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MODELING AND SIMULATION OF AN AIR MOTOR SYSTEM USING EXTENDED RADIAL BASIS ALGORITHM

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ABSTRACT

The past decade has seen an increase in research activities in the control of pneumatic drives. Motivation for this kind of study is that, the response of a pneumatic drive is very slow which leads to inability of the system to attain set points due to high hysteresis. Also the dynamic model of the pneumatic system is highly nonlinear, which greatly complicates controller design and development. To address these problem areas, two streams of research efforts have evolved using conventional methods to develop a modelling and control strategy and adopting a strategy that does not require mathematical model of the system. This paper presents an investigation into the modelling and control of an air motor incorporating a pneumatic equivalent of the electric H-bridge. The pneumatic H-bridge has been devised for speed and direction control of the motor. The system characteristics are divided into three main regions of low speed, medium speed and high speed. The system is highly non-linear in the low speed region and hence a neuro-model with extended radial basis neural networks is proposed.

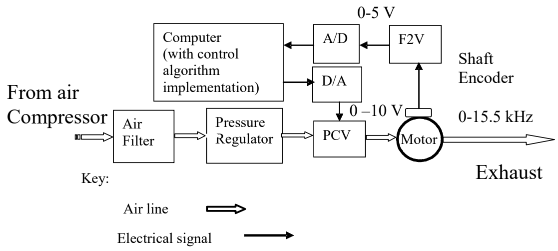
Key words: air motors, modelling, neural networks, radial basis function

1. INTRODUCTION

Modelling plays an important role in the development of servo pneumatics or pneumatic motors. The control of pneumatic motors, regardless of which approach is going to be employed, requires sufficient insight into the behaviour of the system. The exact mathematical model for air motor system at low speed is too complex to be handled analytically. Many attempts have been made to introduce simplified models in order to construct a model-based air motor controller [1]. A common method has been to approximate non-linear dynamics of the air motor into linear (ideal) models assumed to have sufficiently small uncertainty [2]. Studies on modelling of pneumatic systems, especially locally linearised modelling, can be found in the literature [3]. Linear and nonlinear dynamics of the dynamic model of a pneumatic actuator forms the platform and the launching pad point of the motion control algorithms of the air motor system in this study. [4]. There are numerous researchers who have focused their efforts on different issues of modelling of pneumatic servo systems. The issues include but are not limited to the following: Air flow: a normal pneumatic valve does not behave like a simple nozzle. The mathematical model of the valve airflow must be produced specifying the flow capacity of the pneumatic fluid power valves [5, 6]. Valve modelling: there is little work found in the literature on this topic. The valve's input/output behaviour has significant influence on the servo control system. Analysis of pneumatic valve model parameters reveals that, the valve model contains two friction parts, namely static part and dynamic part. Friction parameters may be identified using evolutionary strategies [7, 8]. Air motors are compact, lightweight sources of smooth power with relatively less vibration. They are not affected by continuous stalling or over load; they start and almost stop instantly. They play a very significant part as prime movers because they are relatively cheap, easy to maintain and have versatility of variable speed and high starting torque. They are intrinsically safe in hazardous areas and will operate in exceptionally harsh environments [9]. When a state feedback controller is used to control pneumatic servo systems, it is impossible in practice to determine the control gains theoretically because of the high nonlinearities of the system and the uncertainty in system modelling. Knowledge of the system behaviour is required for construction of a scheme for modelling and simulation of the pneumatic system. Radial basis function neural network (RBF-NN) is a special class of multi-layer feed forward networks, widely used with supervised learning algorithms to solve nonlinear engineering problems. This paper reports on recent advances on scientific findings and application of intelligent techniques as alternative methods of modelling and simulation of an air motor. The paper is organized as follows: Section 2 provides a brief description of the experimental set up utilized in this study. Section 3 briefly describes the modelling approach. Section 4 presents experimental assessment of the performance and the implementation of the RBF-NN modelling strategy. The paper is concluded in Section 5.

