Evaluation of 3D radio-frequency electromagnetic fields for any matching and coupling conditions by the use of basis functions

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*Abstract*- A procedure for evaluating radio-frequency electromagnetic fields in anatomical human models for any matching and coupling conditions is introduced. The procedure resorts to the extraction of basis functions: such basis functions, which represent the fields produced by each individual port without any residual coupling, are derived through an algebraic procedure which uses the S parameter matrix and the fields calculated in one (only) full-wave simulation. The basis functions are then used as building-blocks for calculating the fields for any other S parameter matrix. The proposed approach can be used both for volume coil driven in quadrature and for parallel transmission configuration.

*Keywords*: RF full-wave simulations, electromagnetic RF fields, RF coils, ultra high field Magnetic Resonance.

# Introduction

Since the emergence of Magnetic Resonance (MR) there has been a great interest in predicting and characterizing the electromagnetic (EM) behavior of the Radio Frequency (RF) coil and sample under investigations.

As frequency increases, RF fields interact more strongly with the sample, i.e. the human tissues, rendering static approaches [1] no more reliable for EM characterization. Customized analytical methods can be developed to investigate this EM characterization using simplified geometries and assumptions [2,3]. However, the complex interactions between MR coils and sample cannot be usually solved using analytical methods; it follows that numerical techniques based on full-wave 3D solutions to Maxwell's equations are required [4]. In this context, many different numerical methods can be employed, including the Finite-Difference Time-Domain (FDTD) [5-8], the Finite Element Methods (FEM) [9,10], the Method of Moments (MoM) [11,12]. FDTD, or its version involving the integral form of Maxwell's equations in time domain, i.e. the Finite Integration Technique (FIT), has been widely used for the EM characterization of MR coil loaded with anatomic human models. When applied to volume coils having multi-mode resonances (i.e. birdcage resonator [13,14] and TEM [15]), it suffers a convergence problem, leading to time-consuming simulations. To overcome this difficulty many authors use a simplified geometry (without capacitors) which is enforced to operate at the RF and accordingly to the appropriate single-mode resonance [5]. However, the aforementioned approach cannot take into account the effects of high-order modes, leading to an under-estimation of the Specific Absorption Rate (SAR) [16].

FEM and MoM are both frequency-domain procedures; thus they are both inherently quicker than FDTD and FIT (time-domain procedures). However, MoM suffers in treating anatomic human models and it usually limits its applicability to homogeneous loads only.

Each of the mentioned 3D full-wave numerical techniques permits the EM characterization of RF fields, namely the E and B1 field, and the calculation of the SAR distribution, as well as the S parameter matrix. In most cases, it is difficult to have access to the actual matching networks and to correctly reproduce them in the simulation; thus, it follows that the simulated S parameter matrix can be quite different from the actual one. RF fields and SAR distributions depend on the S parameter matrix: in [17] it has been shown that a simple phase-variation in some Sxy elements of the S matrix can lead to a SAR increase up to 30%. The use of 3D full-wave tools by itself becomes a severe limiting factor in the analysis multi-port MR coil when the anatomic human model is included in the simulation domain, because the full highly complex 3D EM problem must be solved for each matching and coupling condition (note that a multi-port MR coil can be either a volume coil driven in quadrature, or a coil for parallel transmission). Alternatively, two-way link between RF circuit and 3D EM simulation tools can be employed [18]: this enables simulation results from the RF circuit domain to be used to drive the 3D EM domain. Two-way link has been introduced in MR in [8], but it requires to substitute all the lumped elements with ports with impedance of 50 ohm. Here we propose a different procedure which can be summarized as follows:

i. The tuned coil with the anatomic human model is simulated by modeling the feeding using equivalent ports with impedance of 50 ohm as sources for a given matching and coupling condition (without removing any lumped elements); S parameter matrix and RF fields are computed at the operating frequency.

ii. The RF field basis functions, i.e. the RF fields produced by each individual ports without any residual coupling between the other ports, are derived through an algebraic procedure.

iii. For any other S parameter matrix, i.e. for any matching and coupling condition, the RF fields are calculated as appropriate combination of the basis functions.

The procedure permits to quickly evaluate RF fields inside an anatomic human model under different matching and coupling condition. The main difference with previous methodologies is contained in step ii), i.e. the extraction of the basis functions. It is known that the RF fields produced by one port contain residual coupling between the other ports as would be the case in physical system. Some authors have introduced the so-called active decoupling matrix to eliminate such residual coupling [19, 20]. However the active decoupling matrix can be found by resorting to full-wave simulations performed under ideally decoupled condition. Moreover, such condition can be implemented only for parallel transmission [17, 19, 20], and not for volume coils. Here instead, the residual coupling will be removed through an algebraic procedure which uses the S parameter matrix and the RF fields calculated in one (only) full-wave simulation; this will allow to calculate the RF field basis functions which can be used as building-blocks for calculating the RF fields for any other S parameter matrix. It follows that, if we have access to the measured S matrix, we can calculate the realistic RF fields and, thus, SAR distribution.

