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Illegal finance and usurer behaviour

*Raffaella Barone**, *Roy Cerqueti**, *Anna Grazia Quaranta**

Abstract

This paper deals with a stochastic dynamic optimization problem in the context of illegal company financing. Our analysis of the usury phenomenon is conducted by searching for the best interest rate which an illegal financier should apply to a company in order to bring about the firm's bankruptcy whilst still securing the maximum wealth for the firm's guarantee. In this case, the company itself can be taken over and used by the financier for illegal activities. Because of the highly complex nature of the problem, the analysis will be performed via simulation studies.

JEL Classification: C61; G32; K42

Keywords: usury, external financing of companies, bankruptcy, stochastic dynamics optimization problem.

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Introduction

Several European countries base their anti-usury laws on the definition of a critical interest rate applied to the borrower by the lender, regardless of the nature of the lender.

The aim of this paper is to shed light on the limits of this approach, emphasizing the importance of the role played by organized crime in usury credit. The problem is more relevant in this period of financial crisis, since many firms in difficulty find themselves rationed by the legitimate sector and may resort to looking for finance from the usury market.

It is well known that the subject who loans money at a usurious interest rate generally comes from organized crime and uses the usury credit as a money laundering technique.

A criminal organization which has committed a crime gains an aggregate illegal monetary return from this activity. If the criminal organization uses these illegal funds directly, it increases the probability of its being detected and charged with the crime, therefore it seeks to launder dirty money. All illegal activities which circumvent government laws and regulations and the income derived from them, potentially all the revenues produced by the underground economy, need to be laundered (Schneider 2005).

One of the most common money laundering techniques used by criminal organizations is usury credit. They utilize these loans to pursue their final goal, namely to gain possession of the collateral offered by the borrower, which has an illegal value peculiar to them.

As Masciandaro (2001) and Europol (2008) have pointed out, organized crime groups are showcasing significant use of legitimate business structures to launder criminal proceeds and establish themselves in legal business.

In this paper, we make a two-fold contribution to the economic literature on the topic. On the one hand, we analyze the relationship between organized crime, money laundering¹ and usury credit, focusing on the optimal interest rate chosen by the illegal financier in order to achieve his aim. On the other hand, we suggest a methodology that is, at this time, a crucial innovation.

We propose a stochastic dynamic optimization problem, with particular reference to the field of optimal control theory.

Several monographs give a complete survey of mathematical control theory. For the deterministic case, we remind the reader to Bardi and Capuzzo Dolcetta (1997). Stochastic control theory is described in Borkar (1989), Fleming and Soner (1993), Krylov (1980), Yong and Zhou (1999).

The key point of optimal control theory is represented by an optimization problem, where the constraints are associated to the properties of some functions (control variables), which are elements

¹ For a survey of the literature see Walker and Unger 2009, Unger 2009, Argentiero et al 2008, Barone and Masciandaro 2008, Schneider 2008. See also Bagella et al. 2009. Micro dynamic models are proposed in Argentiero et al. 2008, Bagella et al. 2009. A static model of money laundering by usury is showed in Barone 2004.

1 of a certain functional space (the admissible region). Dependence on time, besides the control
2 variables, is also represented by the introduction of a state variable, which describes the evolution
3 of the system. Thus, the objective function depends on the state variable and controls, and the
4 optimum with respect to the controls of this objective functional is called the value function.
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6 The stochastic framework is related to the analysis of cases with an admissible region given by
7 stochastic process spaces.
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9 In our context, the objective function, which the usurer needs to maximize, is made up of two terms:
10 the probability of the firm defaulting and the probability of the guarantee related to the borrower
11 being positive when the funded company goes bankrupt. The admissible region contains the suitable
12 loan interest rates applied for the restitution of the debt, and the state equation describes the
13 evolution of the wealth of the funded company.
14

15 The optimal solution will be obtained by performing a simulation study using the Monte Carlo
16 method in order to determine the optimal interest rate required by a criminal creditor to achieve
17 her/his goals.
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19 We stress that the illegal financier is aiming for the borrower to default, since only in this case
20 could the usurer take possession of the collateral, that is, in our analysis, the legitimate firm in need
21 of funds.
22

23 Collateral is a fundamental tool in money laundering and, as such, it is of great intrinsic value to the
24 illegal financier.
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26 Money-laundering enables the criminal organization to disguise the illegal origins of its wealth,
27 throw off suspicions of law enforcement and erase any incriminating traces of illegal activity.
28 (UNODOC Annual Report 2010).
29