2. SYSTEM SET UP

The control system for the air motor is shown schematically in Fig. 1. The computer (PC) with the auxiliary hardware is used to source out and read all plant devices. All electrical devices are externally powered. Coding the control algorithm is straightforward. However, it is always advisable to consider factors such as realization, actuator nonlinearities and computational delay to minimize controller sensitiveness to errors.



A/D: analogue to digital converter
D/A: digital to analogue converter
F2V: frequency to voltage converter
PCV: pressure control valve

Figure 1. A schematic of an air motor control system

The motor speed is measured by a shaft encoder, which represents the measured speed in terms of frequency. The frequency to voltage (F2V) converter transforms the frequency from the shaft encoder to a voltage in the range 0 to 5 V. This analogue voltage is then converted into binary form by an A/D converter, which the computer can now read. The controller uses this measured speed along with other variables to generate a control signal. A D/A converter converts the control signal from binary into analogue voltage. This analogue voltage when applied to the pressure control valve (PCV) either increases or decreases the air pressure to the motor, thus controlling the speed of the motor. Many of the applications of neural networks, particularly in the area of non-linear system identification and control, reduce the problem of approximating unknown functions of one or more variables of discrete measurements [10]. A number of authors have established that multi-layer feed forward neural networks, with a variety of activation functions, serve as universal approximators [11]. In the case of modelling the low frequency dynamics of the air motor, RBF-NN has been chosen. This is a form of neural networks, which can be designed very quickly and find an exact solution. A trade off is that the behaviour of such networks may be extremely complex.

3. SYSTEM IDENTIFICATION

Before controlling the system, the system must be identified. System identification is one of the most fundamental requirements for many engineering and scientific applications. The objective of system identification is to find exact or approximate models of dynamic systems based on observed input and output data. These input and output data can be obtained through experimental work, simulation or directly collected from the plant. Once a model of the physical system is obtained, it can be used for solving various problems such as, to control the physical system or to predict its behaviour under different operating conditions [12]. A number of techniques have been devised by many researchers to determine models that best describe input / output behaviour of a system. In many cases when it is difficult to obtain a model structure for a system with traditional system identification techniques, intelligent techniques are desired that can describe the system in the best possible way [13]. The system characteristics are divided into three main regions, namely low speed (below 390 rev/min), medium speed (390 to 540 rev/min) and high speed (540 to 680 rev/min). The system is highly non-linear in the low speed region and hence a neuro-model using extended radial basis neural networks is proposed.

3.1. RBN-NN-learning algorithms

A generalized regression network (GRNN) is proposed to model the system in the low speed range. The GRNN is basically a two-layer network with a radial basis function in the first layer and a special linear output layer [14]. The task of a learning algorithm for the RBF is to select the center and find a set of weights that make the network perform the desired mapping. A number of learning algorithms commonly used for this purpose include but not limited to the following:

- Random center selection and a least square algorithm

- The orthogonal least square
- Clustering and a least square algorithm
- Nonlinear parameter optimization

Details of these algorithms can be found in [15]. A diagram of the GRNN architecture is shown in Fig 2.

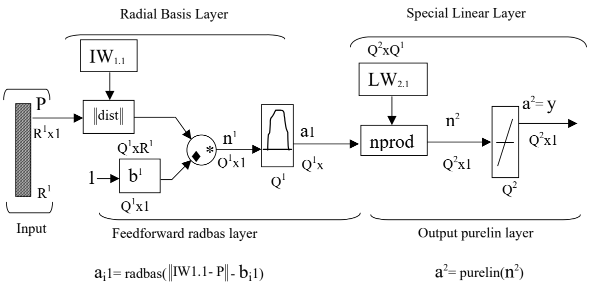


Figure 2. A typical two -layer generalized regression network

The dimensions in Fig. 2 have the following definitions:

