MATHEMATICAL MODELLING OF AIR CYCLE SYSTEMS FOR COMBINED HEATING AND COOLING

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ABSTRACT

Air cycle systems produce significant quantities of relatively high grade heat, allowing them to be employed in combined cooking and cooling of suitable food products.

A recent study assembled an experimental air cycle apparatus from readily available components from the aircraft air conditioning and food industries. Its design was assisted by development of a mathematical model based on performance characteristics of the main components. It was apparent from the model that the available bootstrap was not capable of balanced operation at the extremes of high and low temperatures required. The model was used to identify and assess modifications to balance the flow.

The experimental apparatus was built based on the best option, and test results used to validate the model. The model was then extended in a further study to assess the improvements to system performance likely to result from use of a purpose-built bootstrap and other optimised components (termed ‘ideal’ components).

1. INTRODUCTION

Air cycle refrigeration is a gas cycle rather than a vapour cycle, with no change of phase. It is therefore less efficient at cooling (at typical food processing temperatures) than vapour compression systems but it also produces significant amounts of useful heat at temperatures far higher than those which can be used for heat recovery from conventional systems (Das, 2000). If this heat can be used concurrently with the cooling to meet a heating or cooking requirement, or stored for example in hot water, air cycle can begin to compare favourably with conventional systems with separate cooling and heating equipment. Air cycle is also capable of temperatures much lower than typical (non-cascade) vapour compression systems, providing a feasible alternative to liquid nitrogen.

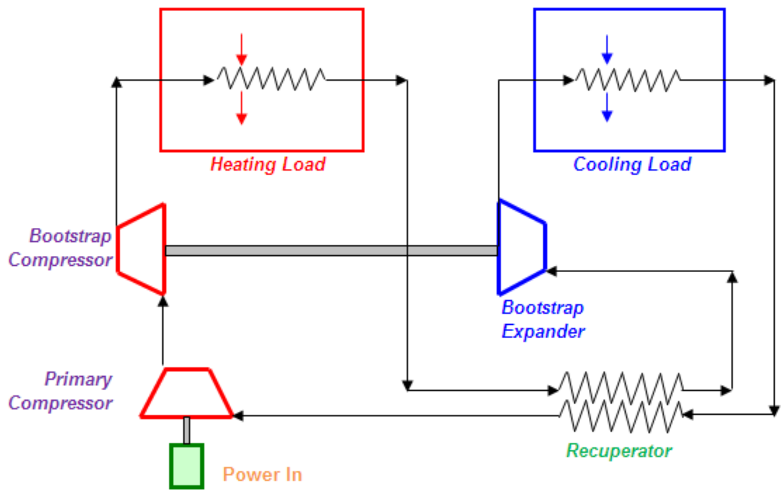

Figure 1. Basic air cycle system with bootstrap and recuperator

Modern air cycle components used in applications such as aircraft air conditioning (Rogers, 1994) are based on high speed turbo-compressors and turbo-expanders, with primary compression coming either from an associated source, such as a bleed of air from the engines in aircraft air conditioning, or from a dedicated primary compressor (Fig. 1). A common technique is to mount the turbo-compressor and turbo-expander on a common shaft in an arrangement known as a bootstrap system, which allows the work released during expansion to be used to drive the bootstrap compressor. A further efficiency improvement is to use a recuperative heat exchanger to pre-cool the air entering the turbine using the cold air returning from the cooling apparatus.

The work reported in this paper was part of two recent studies of the feasibility of using readily available components developed for aircraft air conditioning and food processing in a closed air cycle system to cook and cool suitable food products such as beef burgers. In the case of the bootstrap, this led to the use of an item designed to receive air at the compressor at between 100 and 200°C and at the expander at between 20 and 170°C. However, to achieve the air temperatures required for low temperature freezing in the study, temperatures into the turbine (achieved by use of a recuperator) were well below its design conditions. The density of air passing through the turbine was therefore much higher than it was designed for, resulting in a volumetric flow mismatch between the compressor and the turbine. There was also a thermodynamic mismatch which would mean that Equation 1 below would not be satisfied as insufficient work would be produced by the turbine to drive compression:

$$W_T = \eta_T m_T c_p T_T \left[1 - \left(\frac{1}{r_T} \right)^{(\gamma-1)/\gamma} \right] = W_C = \frac{m_C c_p T_C \left[(r_C)^{(\gamma-1)/\gamma} - 1 \right]}{\eta_c} \qquad \text{(Equation 1)}$$

where W_T and W_C = Work of expansion and compression (J)