30 The use of properly incorporated legal entities, which have a proven financial and commercial track
31 record, is usually one of the most common methods of disguising the true ownership and the origin
32 of the funds used. Companies facing mounting debts are frequently employed to merge illegal
33 capital with the legal capital of previously properly incorporated firms. The investor could request
34 the amendment of the legal documents to empower him/her to manage the finances of the company,
35 or he/she may affect the balance sheet of the company in order to merge the illegal money with the
36 money used for the regular course of business. (FATF – GAFI 2007)
37

38 This criminal strategy increases in those countries where justice is weak and lawlessness and
39 instability prevail. When countries lack strong judicial institutions — such as forceful criminal
40 legislation, reliable law enforcement, a fair judiciary and a humane prison system — criminals find
41 opportunities to profit. (UNODOC Annual Report 2010)
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In the light of our previous remarks, we believe that the choice of setting an interest rate ceiling on the loan contract may be necessary, but is not enough to counteract the illegal practices. When the Authorities design the optimal policy, they should increase their efforts in monitoring loan activity. The main problem should be the fight against money-laundering because, in general, it is the origin of any further illegal activity concerning usury credit.

The paper is organized as follows. In the first section, we analyze the definition of usury in several European countries and in the United States; in section 2, the evolution equation of company wealth is described, while in Section 3, we illustrate the optimization problem. Section 4 is devoted to constructing the optimal strategies via a Monte Carlo simulation and section 5 concludes.

1 Usury laws in Europe and the United States

Several countries base their anti-usury laws on the explicit definition of an objective illegal interest rate threshold. Criminal laws on usury in some other countries are based on the courts' perception of the entity of the restitution rate, when it is compared to the original loan. Generally, one can say that usury regulation focuses on the assumption that usury can be viewed as an onerous credit contract. An overview of the definition of usury in the principal European countries and in the US now follows, mainly referencing to Masciandaro, (2001).

In Austria an objective usury threshold does not exist, and the courts evaluate whether the interest rate applied is clearly disproportionate to the value of the service.

In Belgium the threshold is determined by the King at least every six months, and it depends on the type, amount and duration of the credit.

The law in France identifies different thresholds for corresponding categories of credits as a proportion of the average rates applied by banks to the same types of transactions. The proportion is actually fixed at one-third.

In Germany, as in Austria, a subjective concept of disproportion is also applied to punish illegal financing. However, some objective parameters related to the monthly report of the Deutsche Bundesbank are commonly used to clarify the concept of disproportion.

The legislation in Ireland moved in 1995 from an objective threshold of 39% to a subjective evaluation by the courts, based on the rates applied in the market and on the characteristics of the contract and of the borrower.

The definition of the usury rate in Italy is quite similar to the one applied in France. Some objective thresholds are fixed at one and half times the average rate of the quarterly interest rates for corresponding categories of credit.

1 A subjective mechanism for determining the usury rate is applied in Luxembourg, based on the
2 market interest rates and on the degree of difficulty and inexperience of the borrower.

3 The Netherlands have an administrative regulation providing a rule for the determination of the
4 usury rate based on the duration and the amount of the credit.
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7 In Portugal an objective threshold is fixed as the annual legal interest rate plus 3-5%, which
8 becomes 7-9% in the case of late payment. The exact percentage of increase chosen depends on the
9 existence of real collateral.
10

11 The courts in Spain define the subjective threshold case by case, after analyzing the characteristics
12 of the borrower and the market interest rates.
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14 In Switzerland, the definition of usury rates is a matter for each Canton. However, it is commonly
15 accepted that usury takes place when the annual restitution rate exceeds the legal annual interest
16 rate by 18%.
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18 In the UK, an enormous restitution rate is viewed as a distortion of the credit contract, and the
19 courts can impose corrective action based on a subjective evaluation.
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21 In the US, the situation is quite complicated, since each State of the Union has specific legislation to
22 pin down the illegal interest rate. This is exemplified by the few examples which follow. Some
23 states are more tolerant than others: in Colorado the usury rate is 45%, while in Illinois it is 9%. In
24 New Mexico, the usury rate is fixed by the courts, yet in Wisconsin it depends on the characteristics
25 of the credit.
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36 **2 The evolution equation**

