A joint text mining-rank size investigation of the rhetoric structures of the US Presidents' speeches

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Abstract

This work presents a text mining context and its use for a deep analysis of the messages delivered by the politicians. Specifically, we deal with an expert systems-based exploration of the rhetoric dynamics of a large collection of US Presidents' speeches, ranging from Washington to Trump. In particular, speeches are viewed as complex expert systems whose structures can be effectively analyzed through rank-size laws. The methodological contribution of the paper is twofold. First, we develop a text mining-based procedure for the construction of the dataset by using a web scraping routine on the Miller Center website – the repository collecting the speeches. Second, we explore the implicit structure of the discourse data by implementing a rank-size procedure over the individual speeches, being the words of each speech ranked in terms of their frequencies. The scientific significance of the proposed combination of text-mining and rank-size approaches can be found in its flexibility and generality, which let it be reproducible to a wide set of expert systems and text mining contexts. The usefulness of the proposed method and the speech subsequent analysis is demonstrated by the findings themselves. Indeed, in terms of impact, it is worth noting that interesting conclusions of social, political and linguistic nature on how 45 United States Presidents, from April 30, 1789 till February 28, 2017 delivered political messages can be carried out. Indeed, the proposed analysis shows some remarkable regularities, not only inside a given speech, but also among different speeches. Moreover, under a purely methodological perspective, the presented contribution suggests possible ways of generating a linguistic decision-making algorithm.

Keywords: Text mining, Natural Language Processing, Politics, Rank-size laws.

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1. Introduction

The main changes in schools of thought and political arrangements have been communicated to the public by means of speeches, whence by different rhetoric structures. Politicians, trying to convince their own people, and others, about their opinions, have always used the words as the primary means [1]. The way to build the rhetoric structures has changed with time. Some aspects of changing in languages are described in [2]. Evidences of these changes could be highlighted through text analysis in order to study classes of words as in [3].

This paper moves from this premise. It presents a joint text-mining procedure and ranksize analysis for the description of the political speeches and their structures. In doing so, we are in line with a wide strand of literature where texts are viewed as complex expert systems and text-mining techniques are used for exploring a number of contexts [4; 5; 6; 7; 8; 9; 10; 11; 12].

The political environment here analyzed let this paper be particularly close to [13], where the authors focus on the Belgian political uncertainty in 2011 and how the Flemish newspapers treated it. We here present the paradigmatic case of the speeches of the US politicians, with special attention to US Presidents. Indeed, the United States President is one of the most important people in the world and his speeches are often addressed to a wide audience. So, there is no doubt about the immanent relevance of the words pronounced by a President and it is expected that they bear much influence on the overall economic and social contexts.

In this work, we deal with analysis of the US presidential speeches in a larger sense than the mere focus on the meaning of the single words as it happens for exploring the topics present in a text [14]. Specifically, we analyze the constitutive rhetoric structures of the speeches to explore the way of creating speeches. The basic scientific motivation lies in the evident accuracy of words selection by the US Presidents and their collaborators, which leads to the worthiness of understanding if a speech repeats obsessively few words or is more scattered among a large numbered of tokens. In this respect, words can be ranked according to their frequencies, so that each talk represents a structured system that contains information about the way of communicating messages to the audience. The inner structure of the speeches can be inferred by applying a proper best-fit procedure for a rank-size law.

Our approach is in line with the languages studies investigating structures of texts, speeches or languages [15; 16; 17; 18; 19]. In our peculiar context, we follow the route traced by the studies on the connection between political speeches and Government policies as in [20] for the case of UK. The contribution [21] is an example of the investigation of news' impact on the financial market starting from premises close to ours. It is also worth mentioning [22] where the author identifies the main changes in the rhetoric of the President Inaugural Addresses and Annual Messages from George Washington to Bill Clinton. As we will see, the present paper is radically different from [22]: (i) the present dataset is remarkably larger than the [22]'s one; (ii) the employed methodology is different: as we will see in details, we use a rank-size approach, while the quoted paper adopts the General Inquirer for the specific assessment of words categories; (iii) the target is different: we here aim at giving a view of the structure of the speeches through the tokens frequencies and

their ranks as in [23], while [22] pays special attention to the meaning of the words.