- PR represents an $R \times 2$ matrix defining the minimum and maximum values of R inputs
- IW represents the new input weight matrix
- Q is the number of neurons in the layer
- LW is the new $\mathrm{Q} \times \mathrm{R}$ weight matrix
- b is a new $\mathrm{Q} \times 1$ bias vector
- n is the number of network layers

In Fig. 2, the $\|\mathrm{dist}\|$ box accepts the input P, plus the input matrix $\mathrm{IW}^{1.1}$ and produces a vector having Q^1 elements. The elements are the distances between the input vector and vectors $\mathrm{IW}^{1.1}$ formed from the weight matrix. The bias vector b^1 and the output of the $\|dist\|$ are combined with the MATLAB operator (dot*), which does the element-by-element multiplication. The selection of a representative training set is very important when training a neural network (NN). The most critical ability of an NN is its ability to generalize to data to which it has never been exposed [16]. For this generalization to be realized, a training set must be constructed, which is very representative of the entire dataset. For this study, the generation of the training set was done through an experimental data collected from the air motor rig.

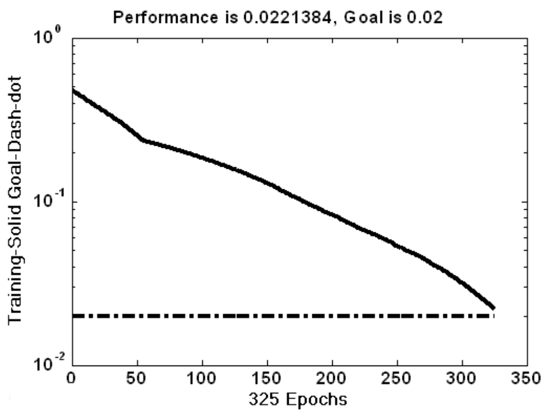


Figure 3: Performance curve showing RBF convergence

Fig. 3 shows the net performance curve on training set. It can be observed that the network tends to converge with sum-squared error (SSE) of 0.022 after 325 epochs. After convergence has been achieved, the network is tested with estimating data set of the remaining 500-input/output data set that the network has never seen. An SSE of 0.037 was obtained after 325 epochs, indicating a 1.5% error. The results are convincingly promising and show that RBF network arbitrarily approximates and learns the parameters in the hidden layer together with those in the output layer and can hence be implemented for further model development. It must be noted that for best network measurement and performance results, the mean squared error is a good criterion to use. However, in this study, SSE produced performance measurement results are adequate enough to be used for further model development.

4. MODEL ANALYSIS AND IMPLEMENTATION

The input variables of the NNs were chosen on the basis of physical laws that describe the behaviour of the pneumatic motor and on the basis of the effect of the inputs on parameters and performance of NNs. Moreover, the effects of functional form of input variables were tested. Following these tests, together with the maximum air pressure (4.5 bar), some combinations of input parameters, measured at variable load were chosen: maximum speed, medium speed and low speed. Maximum pressure was chosen because it is a good indicator of the fluid state while the value of quantities of speed inputs change due to variable load demand. Three different RBF algorithms were investigated in order to determine the type of network most suitable for modelling the air motor system.

4.1. Excitation signal

In a system identification exercise the input signals must be persistently exciting so that they provide sufficient information in the response. Psuedo random binary sequence (PRBS) signals are excellent for dynamic analysis. They can be tailored to the process by proper selection of the variables. PRBS should be tailored such that the frequency band of interest is covered. The PRBS used in this study satisfies the binary polynomial equation:

$$\left(1 \oplus D^3 \oplus D^{10}\right) u\left(t\right) = 0 \tag{1}$$

or

$$u\left(t\right) = \left(D^3 \oplus D^{10}\right) u\left(t\right) \tag{2}$$

where the above equation is irreducible and primitive D^m means a delay of m units and $\oplus$ is modul-2 summation. The variance of the estimates is often proportional to the inverse of the experimental length, which should be as long as possible to increase the accuracy of identification. Typically 1000 data pairs will be adequate for parameter estimation.