η_T and η_C = Efficiency of expansion and compression

m_T and m_C = Mass flow through expander and compressor ($kg.s^{-1}$)

c_p and c_v = Specific heat of air at constant pressure and constant volume ($J.kg^{-1}.K^{-1}$)

T_T and T_C = Air temperatures at expander and compressor inlets (K)

r_T and r_C = Pressure ratio for expansion and compression

γ = Ratio of specific heats (c_p/c_v)

A steady state mathematical model was therefore developed to assist in evaluating approaches to dealing with this mismatch, some of which are described below. The chosen approach was then assembled and used for cooking and cooling of burgers as an example product, and for measurements of Coefficient of Performance (COP = Cooling or Heating Duty / Power Consumed). Experimental results were compared with predicted performance to validate the model. The model was then extended to look at the potential for improved performance offered both by a bootstrap designed to avoid the flow mismatch, and by the use of an optimised primary compressor and heat exchangers.

2. METHOD

2.1 Basic description of model

A steady state mathematical model of a closed air cycle system with ability to heat and cool food products and to produce hot water was developed. It was programmed in Microsoft C++, with results being transferred to Microsoft Excel for graphical and numerical analysis.

The basis of the model was the ability to navigate 3-dimensional bootstrap compressor and expander performance characteristics to arrive at stable operating conditions under varying cooling and heating loads and speed of operation of a high-speed rotary primary compressor, while satisfying the work balance presented in Equation 1. These characteristics were provided by the bootstrap manufacturer. The compressor characteristic used mapped compression efficiency as a function of pressure ratio, 'corrected' mass flow (actual mass flow * square root of inlet temperature / inlet pressure) and 'corrected' speed (rotational speed / square root of inlet temperature). For expansion, two characteristics were used, the first mapping expansion efficiency as a function of velocity ratio (rotational speed / (manufacturer's component factor * square root of isentropic temperature drop)) and the second mapping turbine flow factor as a function of 'corrected' turbine speed and pressure ratio.

Around this basis, an iterative model was used primarily because of the inclusion in the system of a recuperator. Transferring heat between the high and low pressure sides of the system on either side of the expander, the recuperator reduces the temperature of air entering the expander by cooling it with air returning from the expander via the cooling tunnel (see Figure 1). The circular nature of this heat transfer, together with the need to balance loads around the system, required several iterative loops.

A simplified description of these loops and some of the model data flow is as follows:

Enter initial input data, including limits for power input.

LOOP 1 > Iterate to adjust expander inlet pressure until primary compressor power is within above limits

> LOOP 2 > Iterate to adjust low pressure inlet temperature to recuperator based on cooling load tunnel exit temperature (due to circular nature of heat exchange in recuperator)
>
> > LOOP 3 > Iterate to adjust primary compressor outlet pressure until bootstrap compressor outlet pressure is within error limits of expander inlet pressure plus pressure drops in components and pipes in between
> >
> > Check for choked flow, calculate expander mass flow and pressure ratio
> >
> > > LOOP 4 > Balance work form expansion with work in bootstrap compressor, allowing for mismatch due to losses
> > >
> > > Calculate values to navigate bootstrap characteristics
> > >
> > > Derive efficiencies of bootstrap compression and expansion from characteristics
> > >
> > > Calculate temperatures and pressures around bootstrap, including iterations to balance heating and cooling loads based on new air temperatures and flow rates

Calculate end values including COPs and output to file.

2.2 Development of design for experimental system

To operate the available bootstrap under conditions of flow mismatch, several options were assessed. One of these was to block a proportion of the nozzles in the expander to reduce the flow to a level which could be handled by the bootstrap compressor. While successful, as might be expected the model indicated that this resulted in unacceptable decreases in system cooling capacity as system mass flow needed to be reduced to 60% or less compared to the original expander flow. Another option was the addition of a further compressor in parallel with the bootstrap compressor, through which a varying proportion of the mass flow could be diverted away from the bootstrap compressor. The model indicated that this approach, while requiring an increase in power to run the parallel compressor, nevertheless gave satisfactory balance and acceptable performance. It was therefore chosen as the design for an experimental test apparatus based on components supplied by industrial partners in the feasibility study.

Model results also showed the need for several heat exchangers, including an intercooler between the primary and parallel compressor to avoid excessive temperatures in the parallel compressor. Two other exchangers were required to remove some of the large amounts of heat produced by compression, as shown in Figure 2. The first (the water heater) was found to be best positioned after the heating load tunnel but before re-combination of the parallel compressor bypass, where air temperatures were still high enough to produce hot water at temperatures up to 95°C. The second exchanger (the reject heater) was added after the parallel compressor bypass was re-combined with the bootstrap compressor flow, to reject surplus heat from the high pressure air before it entered the recuperator. Varying the amount of heat rejected was found to be one approach to controlling temperatures on the low pressure side of the system.