As anticipated, we investigate the rhetorical structure of the US Presidents by using a different method with respect to the proposals adopting a computer science perspective. In this framework, we mention [24] for the case of web documents summarization, [25] where a deep learning algorithm is used to run a semantic analysis or [26], that deals with a rhetoric study of legal documents. Our work falls in the context of the processes that have to be implemented before the application of machine learning models or automated discourse processing methods (see e.g. [27], where the authors discuss the data preparation phases). In this specific area, we highlight the role of the rank-size law parameters as texts characters keepers, hence reducing the dimension of the textual data. Under a different perspective, the calibrated parameters of the rank-size law generate a three-dimensional space which is able to represent the data (refer to [23] as an example of efforts done to reach text characters representativeness in the pre-processing phase of non-supervised clusterization of corpus). Furthermore, observing the features of the texts from a rank-size perspective, it is possible to avoid restrictive assumptions on languages changes and semantic homogeneity, as in [23]. The followed macro-view approach of the US Presidents speeches transcripts – which are observed as if they constitute a rhetoric framework – leads to a different representation of those documents with respect to the classical bag of word methods.

N-grams models are predominant in Natural Language Processing field. Such methodological proposals are employed to represent texts in vector spaces where similarity and distance measures are applied to determine document closeness. In these cases, the analysis of the words frequencies is the connection between texts processing and successive analysis. The present research can be effectively inserted in the set of studies performing pure dataset exploration with efficient models (see some examples in Table 1) and studies like [23; 28], where new methodological tools are developed). Indeed, we here present an efficient way to extract the information underlying the speeches transcripts. In particular, we are far from the filed in which semantic and syntax are involved and rather deal with a model in the Information Extraction and Information Theory fields.

During the last years, several stylometric researches have contributed to the creation of measures to feed the authorship attribution algorithms (see [29] for a wide review of this theme). In this context, it is important to take into account the lexical aspects of the writings. Indeed, it is well known that the assessment of the most frequent words in a writing foster the identification of its author. Therefore, our procedure can express some potentialities in authorship attribution since we deal with the distribution of the terms within documents, by quantifying the presence of tokens along ranks thanks to the rank-size parametrization. So, it could be possible to create a rhetorical profile for each speaker on the basis of the rank-size analysis of the speeches.

In order to clarify the context in which we are operating we refer to Table 1, where references are related to the different approaches used in text mining and their applications. In this way we offer a summarized view of the state of the art.

A key step of this research is the text-mining procedure for the creation of the dataset which contains about 1000 Presidents' speeches (see e.g. [13] as reference of a similar way for collecting data from internet). Rough data has been taken from the website: http:

Authors	Approach	Description
[18]	Binary transformation of texts; Correlation analysis	An analysis of long-range correlations in texts
[4]	Supervised classification based on machine learning; Naive Bayes; Random forest; Support Vector Machine	Identification of child abuse
[24]	Lantent Semantic Analysis; Singular Value Decomposition; Conditional random field classifier; K-mixture model	New multi-document summarization methods applied to web-based news
[16; 17; 30; 31]	Zipf and Zipf-Mandelbrot law; GrassbergerProccacia method; Multi-fractal analysis and box counting	Comparison between natural and artificial languages
[32]	Rank-Size analysis; Zipf's law; Exponential law; Shannon entropy	Analysis of referees responses in a peer review system
[15]	Frequencies analysis of words from Swadesh fundamental vocabulary; Principal component analysis; Linear regression model; Bayesian Markov chain Monte Carlo model;	Common words usages in different languages
[13]	Frequencies analysis of words belonging to different sentiment classes; Knowledge discovery in databases; Polarity analysis	Sentiment analysis of political related articles
[33]	Random texts generation; Zipf's law; Parametric distance measures; Statistical tests	Comparison between real and random texts
[34]	Frequencies analysis of words belonging to different classes; Extreme gradient boosting; Principal component analysis and regression; Random forest; LASSO and Ridge regressions; Elastic Net	Text mining of regulatory disclosures for finance applications
[14]	Latent Dirichlet Allocation; Markov chain Monte Carlo and heat bath algorithms	Topic modeling for PNAS papers abstracts
[35; 36]	Literature and methodological review	Statistical linguistic exploration using many languages
[37]	Information theory; Zipf's law; Shannon entropy	Questioning language evolution and Zipf's law principle
[25]	Discourse Parsing ([38]); Rhetorical structure theory; Long short-term memory; Deep Learning and Neural Network	A methodological proposal in sentiment analysis
[22]	Frequencies analysis of words belonging to different classes; General Inquirer; Key words analysis	Content analysis of the main US Presidents speeches
[20]	Frequencies analysis of words belonging to different classes; Custom words classes based on geography and political criterion	Sentiment analysis of UK political leaders
[10]	Frequencies analysis of words belonging to different classes; Latent Dirichlet allocation	Research of trends in economics literature
[7]	Frequencies analysis of words belonging to different classes; Lexicon based method; QDA Miner 4.0 software package; Multidimensional scaling	Sentiment analysis for consumer behaviors
[6]	Literature review	Sentiment analysis for finance application
[5]	Text analyst 2.1 software; Clustering; K-means; ANOVA analysis	Keyword-based patent analysis
[8]	Uni-grams analysis; Texts representation through vectors of frequencies; Text distance measurement; Clustering	Intrinsic plagiarism detection model
[39]	Literature review	Rank-size and Zipf's law in natural language
[40]	Literature and methodological review	Words frequencies applications
[28]	Term frequency - Inverse document frequency; K-mixture model	Multi-document summarizer proposal
[26]	Term distribution model; Conditional random field; Rule-learning system SLIPPER	Rhetorical role identification from legal documents
[27]	syntactic n-grams analysis; Part of speech tags; Support vector machine; Naive Bayes; J48 classifier; WEKA approach, see [41]	Authorship attribution
[29]	Literature review	Authorship attribution problems
[23]	Power-law analysis of words frequencies; Frequency Rank Ratio (FRR); Influence Language model	Analysis of words collocation within texts
[3; 21]	Frequencies analysis of words belonging to different classes; General Inquirer; Principal component analysis; Regression analysis;	Sentiment analysis for finance application
[2]	Frequencies analysis of words belonging to different classes; Regression analysis	Lexical replacement analysis in different languages
[12]	Named Entity Recognition; Dictionary based analysis; Regression analysis	Text mining of biomedical literature
		Topic modeling with the Zipf's law

Table 1: List of references about text mining researches. Each row represents a study, starting from the left side, in the first column there are the references, in the second the approaches used and in the third the fields of application of the respective papers.

//millercenter.org the 30^{th} of July, 2017, i.e. a set of 978 speeches, ranging from the Inaugural address of George Washington (1789) to the Donald Trump's speech Address to Joint Session of Congress (2017) (as explained in the Section 3, the number of speeches is reduced to 951 following data collection and treatment phases).

Two classes of results are derived: the first one is associated to the dataset and the second class of findings relies on the rank-size analysis, whose parameters have peculiar meanings. The former class comes from the building procedure presented in a phase-wise including the pre-process phases, to ensure a comfortable replicability of it in other contexts and for further studies. In the respect of the latter case, we get non-linear regression on the Zipf-Mandelbrot law and the respective goodness of fit, hence arguing a common macro-structure among the speeches of the US Presidents.

To the best of our knowledge, this paper is the first one dealing with the US Presidential speeches with a so large dataset.