4.2. Validating the model

An identified model should never be accepted until it has been thoroughly validated. Some standard tests, which can be applied, include the following:

4.2.1 Parameter significance

If the estimate is $\theta_{Est} \pm \sigma$ where θ_{Est} is the estimated values and σ is the standard deviation, then if $\sigma \geq \theta_{Est}$ the estimate is insignificant. The parameters should be excluded from the model and the estimation algorithm should be re-run. Parameter significance test method has some limitations especially when dealing with non-linear models. It has been evidently shown that air motor systems are non-linear at low speed (Al-Miskiry, 1999; Reynolds, 2000). The essence of this research is therefore to address these non-linear issues. It is therefore appropriate to use a validity correlation test method that will handle non-linear the dynamics of the air motor.

4.3. Correlation functions

An alternative approach for model validation constitutes auto correlation and the cross correlation tests. If a model is adequate then the residual or prediction errors $\varepsilon(t)$ should be unpredictable from (uncorrelated with) all linear and non-linear combinations of past inputs and outputs. Derivation of simple tests, which can detect these conditions, is complex, but it can be shown that for non-linear systems the following five-correlation conditions should hold (Billings and Voon, 1986). A pseudo-random binary sequence (PRBS) input signal is used to excite the system and 1000 input/output data points are collected for estimation of the model parameters. To ensure that the model is an adequate representation of the characteristics of the system, it is validated through a number of tests. These include:

Significance of parameters: An estimated parameter is significant if it is greater in magnitude than its corresponding standard deviation.

Correlation tests: For a model to be adequate, the correlation tests in equations (3) to (7) have to be satisfied.

$$\phi_{\varepsilon\varepsilon}(\tau) = E[\varepsilon(t-\tau)\varepsilon(t)] = \delta(\tau) \qquad (3)$$

$$\phi_{u\varepsilon}(\tau) = E[u(t-\tau)\varepsilon(t)] = 0 \quad \forall\tau \qquad (4)$$

$$\phi_{u^2\varepsilon}(\tau) = E[(u^2(t-\tau) - u^2(t))\varepsilon(t)] = 0 \quad \forall\tau \qquad (5)$$

$$\phi_{u^2\varepsilon^2}(\tau) = E[(u^2(t-\tau) - u^2(t))\varepsilon^2(t)] = 0 \quad \forall\tau \qquad (6)$$

$$\phi_{\varepsilon(u\varepsilon)}(\tau) = E[(\varepsilon(t)\varepsilon(t-1-\tau) - u(t-1-\tau)] = 0 \quad \tau \geq 0 \qquad (7)$$

where, $\phi_{u\varepsilon}(\tau)$ indicates the cross-correlation function between $u(t)$ and $\varepsilon(t)$, $\varepsilon u(t) = \varepsilon(t+1)u(t+1)$ and $\delta(\tau)$ is an impulse function.

For linear models, however, only the first three tests above are sufficient. Theoretically, the above tests indicate that the auto correlation function (ACF) of the residuals should be white, and the cross-correlation (CCF) between the input sequence and a white noise sequence should be zero. In practice, the approximate 95% confidence interval at $\pm 1.96/\sqrt{N}$ can be used to test the above. Testing model variance over a different data sequence. It is important to note that it is easy to fit a model to data that appears to predict well over the data set used for estimation. Even bad models exhibit this property. It is much more difficult to get correct model, the model that describes the dynamics of the underlying system and not one data set.

4.4. RBF model development

Since all validity correlation tests are acceptable, the next step is to develop the system model using real input/output data.

