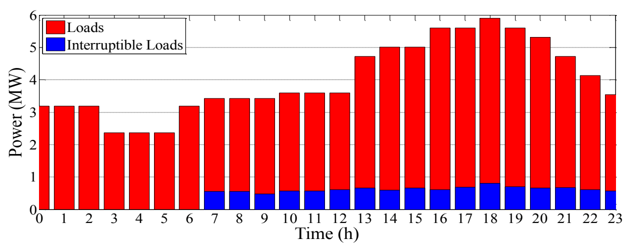

Fig. 10. loads reduction in grid-connected mode

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