37 We assume that a criminal subject needs funds for an investment project. When the contract
38 expires, the borrower should repay the principal and the interest. If the borrower defaults, then s/he
39 has to transfer ownership of the collateral to the lender. At the beginning of the contract, the initial
40 value of the capital borrowed plus interest is lower than the actual value of the good offered as
41 collateral, therefore the borrower prefers to repay the debt, if s/he is able to do so.
42

43 To obtain funds, it is possible to apply to a legal or illegal creditor. If the contract is drawn up with
44 a legal lender, the borrower has the opportunity to obtain a loan with a lower interest rate than the
45 illegal rate. Both types of contract stipulate transfer of ownership of the collateral if the borrower is
46 not able to repay the loan. This guarantee has a different value for the legal creditor, in particular a
47 bank, and for the usurer; the legal costs of liquidating the good for a bank are higher than for the
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1 usurer.² The latter assigns a higher illegal value to the good as it can be used for money laundering
2 purposes.

3 If the borrower is successful, s/he repays the capital plus the interests. In the following, we assume
4 that at t_0 , i.e. the starting period, the borrower applies to a usurer because s/he has been rationed by
5 the legal market or for a personal choice of convenience (Masciandaro, 2001).
6

7 The borrower is obliged to pledge the company in need of finance to the usurer as collateral.
8

9 At time t the wealth of the company amounts to $X(t)$, and it is described by a controlled stochastic
10 differential equation, as we shall see.
11

12 We also introduce a probability space with filtration $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$, where the filtration \mathcal{F}_t is
13 assumed to be cadlag and is constructed as
14

$$15 \mathcal{F}_t = \sigma(X(s), 0 \leq s \leq t) \cup N, \quad \forall t \geq 0,$$

16 with
17

$$18 N := \{A \in \mathcal{F} | P(A) = 0\}.$$

19 The value of the firm could change over time and its wealth could take on the lowest value of 0 or
20 the maximum amount equal to K . When the company's wealth reaches level K , the company is able
21 to repay its debt.
22

23 The state equation describes the stochastic evolution of the dynamic associated to the wealth of the
24 firm. It is given by the following controlled stochastic differential equation with initial data.
25

$$26 \begin{cases} dX(t) = (\mu - \alpha(t))X(t)dt + \sigma X(t)dW(t), \\ X(0) = X_0 \end{cases} \quad (1)$$

27 where

- 28 • $\mu, \sigma \in \mathbb{R}$ are related, respectively, to the deterministic and stochastic evolution of the firm's
29 wealth.
- 30 • $\alpha(\cdot)$ is an \mathcal{F}_t -adapted stochastic process, and it represents the loan interest rate applied by the
31 financier to the funded firm.
- 32 • $X_0 \in [0, K]$ is the initial wealth of the firm. Formally, it should be an integrable random
33 variable in $[0, K]$ with law π_0 , that is measurable with respect to \mathcal{F}_0 . Since it is reasonable that the
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35 ² Bester (1994) shows that the optimal contract for a bank is a loan without guarantee. With this contract, the borrower
36 should pay a higher interest rate on the loan, but if s/he defaults, s/he doesn't lose the collateral. For a conservative
37 analysis in the following we will assume a banking contract with collateral.
38

1 initial situation of the funded company is known, we can assume that $X_0 = x \in [0, K]$, x non
 2 random.

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 4 • $W(\cdot)$ is a standard 1-dimensional Brownian motion which is independent of π_0 . It drives the
 5 stochastic term of the firm's wealth evolution.
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7 **Remark 1** *The bound values 0 and K are absorbing barriers for the firm's wealth dynamic, which*
 8 *evolves under the pressure of debt repayment.*
 9

10 *When the wealth of the firm reaches the value 0, then we have the company's failure; if the firm's*
 11 *wealth reaches the value K, then the company is able to extinguish the loan.*
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13 **Remark 2** *There exists a unique solution for the controlled equation (1) (we remind the reader, for*
 14 *example, to Øksendal (1995)).*
 15

16 By Remark 2, and fixed $x \in (0, K)$ and $\alpha \in \mathbb{R}$, we denote the unique solution of (1) as $X_x^\alpha(\cdot)$.
 17