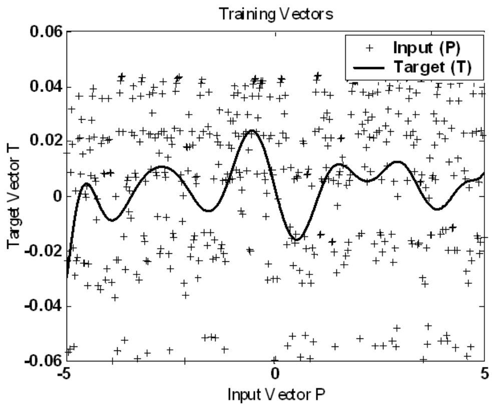


Figure 6. Radial basis function

Fig. 6 shows how as many radbas neurons as there are input vectors in P can be created. This provides a set of radbas neurons in which each neuron acts as a detector for a different input vector. Therefore, if there are Q input vectors, then, there will be Q neurons. In this case, there are 500 input/output data points providing a set of 500 radbas neurons. It is observed that untrained radbas neuron vaguely detect input data vectors presented to them but cannot arbitrarily interpolate reasonably well, to create an explicit radbas profile. The next step is to train 500-input/output data set out of 1000. The trained data is then simulated with the remaining 500 unseen data set to produce input/output radbas network (PT). The transfer function for the radial basis neuron is:

Radbas: (PT) = $e^{-(PT)^2}$ (8)

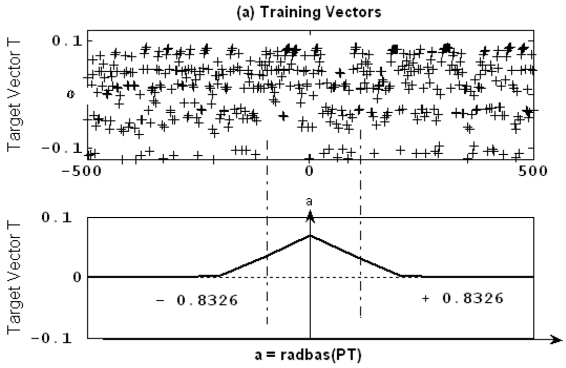


Figure 7. RBF (P, T)

The radial basis function has a maximum output of 0.8 when its input is 0. As the distance between weight and input decreases, the output increases. During training, the neuron was given a bias 'b' of 0.1 and a spread of 8.326. The net input can be expressed as: $\mathrm{sqrt}(-\log(0.4))$. Therefore its output would be: $0.1 \times 8.326 \times 0.6$ i.e. 0.5 for any input vector P at vector distance of 8.326 ($0.8326/b$) from its weight W. Each bias in the first layer was set to 0.8326 per spread. This gives radial basis functions that cross 0.5 at weighted inputs of $\pm$ spread. This determines the width of an area in the input space to which each neuron responds. Fig. 7 shows how a single radial basis transfer function (RBTF) can be utilized to represent a radial basis output for a selected data set. This is the initial stage to design RBF networks and subsequent radbas neurons are then created one after the other until a satisfactory radial basis model is achieved. For this study, 3 data sets were selected; representing three ranges of the air motor speed and their respective outputs are as presented in Fig.8.

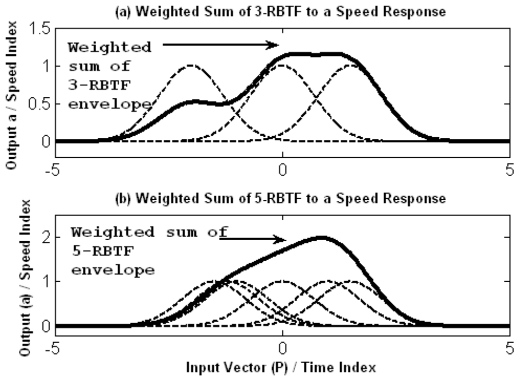


Figure 8. Three & five - RBF model

Fig.8 shows radial basis outputs for these data sets with their weighted sum of responses. Three radial basis neurons do not give a perfectly smooth weighted sum of RBTF envelope and to overcome this shortfall, a radial basis neuron insertion technique is adopted, whereby extra two RBTFs are added at the beginning and the end of the weighted sum of RBTF envelope. The results give a linear five-RBTF model, which is adequate to represent the response of the air motor for the whole data range (low, medium and high speed range). Also, in Fig. 8 a comparison of a three - RBTF model between a five - RBTF model and their respective weighted sum of RBTF responses is shown. A response with many radial basis neurons, give smoother perfectly more linear weighted sum of RBTF output. Therefore, for this study a five - RBTF model (Fig.8 (b)) is the obvious choice. However, there is a trade off between the two choices, a three – RBTF model gives faster response with nonlinear weighted sum of RBTF envelope while a five – RBTF model gives a slower response with a perfectly linear RBTF envelope. During training, three – RBTFs converged in 100 seconds while five – RBTFs converged in 325 seconds. This means that five - RBTFs are slower to operate because they use more computation to do their function approximation or classification.