Interesting implications of social, political and linguistic nature on the speeches delivered by the US Presidents can be carried out. In fact, some noticeable regularities arise. First of all, the number of words used in the speeches form two different regimes in the periods 1800-1850 and 1850-1900. Speeches are more scattered – in terms of the number of employed words – in the last period of investigation. However, after 1900, the frequencies of the words seem to converge toward more homogeneous distributions. Furthermore, a decreasing trend in the highest relative frequencies of the speeches can be observed. All the speeches are associated to best-fit curves with similar decays from the high to the low ranks. All in all, results suggest the presence of common behaviors of the Presidents when delivering a talk – sometimes, associated to specific historical periods – hence pointing to waves of imitative behaviors and institutional universal channels of communication.

The rest of the paper is organized as follows. Next section provides the details of the method used. In Section 3 a description of the dataset building procedure is reported. Then we propose a description of the collected data. Next, we show the main features of the employed rank-size analysis. In Section 6, the results of the analysis along with a discussion of them are reported. Finally, some conclusive remarks are presented and future research directions are also traced.

2. Methodology

The present study is performed through a rank-size analysis on a large corpus constituted by US Presidents speeches transcripts. The size is defined as the (absolute or relative) words' frequencies in each discourse, while the words rank is its position in the decreasing sorted list, so that rank 1 is that for the most often pronounced word in a speech. The rank-size analysis technique is a well-recognized method to explore the property of a large set of data when the data spans several decades and when error bars are not precisely defined due to sampling conditions.

The power law and Pareto distribution with unitary coefficient were originally introduced in [42; 43] to explore the rank-size relationship in the field of linguistics. After its debut, several contributions have investigated the properties of the power law. We report some

examples: [44; 45; 46; 47] in the context of economic geography; [48; 49; 50] in the business size field; [51] in biology; [52; 53] in informatics; [54; 55], in the context of music; [56] in the context of fraud detection; [57] in the gaming field. For a wide review of rank-size analysis see [58]. See Table 2 for a summary of the just mentioned fields of applications.

For what concerns the specific case of text analysis, the rank-size method has been applied in a very wide range of cases, see e.g. [23; 30; 31; 32; 37; 39; 59]. The approach with Zipf's law for text analysis has been criticized by some scientists (see [35; 36]) but, recently, [33; 40] had provided clarifications on some controversial aspects.

The law has been extended by many researchers by introducing methodological modifications as in [60], or in the Zipf-Mandelbrot law (ZML, hereafter) presented in [61; 62] and [63] and in the Lavalette law (see [64]), which have been proven to provide a spectacular fit of rank-size relations, even when Zipf law fails to do it (see e.g. [47; 65]). In Table 3 there is a summary of the methodological works here reported.

These references constitute a non exhaustive list of the wide number of contributions on this topic. Indeed, the families of laws proposed after the first Zipf's paper has massively grown. The researchers have tailored solutions for exploring dataset that, due to their peculiarities, leads to modification of the power laws. Part of this phenomenon originate from the lack of a unique theoretical model that can justify the origination of rank-size regularities. Consequently, this paper moves from the preliminary decision of the law for assessing the rank-size relation among many functions in the literature. In doing so, we have restricted the selection to researches grounded on a similar approach and stemming from analogous theoretical premises, finding e.g. [40; 66; 67]. Finally, we follow the steps of [40] (Chapter 9) in which the authors have evaluated the performance of different comparable rank-size functions, which are suitable even in our cases. Differently from the quoted paper, we do not study texts in different languages and focus only on English texts. Therefore, the best function for these authors – a truncated Zeta distribution – is not the best one for us, because of the different valuation criteria.

In our study, the Presidents speeches transcripts are investigated as a unique expert system of communication. Hence, the corpus is analyzed as a whole framework through a set of parameters able to capture the regularities and the patterns of the communication system. Such patterns are characterized basically by three elements: the words frequencies at high ranks, the changes in occurrences from high to low ranks and the weights of the tail (at low ranks) for each speech. A detailed explanation of these elements is provided in the following sections. This said, it is clear that three parameters are needed to reach such an objective. Indeed, three parameters effectively summarize the state of the communication scheme at speech level and, furthermore, are appropriate for representing the system in a three dimensional space where each dimension corresponds to an aforementioned feature. We notice that we do not fully explore here the potentiality of such a representation. However, we recognize the importance of this point for making differences with respect to the bag-of-word, count vector or term frequency-inverse document frequency (TF-IDF) approaches (e.g. [23]) and with respect the PCA and LSA techniques for dimensional reductions (e.g. [34]). Given these premises, we exclude all the functions with less than three parameters.

