A Fourier-series-based Virtual Fields Method for the identification of modulus distributions

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INTRODUCTION

- ✓ Requirement of a fast and accurate inverse technique to determine spatially-varying properties of materials has arisen in many research fields.
- ✓ Non-contact full-field measurement techniques have become more attractive to research community.
- ✓ A modest number of inverse techniques using full-field measurement data available in the literature.

FROM THE CLASSICAL VIRTUAL FIELDS METHOD...

Equation (general form) of the principle of virtual work

$$-\int_V \boldsymbol{\sigma} : \boldsymbol{\epsilon}^* dV + \int_{S_f} \mathbf{T} \cdot \mathbf{u}^* dS_f + \int_V \mathbf{f} \cdot \mathbf{u}^* dV = \int_V \rho \mathbf{a} \cdot \mathbf{u}^* dV$$

written for a 2-D linear elastic isotropic case as

$$\int_S \left((\epsilon_{xx} + \nu \epsilon_{yy}) \epsilon_{xx}^* + (\epsilon_{yy} + \nu \epsilon_{xx}) \epsilon_{yy}^* + \frac{1-\nu}{2} \epsilon_{ss} \epsilon_{ss}^* \right) Q_{xx} dS = \int_\ell (T_x u_x^* + T_y u_y^*) d\ell$$

...TO THE FOURIER VIRTUAL FIELDS METHOD

- ✓ Parameterisation of spatially-varying modulus/stiffness by a Fourier series expansion:

$$Q_{xx}(x,y) = \sum_{m=0}^{M} \sum_{n=-N}^{N} a_{m,n} \cos 2\pi \left(\frac{mx}{L_x} + \frac{ny}{L_y} \right) + \sum_{m=0^*}^{M} \sum_{n=-N^*}^{N} b_{m,n} \sin 2\pi \left(\frac{mx}{L_x} + \frac{ny}{L_y} \right)$$

- ✓ Selection of virtual deformation fields as cosine/sine functions of spatial variables.

MODULUS RECONSTRUCTION WITH UNSPECIFIED BOUNDARY CONDITIONS

- ✓ Application of an appropriate window function *W* (*x*,*y*) to zero unknown traction components on the boundary:

$$\int_\ell (T_x \hat{u}_x^* + T_y \hat{u}_y^*) d\ell = \int_\ell (T_x W u_x^* + T_y W u_y^*) d\ell = 0$$

APPLICATION TO EXPERIMENTAL DATA

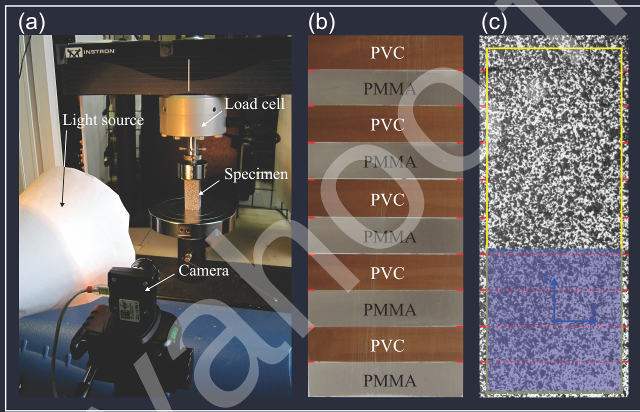

Figure 1: (a) Experimental setup of a multi-layered prismatic plastic specimen under uniaxial compression. (b) Specimen's layer-up. (c) ROI highlighted.

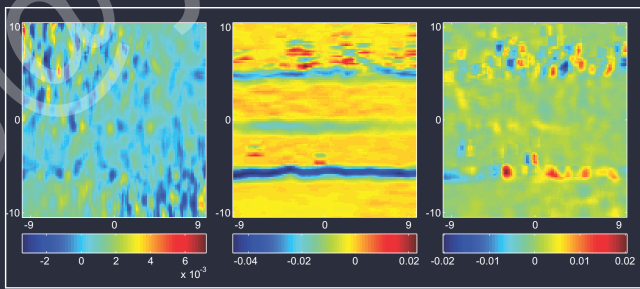

Figure 2: Experimental strain fields measured within the ROI by 2-D DIC. Left to right: $\epsilon_{xx}, \epsilon_{yy}$ and ϵ_{ss}.

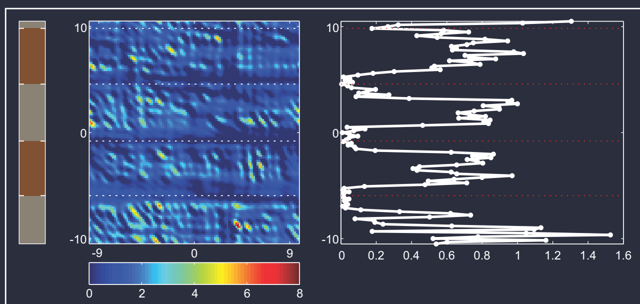

Figure 3: Modulus map reconstructed by the F-VFM. The graph on the right shows mean values of every row of the modulus map. The modulus ratio of the two materials is determined ~1.05 ± 0.1 and compared with the real ratio of 1.2.

CONCLUSIONS

- ✓ Development of a Fourier-series-based method able to reconstruct spatially-varying modulus distributions.
- ✓ Adaptation of the method to challenging situation of limited knowledge of the boundary conditions.
- ✓ Computational efficiency achieved by using the fast algorithm of the proposed technique, which returns nearly a thousand of variables in ~3 seconds.

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