The proposed approach can be therefore used both for volume coils driven in quadrature and for any parallel transmission configuration. Here, it has been validated by using a 7.0 T volume coil driven in quadrature loaded by an anatomical human head. The term basis functions is used in this paper to denote a set of functions for a give space through which it is possible to express any other functions of that space by linear combination. The basis functions are both frequency and load dependent; this means that they have to be calculated anew if changing the tuning and/or the load.

# Methods

The first step for extracting the RF fields basis functions is to simulate the tuned coil with the anatomic human model through a 3D full-wave numerical tool, modeling the feeding using equivalent ports with impedance of 50 ohm as sources for a given matching and coupling condition (without removing any lumped elements). Thus, if we have *N* sources (*N* can be the number of feeding sources of a volume coil or the number of elements in a coil for parallel transmission), the S parameter matrix will be *N×N*. The *N* RF fields are computed at the operating frequency, assuming each source having default input of 1W and zero phase.

Let us indicate with:

 (1a)

 (1b)

two 1*×N* array constituted by the vector E and B1 RF fields, respectively, produced by the sources and computed at the operating frequency and on a given grid of points. Note that the total RF fields can be easily determined by summing the contribution of all the sources, according to the corresponding power input and driving phase. As stated in the previous section, the RF fields produced by each source contain residual coupling between the other ports as would be the case in physical system. Let us now indicate with:

 (2a)

 (2b)

two 1*×N* array constituted by the vector basis functions for E and B1 RF fields, i.e. they represent the vector fields produced by each source which do not contain the residual coupling between the other ports. It holds:

 (3a)

 (3b)

where, in eq. (3), we indicate with  the *N×N* S parameters matrix, with  the *N×N* identity matrix. Thus the basis function at the operating frequency and on a given grid of points can be determined as:

 (4a)

 (4b)

Once the basis functions have been determined, the RF fields can be quickly determined for any other S parameter matrix by applying eq. (3), i.e. avoiding any other 3D full-wave simulation.

# Results

The procedure has been validated considering a volume coil loaded by an anatomical human head. Specifically, we resorted to the Finite Integration Technique (FIT) in Time-Domain employed in CST MW Suite to simulate a transmit-receive shielded 16 elements 1H high-pass birdcage head coil manufactured by Nova Medical (Wilmington, MA, USA), operating in quadrature at 298 MHz. The elements (copper flat strips) of the coil are placed equally spaced along a circle of radius of 29.5 cm; the radius of the shield is 37.5 cm, the length of the coil is 27.5 cm. The coils has been loaded by a human head extracted from the 2×2×2 mm3 voxel-size anatomic adult human model Billie (Virtual population, ITIS foundation), as shown in Fig. 1.

Quadrature feeding has been employed by using 4 sources having equal input power, equally azimuthally displaced by π/2, with a relative electrical phase shift of π/2.

The coil has been tuned to the frequency of 298 MHz and matched using a capacitive matching circuit [21], achieving: . (5)

In the CST simulations, 12.5 million mesh node have been used (simulation time for all 4 ports= 48 hours on one Intel (R) Xeon (R) CPU E31255- 3.10GHz/32GB-RAM workstation).

The vector E and B1 RF fields produced by the four sources and computed at the operating frequency and on the axial slice crossing the eyes are calculated when applying 1 W of input power (at each source); the corresponding z-components of E and the counter-rotating components of B1 (i.e. B1+) are shown in Fig. 2 and Fig. 3 (for the sake of brevity, the x/y components of E and the z/anticounter-rotating components of B1 are not shown). The total B1+ field for quadrature feeding, i.e. determined by summing the contribution of all the sources accordingly to the correspondent driving phase, is also given in Fig. 4. Fig. 5 and Fig. 6 shows the vector basis functions for E (z-components) and B1 (counter-rotating components).

With the aim of validating the procedure, we performed another 3D full-wave simulation with different matching and coupling conditions: this was achieved by reducing three times the matching capacitors, leading to:

. (6)

In this context, Fig. 7a and 7c show the magnitude and phase of total B1+ field, quadrature feeding, determined by using a 3D full-wave simulation with the different matching and coupling conditions. Fig. 7b and 7d show the magnitude and phase of total B1+ field, quadrature feeding, obtained by the using of eq. (3), i.e. as combination of the basis functions of Fig. 6 weighted accordingly to  (with the S parameter matrix given in eq. (6)).

# Discussions and Conclusions

As expected from the geometry of the volume resonator, we found that the z-component of the E fields produced by the single ports is greater than x and y-components.

The total B1+ field determined by summing the contribution of all the sources accordingly to the correspondent driving phase shows the typical central focusing behavior [2].