18 Let us denote with \mathbb{T} the set of the stopping times in $[0; +\infty)$, i.e.
 19

$$20 \quad \mathbb{T} := \{ \tau : \Omega \rightarrow [0, +\infty) \mid \tau \leq t \in \mathcal{F}_t, \forall t \geq 0 \} \quad (2)$$

21 and let us define the exit time $\tau_{(0,K)}$ of the dynamic from $(0; K)$ as
 22

$$23 \quad \tau_{(0,K)} := \inf \{ t \geq 0 \mid X_x^\alpha(t) \notin (0, K) \} \quad (3)$$

24 Since \mathcal{F}_t is cadlag, then $\tau_{(0,K)} \in \mathbb{T}$.
 25

26 3 The optimization problem

27 Usury credit appears as a short term loan of a low aggregate amount. The short-term nature of the
 28 due date makes it unlikely that the borrower will be in a position to pay back the loan plus the
 29 interest. Sometimes, at the end of the contract, the borrower has the opportunity of renegotiation if
 30 s/he is insolvent (Caperna and Lotti, 1995; Battaglini and Masciandaro, 2000). The more the debtor
 31 renegotiates, the higher the amount due becomes until the debtor is unable to fulfill his obligations.
 32 At the beginning of the contract, the usurer's behavior could be similar to that of a bank, in order to
 33 set a trap for the borrower. As Unger (2007) pointed out, the rate at which the loan is released might
 34 not necessarily be higher than the legal rate and might even be lower, simply because often, behind
 35 usury credit - an illegal activity by its very nature - there is a further hidden illegal activity, namely
 36 financial money laundering³. At the renegotiation stage, no further traps are needed: the real nature
 37 of the illegal financier comes out, and the loan interest rates increase.
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 59 ³ Recently, some economists have proposed estimation of money laundering activity and some others have analyzed the
 60 usury markets. More specifically, Argentiero et al. (2008), Schneider (2008), Barone and Masciandaro (2008), Unger
 61 (2007), Walker (1999) using different methodologies provided measurement of the volume of money laundering. The
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1 In our model, the due date is the definitive deadline for debt repayment, and no renegotiation
 2 opportunities are allowed. In doing this, we aim at focusing on the final target of the illegal
 3 financier, without stressing the strategies implemented to entice people into financial distress.
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5 A key role in the usury model is played by the guarantee g which is evaluated by the usurer
 6 differently from a bank, in particular when the borrower is an entrepreneur.⁴ Banks base the
 7 creditworthiness of a firm needing finance on the real guarantee instead of its projected evaluation.
 8 The aim of the banks is to raise the loans and they are not interested in involving themselves in any
 9 market where the firm could produce wealth. The aim of the usurer, on the other hand, is to take
 10 ownership of the borrower's firm, because it is useful for laundering illegal capital. The value of the
 11 guarantee is therefore greater for a usurer than for a bank.
 12

13 The usurer's strategy is, therefore, to bring about the borrower's default in order to take possession
 14 of the collateral.
 15

16 The guarantee is related to the wealth of the firm and takes into account the eventual income
 17 obtained by the financier in the case of the company's bankruptcy. Therefore, it seems obvious to
 18 assume the guarantee to be nonnegative. We define
 19

$$20 \quad g : [0, +\infty) \times [0, K] \times A \rightarrow [0, +\infty) \quad (4)$$

21 a general function, with

$$22 \quad A := \{\alpha : [0, +\infty) \times \Omega \rightarrow [\delta_1, \delta_2) \text{ such that } \alpha(t) \in F_t, \forall t \geq 0\}. \quad (5)$$

23 Let us define an endogenous time threshold T^* such that g satisfies the following boundary
 24 condition:
 25

$$26 \quad g((\tau_{(0,K)}), 0, \alpha) = \begin{cases} 0 & \text{if } \tau_{(0,K)} \leq T^* \\ \gamma & \text{if } \tau_{(0,K)} > T^* \end{cases} \quad (6)$$

27 where γ is the positive value obtained by the illegal subject when the firm goes to bankruptcy. T^* is
 28 assumed to be a short maturation time for the guarantee g , and we will explain its form in the Monte
 29 Carlo simulation results section. The threshold T^* formalizes that the failure of the firm is not
 30 profitable for the lender at the beginning of the contract. The usurer needs to wait till the moment
 31 when such profitability is achieved, and the guarantee's wealth becomes positive.
 32

33 The illegal financier aims to maximize the probability of company default and, simultaneously, the
 34 probability of the guarantee being positive.
 35

36 Therefore, the value function is:
 37

38 result of the more conservative estimation showed that the value of money laundering activity in 2004 was equal to
 39 US1.2trn.