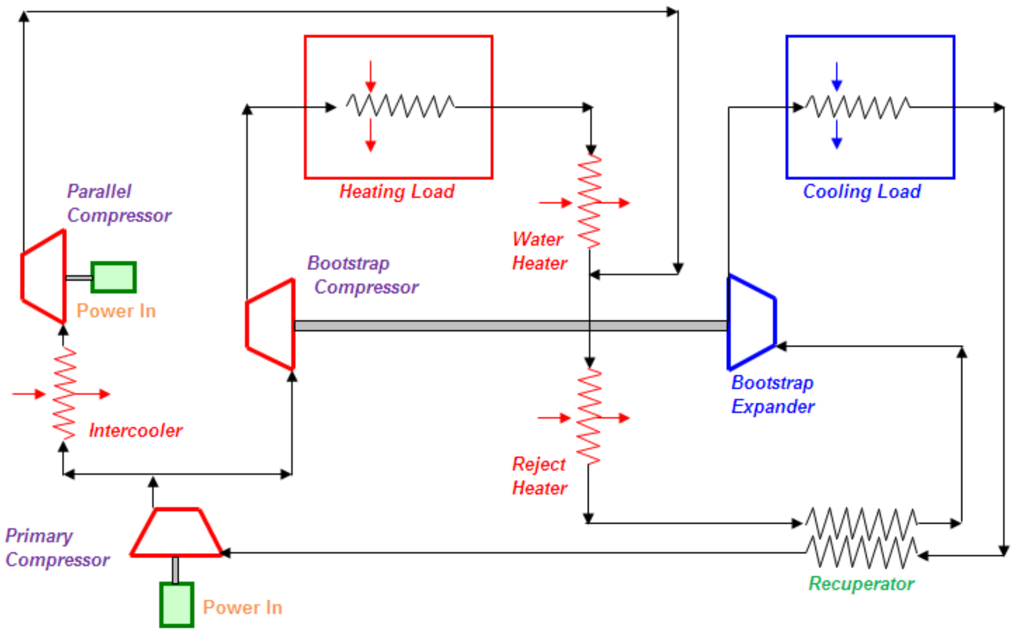

Figure 2. Experimental design with parallel compressor

2.3 Validation

The experimental apparatus was assembled from components supplied by project partners and from additional purchased items as described by Foster et al (2010). Extensive measurement and recording of temperature, pressure, power, mass flow, rotational speed and other data were implemented, and results used to compare with key predicted values from the model.

2.4 Ideal components

The potential for improved performance offered by use of components designed and optimised for the conditions required was assessed as part of a second study by replacing the bootstrap performance characteristics compressor and expander data based on matched performance at the temperatures required, and by varying the efficiency or effectiveness of each of the components. In practice, the matched bootstrap data gave a simplified representation of a bootstrap with a re-sized compressor.

3. RESULTS AND DISCUSSION

3.1 Validation

A series of tests using the experimental apparatus (Foster et al, 2010) determined the COPs of cooling and heating for nine different sets of operating conditions. The model was run with these same operating conditions and key performance parameters compared. Differences between experimental and model results resulted from several sources. These included measurement inaccuracies (for example, air temperatures before and after each component were measured using thermocouples positioned in pockets inserted to the centre near the inlet and outlet of connecting pipe work, and it was apparent that some of the measurements from these positions did not accurately reflect the average temperature of the air due to non-uniformity of flow, radiation etc.), factors not modelled (such as leakage, dehumidification etc.) and factors modelled in relatively unsophisticated ways (such as thermal gains to pipes, pressure drops in pipes).

Table 1 presents a comparison of selected parameters for an example of one of the nine runs.

Differences between experimental and model results resulted from several sources. These included measurement inaccuracies (for example, air temperatures before and after each component were measured using thermocouples positioned in pockets inserted to the centre near the inlet and outlet of connecting pipe work, and it was apparent that some of the measurements from these positions did not accurately reflect the average temperature of the air due to non-uniformity of flow, radiation etc.), factors not modelled (such as leakage, dehumidification etc.) and factors modelled in relatively unsophisticated ways (such as thermal gains to pipes, pressure drops in pipes).

Table 1. Comparison of experimental and model results.