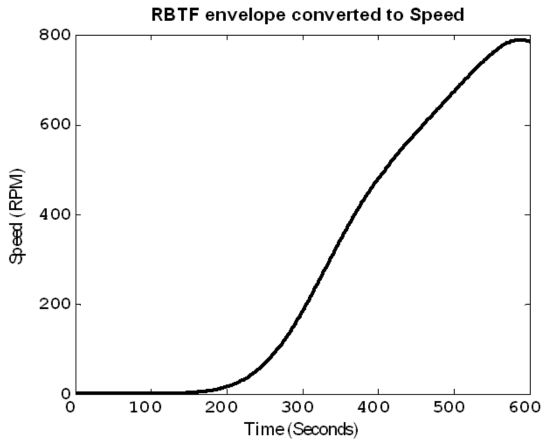


Figure 9. A five - RBTF linear output

Fig. 9 shows the output of the five - RBTF, which represents the air motor speed over the whole data set. Further observations and analysis of Fig. 9 show that using radial basis algorithms enables the air motor system to attain linear operating conditions in less than 200 seconds. In a practical context, these results exhibit instantaneous starting characteristics. These results demonstrate that neural networks are able to handle nonlinearities due to system's hysterisis and can thus be used to control the air motor to attain set point speeds.

4.5. Extended RBF recurrent model

Simple radial basis function network produced reasonably good and promising results. In analogy with conventional RBF network and Elman recurrent networks, the development of RBF recurrent networks then suffices [18].

A diagram of the extended RBF architecture is shown in Fig 10.

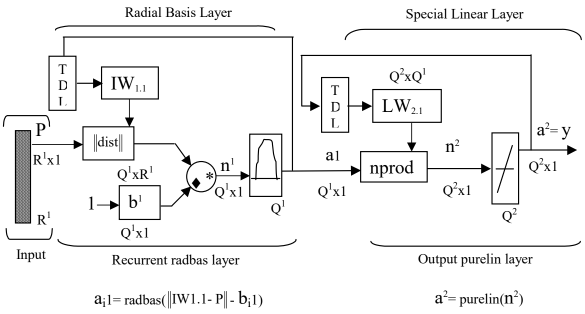


Figure 10. *A typical two -layer recurrent RBF network*

The design of a context layer to which patterns can be copied directly following the learning algorithm, provides for a good comparison of trained inputs of the second layer. This is achieved by the design of the staggered two-delay outputs fed back as inputs of the first layer and second layer respectively. Tapped delay lines consist of a complete memory temporal encoding stage followed by a (dot*) radbas operator. The tapped delay line (TDL) is needed to make full use of the designed neural network. The output of the TDL is an N-dimensional vector, made up of the input signal at the current time, or the previous input signal. The combination of a TDL and a linear network such as purelin, which create a good linear filter. This has the advantage of ease of mathematical analysis and training regimes. As a result, the three networks of the compared algorithms all have tapped delay lines.

The dimensions in the above have the following definitions: PR represents $R \times 2$ matrix defining the minimum and maximum values of R inputs, TDL tapped delay lines, S number of neurons in the layer, IW, new input weight matrix, LW the layer matrix, output vector, b new $S \times 1$ bias vector, n number of network layers and D number of delay lines.