40 ⁴ For an analysis of the differences between legal and illegal credit see also Masciandaro (2001), Crosato-Dalla
 41 Pellegrina (2008) and Dalla Pellegrina (2008).
 42

$$V(x) := \max_{\alpha \in A} \left[P(X^\alpha(\tau_{(0,K)}) = 0) + P(g((\tau_{(0,K)}), X^\alpha(\tau_{(0,K)}), \alpha) > 0 | X^\alpha(\tau_{(0,K)}) = 0) \right] \quad (7)$$

In Table 1, we summarize the main variables used in the previous theoretical analysis and their significance in order to facilitate the reading of the following section.

Table 1. The main theoretical variables and their significance.

| Variable | Significance |
|----------------|---|
| $\alpha(t)$ | Loan interest rate at time t : it is the control variable of the optimization problem |
| δ_1 | Lower bound of the usury interest rate |
| δ_2 | Upper bound of the usury interest rate |
| $X(0)$ | Initial wealth of the firm |
| $X(t)$ | Value of the firm at time t , $X(t) \in [0, K]$. It is the state variable |
| G | Guarantee related to the wealth of the firm |
| T | Set of the stopping times |
| $\tau_{(0,K)}$ | Exit time of the dynamic from $(0, K)$ |
| T^* | Maturation time for guarantee |

4 Monte Carlo simulation results

Our goal is to derive the interest rate α which can maximize the objective function $V(x)$ defined in (7). Via a Monte Carlo simulation, we build 1000 different trajectories of the firm's value X of equation (1); to this end, α , accordingly with the empirical literature, is assumed to vary in an appropriate band while the other parameters' values are fixed. More precisely:

- although the upper bound of the usury interest rate is generally infinity, empirical evidence shows that $\delta_2 = 500\%$ with only 9% of the event surpassing such a high threshold.⁵ As a result, the lower bound is assumed to be $\delta_1 = 0$ while, in line with the aims of the illegal financier (to construct a trap for the company, to reinvest illegal money), the upper bound is fixed to $\delta_2 = 5$;
- $\mu = (1 + \rho) = 1.001$, where ρ is the revaluation rate of the company;
- $\sigma = 0.01$;

⁵ Centro Studi e Ricerche sulla Legalita' e Criminalita' Economica, L'usura tra vecchi confini e nuovi mercati, Roma - 2002.

- in relation to the starting value X_0 we consider three different starting points, i.e. $X_0=100$, $X_0 = 500$ and $X_0 = 1000$ respectively to represent small, medium and large enterprises;
- the restitution threshold is related to the initial amount of the loan, namely D . We assume that D is given by 20% of the value $X(0)$. Hence we have $D = 20$, $D = 100$ and $D = 200$ for small, medium and large companies, respectively;
- a prudential restitution threshold K can be given as the sum of the initial wealth of the firm and more than double the debt amount D . So we assume $K = 150$, $K = 750$ and $K = 1500$ for small, medium and large companies, respectively.

The simulation procedure for the three cases of small, medium and large companies is implemented as follows:

- the Brownian Motion is discretized as $dW(t) = \Lambda * \sqrt{\Delta t}$, where Λ is a random number extracted by a centered normal distribution and $\Delta t = 1$;
- we consider a discretization of the range $[0, 5]$ of the interest rate α with a step equal to 0.01; we denote each value of α as α_i ($i = 1, \dots, 50.000$);
- we identify time-points as days and we consider 1000 points to construct each trajectory in order to analyze the evolution of the firm's wealth for approximately three years;
- fixed α_i , 1000 trajectories $X_j^{\alpha_i}$ ($j = 1, \dots, 1000$) are built ⁶;
- for each $X_j^{\alpha_i}$ we derive the time $\tau_j^{\alpha_i}$ in which, for the first time, the trajectory of $X_j^{\alpha_i}$ hits the barrier $\{0, K\}$.