The decision making process to chose among the rank-size laws requires a further cost-

benefit valuation. Indeed, it is necessary to take into consideration the potential bias coming from the estimations of three or more parameters in a non-linear relationship due to the risk of falling in local minima. In particular, the selection of laws dependent on a large number of parameters is apparently more informative in its final result. In fact, theoretically, the presence of a large number of parameters to be calibrated leads to a wider set where identifying the best-fit curve. Despite this mathematical evidence, the computational experience leads to a different reasoning. Indeed, a large number of parameters has the severe flaw of the probable inconsistency of the obtained calibrated parameters, which are noticeably affected by the choice of the initial point of the calibration exercise. Thus, local minima of a given error function are likely found, and the computational complexity of a regression procedure with a large number of parameters leads to the practical impossibility of identifying the global minimum, hence leading to an estimation bias (see e.g. [65] for discussion on this point).

Moreover, literature reports rare cases where the ZML has been criticized for the estimation bias problem. As already said, the main criticisms are grounded on the debate on the theoretical ground of any rank-size regularity (see e.g. the striking cases of [35; 36]). Concluding, three parameters are enough for describing the system we are dealing with and do not lead to bias estimation problems. Hence, Zipf-Mandelbrot law, which has also been commonly used in this field, is appropriate for our analysis.

We conclude this methodological section with some technical details. We implement the best fit procedure by using the Levenberg-Marquardt Non-linear Least-Squares Algorithm in order to derive the ZML parameters for each speech (in Section 5 the details are reported). In so doing, we obtain a collection of 951 triples of best-fit parameters on the absolute frequencies as much as for the analysis over the relative frequencies (see the next section for a justification of the speeches number reduction).

3. Dataset building

This section is devoted to describe the dataset building process, namely we present the routine's actions imposed by using R. The commands used to build the dataset are provided by the libraries "xml2", "rvest", "stringr", "xlsxjars", "xlsx" along with their respective dependencies. From a text mining point of view, a part for the web-scraping phases, the following steps are in line with [8; 23].

The building procedure is divided in 13 phases.

In the first step, the considered website was visually examined in order to understand the structure of the contents. In particular, since one is looking for the presidential talks transcripts, it is important to find the addresses' pages where they are listed. In the case of the Miller Center web site, the hyperlinks to each speech are dynamically showed in the following page: https://millercenter.org/the-presidency/presidential-speeches. Consequently, one has to inspect the HTML source code for finding the objects of interest and to decide how to select them. In this case, it means that one needs to save the whole source HTML code and extract the located links as showed in the next phase.

Authors	Model	Description
[48]	Zipf's law	Analysis of distribution of U.S. Firms; Economics
[57]	Zipf's law;	Analysis of chess opening distribution; Gaming
[50]	Zipf's law; Hill estimator; Literature review	Critical review of firm size distribution studies; Economics
[47]	Zipf, Zipf-Mandelbrot and Lavalette's law	Italian cities tax income distribution analysis; Economic Geography
[46]	Zipf's law; Length ratio	Bulgarian Urban system investigation across years; Economic Geography
[45]	Zipf's law; Gibrat's law; Hill estimator	Urban growth and City size distribution analysis; Economic Geography
[56]	Zipf's law; Benford's law	Proposal of a fraud detection model
[44]	Zipf's law; Gibrat's law	Urban growth and City size distribution analysis for the US case; Economic Geography
[52]	Zipf's law; Markov chain	Web surfer behavior modeling; Informatics
[51]	Zipf's law	Genes distribution; Biology
[53]	Zipf's law	Linux distributions diffusion analysis; Informatics
[54]	Zipf's law; Zipf-Mandelbrot law	Authorship attribution model; Music
[58]	Power law;	Applications review
[49]	Zipf's law	Japanese firms bankruptcy distribution analysis; Economics
[55]	Zipf's law; Simon's model	Connection between linguistic and music investigation; Music

Table 2: List of references of different rank-size applications (excluding text mining). Starting from the left side, the first column reports the references of the papers, the second one lists the employed models and the third column contains a small summary and the scientific fields in which the studies are collocated.