4.6. Implementations and results

The air motor system has three approximate speed regions. The low speed region ranges from 0 RPM to 350 RPM, non-linear region. The medium and high speed regions are approximated from 350 RPM to 540 RPM and 540 RPM to 600 RPM respectively, linear region. Collected data, corresponding to the above speed ranges are chosen to be a set of PRBS shifting between [-850 and –900, -900 and –1050, -1050 and 1300] counts for low, medium and high speed respectively. Figure 11 shows the ramping up and ramping down characterisation of the air motor speed within predefined DC counts regions. This characterisation is desirable to because when the ramp matches the ramp down characterisation, it is a good indication that the system is identifiable by conventional methods, especially within the bounded regions. Furthermore, the bounded regions help to determine the operating regions of the air motor.

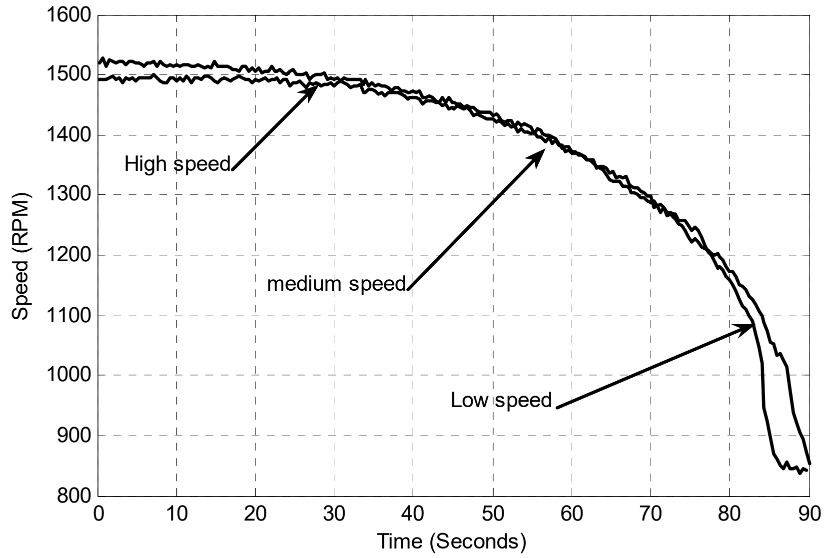


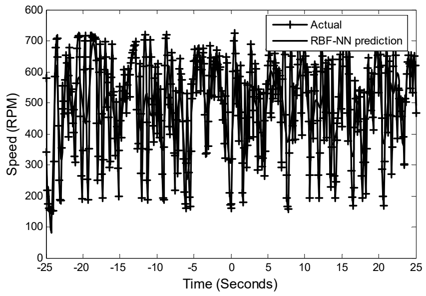
Figure 11 Ramp up and ramp down characterisation of air motor system

For ease of clarity, parameters shown in Fig. 11 can be presented in a tabular form. Table 1 shows the three regions in terms of ADC count, DAC count and RPM.

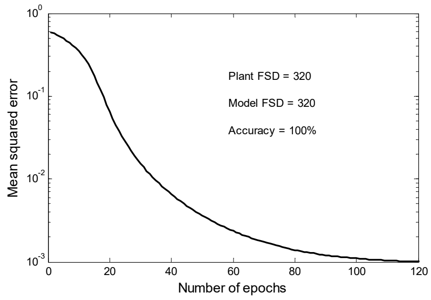
Table 1: Boundary definitions of three speed regions

Region	Input pressure (DAC counts)	Output speed (ADC counts)	RPM
High speed	1100 to 1300	1450 to 1500	450 to 700
Medium speed	900 to 1100	1200 to 1450	350 to 430
Low speed	700 to 900	800 to 1200	0 to350

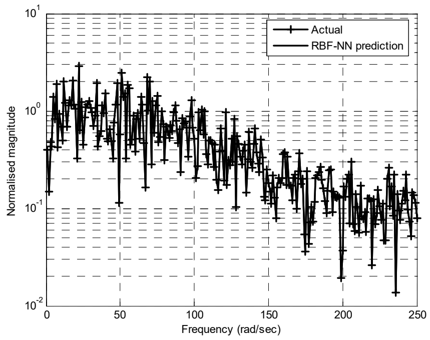
In this investigation, parametric identification of the air motor system with simple least squares and recursive least squares techniques is considered. The experiments involve development of plant model, predicted model, computation of error between the plant model and the predicted model and analysis of correlation function tests.