Let be n^{α_i} the number of the $\tau_j^{\alpha_i}$ such that $X_j^{\alpha_i}(\tau_{(0,K)}) = 0$; we calculate for each value of α_i the first probability considered in equation (7) as follows

$$P(X_j^{\alpha_i}(\tau_{(0,K)}) = 0) = \frac{n^{\alpha_i}}{1000} = h^{\alpha_i} \quad (8)$$

For each value of α_i the average $\bar{\tau}^{\alpha}$ of the n^{α_i} values $\tau_j^{\alpha_i}$ for which $X_j^{\alpha_i}(\tau_{(0,K)}) = 0$ is also derived.

To determine the second component of the value function (7), we argue that:

$$\left| P(g^\alpha(\tau_{(0,K)}) > 0 \mid X^\alpha(\tau_{(0,K)}) = 0) = 1 - P(g^\alpha(\tau_{(0,K)}) = 0 \mid X^\alpha(\tau(0, K = 0))) \right| \quad (9)$$

Therefore, $P(g^\alpha(\tau_{(0,K)}) = 0 \mid X^\alpha(\tau_{(0,K)}) = 0)$ can be written as K^{α_i} , where

⁶ When the 1000 trajectories $X_j^{\alpha_i}$, each made up of 1000 points, are traced, the value α_i , increases by 0.01 and then in relation to this new value of the interest rate we determine another 1000 trajectories of 1000 points, and so on.

$$k^{\alpha_i} = \frac{\#\{\tau_j^{\alpha_i} \leq T^*\}}{n^{\alpha_i}} \quad (10)$$

The value of T^* can be obtained starting from the previous results; in particular, considering all the values assumed by α , we calculate the semi-average of all the $\tau_j^{\alpha_i}$ for which $X_j^{\alpha_i}(\tau_{(0,K)}) = 0$. Then, for each value of α_i we calculate the number of the $\tau_j^{\alpha_i}$ that are smaller or equal to T^* and we divide this number by $n_j^{\alpha_i}$.

In order to solve our optimization problem, we then need to determine the (optimal) level of α which satisfies the maximization of the sum $h^{\alpha_i} + (1 - k^{\alpha_i})$.

As a result we obtain that

1. The optimizing level α^* of the loan interest rate, which is needed for the purposes of the illegal financier (i.e. to maximize the default probability of the company and, simultaneously, the probability that the guarantee related to the firm is positive when the company is bankrupt) is very high and close to the upper bound of the interest rate variation range. In particular, we obtain that α^* is equal to 4.7909, 4.8990 and 4.9103 respectively for the small, the medium and large enterprises. We can also argue that the level of the interest rate α^* increases with respect to the size of the firm. Our explanation is that the value function in (7) is the maximum of the sum of two terms: the probability of default and the probability of a positive guarantee when the company fails. When the loan interest rate increases, then company default should become more probable and the firm's life-time before default shorter. Therefore, the financial distress period should be long enough to allow the guarantee to become positive, but not long enough to wait for exploitation of the guarantee's wealth. Hence, we reasonably have that the optimizing interest rate is close to the upper bound of the variation range;

2. after performing a correlation analysis⁷ of the results obtained for $P(X^\alpha(\tau_{(0,K)})=0)$, $\bar{\tau}^\alpha$,

$$P(g^\alpha(\tau_{(0,K)}) > 0 | X^\alpha(\tau_{(0,K)}) = 0) \text{ we have that}$$

⁷ It is possible to analyze in detail such results for each firm size in Table A1 of the Appendix. Since the correlation values are small, as standard econometric theory suggests when referring to a considerable amount of information as in this case, we also performed a simple linear regression of $P(X^\alpha(\tau_{(0,K)})=0)$, $\bar{\tau}^\alpha$ and $P(g^\alpha(\tau_{(0,K)}) > 0 | X^\alpha(\tau_{(0,K)})=0)$ with respect to α , in order to confirm the evidence in the correlation's direction; the results obtained are classified by firm size and reported in Table A2 of the Appendix.

Once more, having a considerable amount of data and because the aim of these regressions is to receive confirmation about the sign (and subsequently about the causality direction) of the regression parameters, it is possible to understand why there is a very low R^2 associated to any simple linear regression. In any case, the t-tests allow us to be confident

- the probability $P(X^\alpha(\tau_{(0,K)})=0)$ increases with respect to the interest rate α . This result is to be expected, since the term α has a negative effect on the evolution of the firm's wealth;
- the average $\bar{\tau}^\alpha$ of the values of the $\bar{\tau}_j^{\alpha_i}$ for which $X^\alpha(\tau_{(0,K)})=0$ decreases with respect to α . The result is also as expected in this case, since the growth in the loan interest rate has accelerated the failure of the company;
- the probability $P(g^\alpha(\tau_{(0,K)})>0|X^\alpha(\tau_{(0,K)})=0)$ increases with respect to α . The userer is not interested in the real value of the firm but rather in his/her own personal use of the guarantee. The userer could request the transfer of ownership of the company from the borrower to him/herself in order to merge the illegal money with the money used in the course of legal business, in other words laundering dirty money.