Authors	Approach	Description
[65]	Zipf, Zipf-Mandelbrot and Lavalette	Proposal of a new, more general,
[00]	laws	rank-size law
[63]	Bradford-Zipf-Mandelbrot laws	Literature review of the empirical
[00]	Bradiord-Zipi-Mandelbrot laws	hyperbolic distributions
		Signal model for modeling
[67]	Zipf's law	communication considering syntax
		and semantic and new proposal
[66]	Zipf's law	Signal model for modeling
[00]	Zipi s law	communication and new proposal
[64]	Lavalette's law	Paper from Lavalette stating his
[04]	Davalette S law	famous law
		Papers from Mandelbrot proposing
[61; 62]	Zipf-Mandelbrot law	the modification of the famous
		Zipf's law
		Proposal of a model with different
[60]	Rank-size laws	rank-size regimes within Zipf-like
		distributed data
[42; 43]	Zipf's law	Papers from Zipf stating his famous
[42, 49]	Zipi s iaw	law

Table 3: List of relevant references included in the present paper associated to methodological proposal, survey or text mining rank-size techniques. Staring from the left side, in the first column one can find the references of the papers; the second one contains the discussed methodological tool(s) and the last column summarizes the nature of the contributions.

The second step is devoted to the first moves of the web scraping technique as described even in [68; 69] for the specific case of R. Such a technique is an automated computer science procedure for extracting information from websites through a combination of dedicated commands. Thanks to them, it is possible to systematically access web pages in order to extract data of interest. The phases of a web scraping routine are twofolds: the first stage is characterized by the saving of the HTML source code of the web pages; the second stage consists of the extraction of the portion of the code where the needed information is reported. These actions are performed thanks to the functions: $read_html(), html_nodes(), html_attr()$ and html_text(). Such functions are available in the "rvest" library, which is employed for systematically finding the links to speeches' transcripts and other contents of interest. So, in the second step, one grabs all the URLs (the acronym standing for: Uniform Resource Locators) of the speeches in order to prepare a list of links to be opened. The process of saving the addresses from the reference web page might lead to the occurrence of some errors. Such errors can be including mistakes produced by the web site creator. As an example, the links to the speeches' pages could be reported by a different HTML identifier into the page, and this would lead to empty memorization. A control procedure has been applied in order to face this problem; at the end of this phase, 978 web pages' addresses to the speeches' transcripts were obtained.

The third step consists in the application of another web scraping routine on each page that contains the transcribed words. The list of links pre-loaded in the previous phase is treated to obtain speeches' transcripts, titles, dates, places of the statement, sources, and resumes of the speeches. This step is implemented through a "for" loop over the list of links that point to the pages where the speeches are stored. In each for's cycle, the web scraping routine is applied for the second time in order to read the HTML code of each web page pointed by the for running index. At the same time, one controls for possible discrepancies that could occur through an "if"'s statements inside the loop. This has to be done because sometimes the web pages where the speeches are presented could contain errors like: blank area where the corpus is supposed to be and/or the transcript is reported into another web page's section. In so doing we have realized that the web page that contains the speech: "Campaign speech in Indianapolis, Indiana" stated by Herbert Hoover in October 28, 1932 [78], is one of the remarkable exceptions. Indeed, in the corresponding web page the discourse's transcript is positioned in the section dedicated to the resume. So, to capture it, we have used the same HTML selector used for memorizing the resumes of the talk, which is usually positioned in the top-right side of each web page.