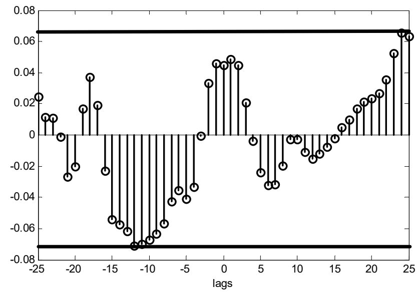
a) Actual and predicted output



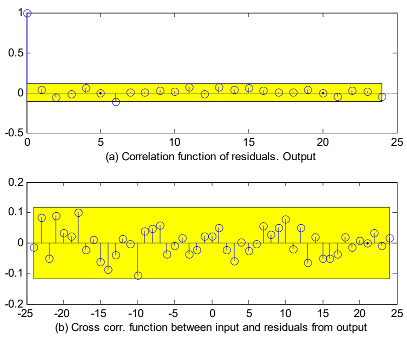
b) Error between actual and predicted output

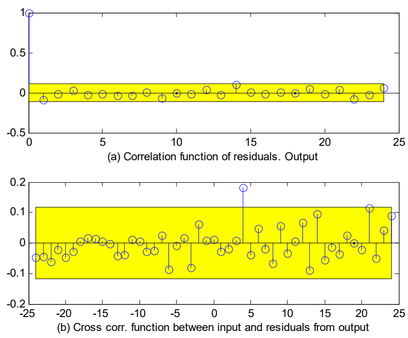


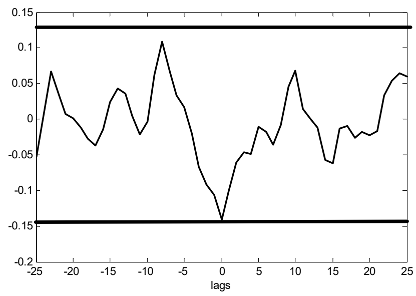
c) Spectral density



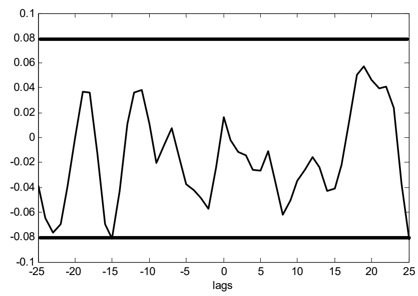
d) Auto-correlation test







f) Cross-correlation of residuals square and input square



g) Cross-correlation of residuals and (input*residuals)

Figure 12 Modelling using RBF at low speed region

5. CONCLUSION

A strategy of applying the radial basis function networks to recognize time varying patterns has been presented. The ability of neural networks has been used in the identification of a pneumatic motor in the low speed region. The effectiveness of this strategy was validated using model validity correlation tests, which were all within the 95% confidence limit. Neural network modelling and simulation techniques presented in this paper show that, given sufficient number of hidden neurons, the RBF-NN can approximate a continuous function to an arbitrary accuracy. However, because the number of radial basis neurons is proportional to the size of the input space, and the complexity of the problem, RBF-NN algorithm can be prohibitively too large. Tuning the various number of parameters, i.e. radius, centers etc, can get quite complicated as is shown in combining regression trees and RBF network insertion. Choosing the right centers (for the hidden layer) is of critical importance although there are a number of ways to solve this unsupervised learning problem, such as using competitive learning. A new recurrent RBF network, which takes the network input and past outputs as augmented input adaptively, learns the parameters in the hidden layer together with those in the output layer. This has the advantage of ease of mathematical analysis and training regimes and outstanding performance in recursive function approximation and estimation.

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