Parameter	Unit	Measured	Model	Variance (mod-meas)	Variance (% of measured)
Primary compressor inlet temperature	°C	24.3	20.0	-4.3	
Primary compressor inlet pressure	kPa	102	102	0	
Primary compressor outlet temperature	°C	153.4	146.0	-7.4	
Primary compressor outlet pressure	kPa	270.0	270.0	0	
Expander inlet temperature	°C	-89.0	-85.0	4	
Expander inlet pressure	kPa	331.0	332.0	1	
Expander outlet temperature	°C	-121.0	-121.0	0	
Expander outlet pressure	kPa	111.0	111.0	0	
Mass flow	$kg.s^{-1}$	0.276	0.26	-0.02	-5.8
Split into parallel	%	38	38	0.0	0.0
Primary compressor efficiency	%	77	78	1.0	1.3
Primary compressor pressure ratio		2.65	2.65	0.0	0.0
Primary compressor power	kW	44.1	43.9	-0.20	-0.5
Parallel compressor power	kW	7.9	7.6	-0.30	-3.8

Expander efficiency	%	76.4	71.9	-4.50	-5.9
Expander pressure ratio		2.98	2.99	0.01	0.3
Recuperator effectiveness		0.85	0.85	0.0	0.0
Heating duty	kW	38.6	37.2	-1.40	-3.6
Heating COP		0.742	0.722	-0.02	-2.7
Cooling duty	kW	3.5	3.4	-0.10	-2.9
Cooling COP		0.067	0.066	0.0	-1.9

However, from the comparison it was concluded that the model successfully informed the design process for the experimental rig, and was within acceptable accuracy compared to measured results. It was felt appropriate therefore to extend the use of the model to examine the potential for improved performance offered by correctly designed / matched and optimised components.

3.2 Ideal components

3.2.1 Bootstrap component matching

In the experimental apparatus, a parallel compressor was introduced to enable operation of the available bootstrap outside of its design conditions by removing some of the mass flow from the compressor. To model a system without this compromise (which requires additional power input, complexity and losses), it was necessary to simulate a bootstrap with a compressor capable of handling the full mass flow at the desired conditions. Modified performance characteristics for use in the model were therefore produced in collaboration with the bootstrap manufacturer, which estimated performance of the same bootstrap expander connected to a bootstrap compressor capable of handling twice the mass flow of the existing unit. While this is deemed acceptable as a design procedure, it does however result in a higher proportion of the total work of compression being transferred to the primary compressor, as insufficient work of expansion is available to drive the bootstrap compressor. Nevertheless, this simulation of the removal of the parallel compressor demonstrated the improvements shown in Table 2 below.

Table 2. Effect of matched bootstrap components

System	Total input power (kW)	Pressure ratios			Cooling			Heating		
		Primary Compressor	Bootstrap Compressor	Expander	Load (kW)	Average temperature (C)	COP	Load (kW)	Average temperature (C)	COP
Experimental	50.0	2.65	1.34	2.99	3.4	-114.5	0.066	37.2	101.9	0.745
Matched bootstrap	50.0	2.87	1.28	3.11	6	-114.9	0.115	38.6	116.4	0.773

3.2.2 Primary compressor efficiency

The impact of improved primary compressor efficiency was modelled on the basis of maintaining a constant cooling load as efficiency improved. A more efficient compressor requires less power to achieve the same compression ratio, so the cooling load can be kept constant simply be varying the primary compressor input power. This can be seen in Table 3, with consequent improvements to COPs of cooling and heating. The only detrimental effect is a slight decrease in high temperatures.

Table 3. Effect of increasing primary compressor efficiency

Primary compressor efficiency (%)	Recup. effectiveness	Bootstrap compressor efficiency (%)	Bootstrap expander efficiency (%)	Total power input (kW)	Cooling Load (kW)	Cooling Average temp. (C)	Cooling COP	Heating Load (kW)	Heating Average temp. (C)	Heating COP
78	0.85	78	85	49.7	12.2	-106.7	0.246	51.0	101.1	1.026
82	0.85	78	85	47.3	12.3	-106.8	0.260	48.9	97.4	1.034
85	0.85	78	85	45.2	12.2	-106.8	0.271	46.9	94.7	1.038

3.2.3 Recuperator effectiveness

A similar approach was taken to model improvements in recuperator effectiveness, where again an increase in effectiveness allows reduced input power to achieve the same cooling duty (Table 4). Although reducing power input lowers the mass flow and pressure ratio produced by the primary compressor, a more efficient recuperator can achieve the same cooling duty by lowering the cooling inlet temperature and extending the temperature rise during cooling (with only a negligible effect on the average cooling temperature).