5 Conclusions

The aim of this paper is to pinpoint the level of interest rate which allows illegal financiers to maximize their target function. We propose a new model specifically for the illegal financier's objective, as the userer aims to maximize the probability of leading the company into bankruptcy and, simultaneously, of obtaining the maximum wealth level of the firm's guarantee.

Due to the particular complexity of the function of the illegal objective, we solve the optimization problem by performing a Monte Carlo simulation procedure. Globally, the results obtained show that (i) the level of interest rate needed for the purposes of the illegal financier is very high and close to the upper bound of the interest rate variation range; (ii) the optimizing loan interest rate increases as the firm's size increases; (iii) the default probability of the company increases with respect to the interest rate; (iv) the average of the time values for which the borrower is in default decreases as the interest rate increases; (v) the probability that the guarantee is positive when the borrower is in default increases with respect to the loan interest rate.

Judicial experience may suggest the presence of a critical date \bar{t} , when the interest rate payable for the restitution of the loan is renegotiated. Indeed, a typical repayment plan consists in a number of small installments at the beginning and a single huge installment, to be repaid by the company owner to the userer at a future date \bar{t} , which appears, to the borrower, to be a long time away. The borrower is often unable to pay such a huge amount and, in this worst-case scenario, the userer imposes different conditions and the debt is renegotiated. The formalization of a model describing this framework is already on our research agenda.

of a probability of 95% in the value, and so in the sign, of any regressor's coefficient and, as a consequence, it is possible to confirm the results reported in previous Table A1 regarding the correlation's direction.

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APPENDIX

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Table A1. Correlations between the quantities obtained via the Monte Carlo Simulation.

| | X(0)=100 | X(0)=500 | X(0)=1000 |
|--|----------|----------|-----------|
| $\rho(P(X^\alpha(\tau_{(0,K)})=0), \alpha)$ | 0.003355 | 0.00382 | 0.00160 |
| $\rho(\tau^{-\alpha}, \alpha)$ | -0.00746 | -0.00246 | -0.00208 |
| $\rho(P(g^\alpha(\tau_{(0,K)}) > 0 X^\alpha(\tau_{(0,K)})=0), \alpha)$ | 0.00656 | 0.00312 | 0.00390 |

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Table A2. Results of the Regressions between the quantities obtained via the Monte Carlo Simulation. Dependent variable: interest rate level (α).

| Independent variable | X(0)=100 | X(0)=500 | X(0)=1000 |
|--|-----------------------|----------------------|--------------------|
| $P(X^\alpha(\tau_{(0,K)})=0)$ | 1.68 (0.776) | 1.90 (0.696) | 0.80 (0.380) |
| $\tau^{-\alpha}$ | -0.0037 (0.000014) | -0.0012 (0.00015) | -0010 (0.00005) |
| $P(g^\alpha(\tau_{(0,K)}) > 0 X^\alpha(\tau_{(0,K)})=0)$ | 3.287 (1.673) | 1.556 (0.192) | 1.94 (0.946) |

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- Each parameter is significative at 5% referring to a bilateral test
 - Standard error in brackets
 - Number of observations = 50000.

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- [1] Argentiero, A., Bagella, M., Busato, F., 2008. Money laundering in a two sector model: using theory for measurement, *European Journal of Law and Economics*, 26(3), 341-359.
- [2] Bagella, M., Busato, F., Argentiero, A., 2009. Money Laundering in a Microfounded Dynamic Model: Simulations for the U.S. and the EU – 15 Economies, *Review of Law and Economics*, 879-902.
- [3] Bardi, M., Capuzzo-Dolcetta, I., 1997. *Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equation*, Birkhäuser Boston, Inc., Boston, MA.
- [4] Barone, R., Masciandaro, D., 2008. Worldwide anti-money laundering regulation: estimating the costs and benefits, *Global Business and Economics Review*, 10(3), 243-264.
- [5] Barone, R., 2004. Riciclaggio Finanziario e Credito d'Usura: Un Modello di Analisi Economica, *Rivista Internazionale di Scienze Sociali*, 2, 119-135.
- [6] Battaglini, M., Masciandaro, D., 2000. The Possibility of Ringing Twice: Banking Contracts, Usury, Property Rights and Renegotiation, *Rivista di Economia Politica*, 27(3), 415-443.