At this stage, a visual inspection of the obtained results highlights that different links in the list point to the same speeches' pages. Consequently, in the third step, some information would be downloaded twice. In order to manage such a case, a control on the speeches' title has been applied. In particular, we have checked for titles duplication and saved their position. In this way, it is possible to eliminate the respective positions into the variables used for each type of information (titles, resumes etc..). There are 7 so obtained duplicated speeches: January 20, 2005: "Second Inaugural Address", April 27, 1961: "President and the Press", June 12, 1895: "Declaration of US Neutrality", December 6, 1892: "Fourth Annual Message", December 9, 1891: "Third Annual Message", December 1, 1890: "Second Annual

Message" and December 3, 1889: "First Annual Message". After this further control, we have 971 stored transcripts.

The fourth phase is employed to manage the presence of typos in the inspected web pages of the Miller Center web site. The typologies of typing errors that mind more for the analysis are all those that contrast a correct division of the text into different tokens. Examples are the situations where the space between two words, number and word or punctuation and word is missing. Such typos, which can generate strings like: "you. Therefore", "10,000of" or "the President" etc., impede the software to divide the text according to the adopted tokenization method.

The procedure for managing such typos is the following: the transcripts are firstly stored as a list of strings in a variable. One looks into each string to find all words divided by points without spaces like: "years.And", "slowed.And". These two elements are for example found in: "2016 State of the Union Address" [79]. The problem is solved by inserting a space between the points and the following word.

Then, one solves the issue of numbers followed and preceded by words without spaces' interruptions. An example of this typo can be found in the "First Annual Message" of December 6, 1981 [80]. There, the following exceptions occur: "June30", "in1881", "length3", "since1860", "of250". With the same method used in the previous phase, this typos are found and corrected as follow: "June 30", "in 1881", "length 3", "since 1860", "of 250".

Lastly, one manages the typos generated by two consecutive words merged without spaces, with the former entirely made by lower case characters and the latter made by the first character in upper case and the rest in lower case. An example is in the "Inaugural Address" of March 4, 1925 [81], where the wrong token " our Constitution" has been transformed through the correction process in " our Constitution".

The procedure described in the fourth step is developed by employing regular expressions, which is a simplified method for searching patterns into strings by means of pseudo-coding languages. A formal definition of the regular expression is reported in [70].

A further problem is related to the interactions of the President. Indeed, the President often talks in front of a wide and active audience. In such cases, it could happen that he is interrupted by applause, laughters, singed slogans or very loud screams at which the President could sometimes respond. These situations are reported into the speeches transcripts: the applause and laughter are sometimes reported between round or square brackets, while other kinds of contents like interactions between Presidents and audience are displayed after the specification of the speaker. A quite complete example of the described situations is given by the speech stated by Barak Obama: "Remarks in Eulogy for the Honorable Reverend Clementa Pickney" [82].

Thus, the fifth phase is devoted to remove the parentheses (square and rounded brackets) and their contents. For this aim, one needs to systematically access the individual speeches. In so doing, for example, there are cases in which the parentheses appear in the text but as typos; see e.g. the "Sixth Annual Message" of December 4, 1928 [83]. This type of exceptional error can be detected by registering the lengths of the parentheses' contents. For the analyzed dataset, the non-suspicious length of parentheses' content amounts to about 600 characters. In order to identify this limit as reasonable for a string length between two

brackets, a visual inspection of all the parentheses' contents has been performed. Above the critical threshold of 600 characters, one can consider that the strings bounded by two parentheses do not constitute a content to be eliminated but, rather, a typo (this is the case of a missing closing bracket). A control is implemented by means of an "if" statement, to check if the eliminated pieces of text do not exceed the 600 characters threshold.

In this context, we need to mention also public events like press conferences, that are typically followed by questions from the public or by the journalists. The questions are denoted by an initial uppercase "Q" followed by a punctuation character or a space.

Another well established type of Presidential public meeting are the debates, which are characterized by a dialogue among candidates and/or journalists.