Table 4. Effect of improved recuperator effectiveness

Primary compressor efficiency (%)	Recup. effectiveness	Bootstrap compressor efficiency (%)	Bootstrap expander efficiency (%)	Total power input (kW)	Cooling Load (kW)	Cooling Average temp. (C)	Cooling COP	Heating Load (kW)	Heating Average temp. (C)	Heating COP
78	**0.85**	78	85	49.7	12.2	-106.7	0.246	51.0	101.1	1.026
78	**0.90**	78	85	46.2	12.2	-107.6	0.265	48.1	100.5	1.042
78	**0.95**	78	85	43.0	12.2	-108.7	0.285	45.6	100.2	1.061

3.2.4 Bootstrap compressor efficiency

Improving the efficiency of the bootstrap compressor allows the bootstrap to produce a greater proportion of the total pressure ratio, and thereby reduces the proportion which the primary compressor must produce. Again, this allows analysis based on reducing primary compressor input power as bootstrap compressor efficiency improves (Table 5). While COPs of cooling are improved with no effect on average cooling temperature, COPs of heating are also improved but heating occurs at lower average heating temperatures (due to less heat being added by the more efficient compression process).

Table 5. Effect of improved bootstrap compressor efficiency

Primary compressor efficiency (%)	Recup. effective ness	Bootstrap compressor efficiency (%)	Bootstrap expander efficiency (%)	Total power input (kW)	Cooling Load (kW)	Cooling Average temp. (C)	Cooling COP	Heating Load (kW)	Heating Average temp. (C)	Heating COP
78	0.85	**60**	85	49.6	11.8	-102.8	0.238	50.9	104.5	1.024
78	0.85	**69**	85	47.6	11.8	-102.0	0.249	49.0	101.6	1.030
78	0.85	**78**	85	46.1	11.8	-102.8	0.257	47.5	99.0	1.031

3.2.5 Expander efficiency

More efficient expansion in the bootstrap expander lowers cooling temperatures produced by any given pressure ratio. For a constant cooling load, the primary compressor pressure ratio and therefore input power can again be reduced as expander efficiency improves. While COPs of cooling increase and average cooling temperatures are lowered, COPs of heating also increase but average heating temperatures are reduced because less compression (and heat generation) is taking place (Table 6).

Table 6. Effect of improved expander efficiency

Primary compressor efficiency (%)	Recup. effective ness	Bootstrap compressor efficiency (%)	Bootstrap expander efficiency (%)	Total power input (kW)	Cooling Load (kW)	Cooling Average temp. (C)	Cooling COP	Heating Load (kW)	Heating Average temp. (C)	Heating COP
78	0.85	78	**75**	49.6	11.2	-92.7	0.226	50.2	102.3	1.012
78	0.85	78	**80**	44.6	11.2	-94.4	0.251	45.8	98.7	1.026
78	0.85	78	**85**	40.6	11.3	-96.2	0.278	42.3	95.5	1.044

3.2.6 An ‘optimum’ system

If the improvements to the above components are all implemented in conjunction with a matched bootstrap, the overall benefits are significant. Table 7 shows that for almost the same input power and the optimised system set for maximum cooling at the same average cooling temperature, typical cooling duty can be increased by a factor of over 4 while heating duty also increases by over 40%. These duties are reflected in the COPs of cooling and heating which rise from 0.066 to 0.295 and 0.728 to 1.074 respectively.

Table 7. Comparison of experimental (first line) and optimised component systems (second line)

Primary compressor efficiency (%)	Recup. effective ness	Bootstrap compressor efficiency (%)	Bootstrap expander efficiency (%)	Total power input (kW)	Cooling Load (kW)	Cooling Average temp. (C)	Cooling COP	Heating Load (kW)	Heating Average temp. (C)	Heating COP
74	0.85	68.8	71.9	51.5	3.4	-114.5	0.066	37.5	101.9	0.728
85	0.95	78	85	49.6	14.6	-114.7	0.295	53.3	102.4	1.074

4. CONCLUSION

The development of a mathematical model of an air cycle system for combined heating and cooling successfully informed the design process leading to assembly of an experimental air cycle apparatus. Data from this apparatus were compared with predicted values from the model and deemed to be within acceptable levels of variance.

The model was extended to predict the impact of using a purpose-built bootstrap and optimised components. In all cases these reduced the power required to run the system at constant cooling loads, and thus increased COPs of cooling and heating. Depending on the temperatures required for cooling and heating, some care should however be taken to ensure that changes to components do not raise or lower temperatures to unacceptable levels.

While the model indicated considerable benefits offered by improved components, the challenges now are to source such components at reasonable cost and implement their use in real rather than modelled systems.

5. ACKNOWLEDGEMENTS

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