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- [7] Bester, H., 1994. The Role of Collateral in a Model of Debt Renegotiation, *Journal of Money Credit and Banking*, 26(1), 72-86.
 - [8] Borkar, V.S., 1989. *Optimal control of diffusion processes*, Pitman Research Notes in Mathematics Series, Longman Scientific & Technical: Harlow.
 - [9] Caperna, A., Lotti L., 1995. Il fenomeno dell'usura tra esperienze giudiziarie e prospettive di un nuovo assetto normativo. *Banca, Borsa e Titoli di Credito*.
 - [10] Cifarelli, M.D., Masciandaro, D., Peccati, L., Salsa, S., Tagliani, A., 2002. Success or failure of a firm under different financing policies: a dynamic stochastic model, *European Journal of Operational Research*, 136(3), 471-482.
 - [11] Crosato, L., Dalla Pellegrina, L., 2008. Improving Bankruptcy Proceedings or Strengthening Punishments? An Assessment on anti-usury policies, "Paolo Baffi" Centre Research Paper Series No. 2008-30.
 - [12] Dalla Pellegrina, L., 2008. Illegal Finance, Interest Rates and Bank Lending: the Missing Supply Side of Usury, *Global Business and Economics Review*, 10(3), 265-283.
 - [13] Europol, 2008. OCTA 2008: EU Organized Crime Threat Assessment, European Police Office, The Netherlands.
 - [14] Financial Action Task Force – Groupe d'action financière (2007), "Complex Money Laundering Techniques: A Regional View" Fatf – gaFI
 - [15] Fleming, W.H., Soner, H.M., 1993. *Controlled Markov processes and viscosity solutions*, Applications of Mathematics (New York), Springer-Verlag: New York.
 - [16] Krylov, N.V., 1980. *Controlled diffusion processes*, Springer-Verlag: New York.
 - [17] Masciandaro, D., 2001a. Why Shylock Can Be Efficient? A Theory of Usury Contracts, *Kredit und Kapital*, 24(3), 381-399.
 - [18] Masciandaro, D., 2001b. In Offense of Usury Laws: Microfoundations of Illegal Credit Contracts, *European Journal of Law and Economics*, 12(3), 193–215.
 - [19] Masciandaro, D., Battaglini, M., 2000. Il vantaggio di bussare due volte: contratti bancari ed usura, diritti di proprietà, valore della garanzia e della rinegoziazione, *Economia Politica*, 3, 415-444.
 - [20] Øksendal, B., 1995. *Stochastic differential equations. An introduction with applications*, Fourth edition, Universitext, Springer-Verlag, Berlin.
 - [21] Schneider, F., 2005. Shadow Economies around the World: What Do You Really Know?, *European Journal of Political Economy*, 21(3), 598-642.
 - [22] Schneider, F., 2008. Money Laundering and Financial Means of Organized Crime: Some Preliminary Empirical Findings, "Paolo Baffi" Centre Research Paper Series, 16, 1-30.

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- [23] Unger, B., 2007. Implementing Money Laundering (Chapter 4), in Masciandaro, D., Takts, E., Unger, B., *Black Finance: The Economics of Money Laundering*, Edward Elgar, Cheltenham UK-Northampton, MA, USA.
 - [24] Unger B., 2009. Money Laundering – A Newly Emerging Topic on the International Agenda, *Review of Law and Economics*, 807-819.
 - [25] United Nations Office on Drugs and Crime, 2010. Promoting health, security and justice. Annual Report 2010.
 - [26] Walker, J., 1999. How big is the global money laundering?, *Journal of Money Laundering Control*, 3(1), 25-37.
 - [27] Walker, J., Unger, B., 2009. Measuring Global Money Laundering: “The Walker Gravity Model”, *Review of Law and Economics*, 821-853.
 - [28] Yong, J., Zhou, X.Y., 1999. *Stochastic controls. Hamiltonian systems and HJB equations*, Applications of Mathematics, Springer-Verlag: New York.