By comparing the basis functions with the fields produced by the single ports it is possible to quantify the impact of the residual coupling: here, we observed that residual coupling affects especially the fields produced by source 4 with a relative modification of the maximum of the magnitude up to 15% (note that such value is in a good agreement with [17]).

As highlighted before, the residual coupling is removed through an algebraic procedure which uses the S parameter matrix and the RF fields calculated in the first (and only) full-wave simulation; this allow to calculate the RF field basis functions which can be used as building-blocks for calculating the RF fields for any other S parameter matrix. In this context, an excellent agreement can be observed by comparing Fig. 7a and Fig. 7b. Specifically, the average relative error is lower than 3% (maximum relative error lower than 4%). An excellent agreement between that phases can be observed too, as shown by Fig. 7c and Fig. 7d. It follows that, once the basis functions have been determined, the RF fields can be quickly determined for any other S parameter matrix by applying eq. (3), i.e. avoiding any other 3D full-wave simulation. We remind the reader that this holds true if the effects of fringe fields of capacitors and other lumped elements are neglected. If the measured S parameter matrix is available, it can be used in eq. (3), allowing the procedure to provide an insight into the realistic RF fields distributions.

Finally, we point out again that the purpose of the paper is to provide a method for determining the change in the fields of a coil due to the change in S-parameters. In this context, we introduce the basis functions, i. e. a set of *N* functions for a give space (*N* can be the number of feeding sources of a volume coil or the number of elements in a coil for parallel transmission) through which it is possible to express any other functions of that space by linear combination. We have shown that: it is possible to extract the basis function using one full-wave simulation; such basis functions are then used as building-blocks for calculating the fields for any other S parameter matrix.

The basis functions are both frequency and load dependent; this means that they have to be calculated anew if changing the tuning and/or the load.

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**Figure Captions**

**Fig. 1.** Billie's head inside the MR birdcage coil. The 4 sources (numbered clockwise) are displayed in red.

**Fig. 2.** Magnitude of the E field, z-component, produced by the 4 sources when applying 1 W of input power at each source (x and y axis are given in mm, magnitude in V/m).

**Fig. 3.** Magnitude of B1+ produced by the 4 sources when applying 1 W of input power at each source (x and y axis are given in mm, magnitude in T).

**Fig. 4.** Magnitude of B1+ produced by all the sources driven accordingly to the correspondent driving phase (x and y axis are given in mm, magnitude in T).

**Fig. 5.** Magnitude of the 4 basis function of the E field, z-component (x and y axis are given in mm, magnitude in V/m).

**Fig. 6.** Magnitude of the 4 basis function of B1+ (x and y axis are given in mm, magnitude in T).

**Fig. 7.** Magnitude (a) and phase (c) of total B1+ field, quadrature feeding, determined by using a 3D full-wave simulation with the different matching and coupling conditions. Magnitude (b) and phase (d) of total B1+ field, quadrature feeding, obtained by the using of eq. (3), i.e. as combination of the basis functions of Fig. 6 weighted accordingly to  (with the S parameter matrix given in eq. (6)). (x and y axis are given in mm, magnitude in T, phase in radiant).

APPENDIX

Let us consider a system comprising *N* sources (where *N* can be the number of feeding sources of a volume coil or the number of elements in a coil for parallel transmission); the system is loaded with a given anatomic human model. The system can be described through the S parameter matrix, which is *N×N*.

Let us now compute the *N* RF fields at the operating frequency; the fields are expressed in eqs. (1). The field  (with ) contains residual coupling between the other ports; thus, it can be assumed to be constituted by *N* contributions, being the number of sources equal to *N*. Denoting the contributions as in eqs. (2), we have

 (A1)

In eq. (A1) we introduced the matrix  representing appropriate (complex) weights. It holds:

. (A2)

Let us expand the first row of eq (A1), i.e. the field produced by source #1; we have

 (A3)

In eq. (A3):

1.  represents the vector field produced by source #1 which does not contain the residual coupling between the other ports; its weight  should account for reflection that occurs at the port: thus, it holds  . The expression correctly reduces to  if the matching capacitor is short-circuited (note that a capacitive matching circuit has been considered throughout the paper).
2.  represents the vector field produced by source #2 which does not contain the residual coupling between the other ports; its weight  should account for coupling between source #1 and source #2: thus, it holds .
3. the procedure highlighted in ii) can be extend to all the others 

Expanding all the rows of eq. (A1), it can be shown that it holds:

 (A4)

Substituting eq. (A4) into eq. (A1), we derive eq. (3a). The same procedure can be applied for eq. (3b).

Finally, we point out that the power balance cannot be applied directly to  matrix, being the complex power proportional to  (see the analogy with transmission lines theory, where the complex power is proportional to  being Γ the reflection coefficient); it follows that the dissipated power, i.e. the real part of the complex power, is proportional to .

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