The above-mentioned elements constitute noises for the analysis of the Presidential statements; therefore, the sixth phase is dedicated to the amendment of the transcripts from the strings that do not come out directly from the Presidents, unique source of interest of this study. In this phase, some speeches with peculiar complexity (like the debates, where the rhetoric structure of the Presidential speech may be questionable, being also driven by the conversation flow) have been directly removed. In particular, we have removed 13 Debates and 1 Conversation from the original Miller Center database. To this end, we have looked for the words "Debate" and "Conversation" into the titles list and removed all the transcripts corresponding to the titles in which such strings appear.

By means of a "for" loop, the transcripts have been fathomed in order to find the presence of "Q" followed by punctuation or space, because such strings are clear signals of the presence of a question (so, out of the President rhetoric, as mentioned above). For consistency, all the text after a question has been removed; indeed, Presidents words are driven by the conversation and are not relevant in the analysis of the rhetoric structure of the speeches. An example is the Ronald Reagan speech: "Speech on Foreign Policy" state in December 16, 1988 [84]. Some exceptions did arise. For example "Q." is often used into the transcripts to report names' abbreviations, like "J. Q. Adams" or "Q. Tilson". Such cases have been treated through a visual inspection of the removed texts.

Furthermore, in the Miller Center speeches' transcripts structure, the most noticeable words pronounced by the public and reported are preceded by strings like: "THE AUDIENCE", "AUDIENCE", "AUDIENCE MEMBER" or "Audience", while the President statements are preceded by strings like: "THE PRESIDENT" or "President" sometimes followed by the President's surname (i.e. see the speech "Remarks at the Democratic National Convention" state by Bill Clinton in August 29, 1996 [85]).

By using the regular expressions, it is possible to remove the reported public interventions, thus leaving just the words pronounced by the President. Such an amendment is done by eliminating the characters between the second (or third in case there were multiple speakers like auditors and journalists) speaker markers (for example "AUDIENCE") and the President markers (for example "THE PRESIDENT"). This means that one has to meet jointly (one after the other) the markers of both speakers in order to select the unnecessary text portions. Consequently, this process might lead to the removal of an excessive piece of text in the unlucky case of an error in the transcript. For example, this is the case of the presence of a second speaker marker that is not followed by the President's marker once he

takes the floor again, and the missing marker appears only later in the text. To manage such error cases, we proceed by further analyzing the texts candidate to be removed. In particular, we consider a threshold of 400 characters above which the selected texts are not canceled. That is decided thanks to a visual inspection of all the portions of the selected texts. A remarkable example of the described exception is the speech by George W. Bush on September 3, 2004, entitled: "Remarks at the Republican National Convention" [86]. In that transcript, there are many audience interventions which are reported and marked with the string "AUDIENCE:" but after one of them there is a lack of marker that should have indicated where the President' words appear again. Thus one would eliminate a bigger portion of text that ends when the string "THE PRESIDENT:" is met again. In order to avoid the loss of large bunch of data, we decided to leave the selected pieces of text longer that 400 characters inside the analyzed sample. After all the included words are so few that they cannot affect the final result; this was confirmed by visually inspecting the few exceptions which were found.

Sometimes, at the very beginning of the speech transcript, it is possible to find a string of the type: "THE PRESIDENT:", "The President:" or "The President" followed by his surname to mark the starting point of the President's words. This string is not captured by the described process because it is the very first and is not coming after the intervention of other speakers. Therefore, at the end of this phase, one has to control for the presence of strings of that type at the opening words of the text, and has to eliminate them in affirmative case.

The seventh phase manages the situations in which the President delivered messages with his wife. In this case, we have applied a control using a string of the type "Mrs." followed by the President's surname. Then, to meet consistency, we have removed all the words of the speech from the starting point of the intervention of President's wife.

At this stage, one has to check that the listed modifications do not reduce a given speech so much that it becomes not suitable for performing a consistent analysis. For exploring the implications of too short corpus in applying ZML fit see [71]. The control of the suitable length of the speeches is the scope of the eighth phase. We have eliminated the speeches whose resulting number of characters is less than 600. This threshold has been identified by the inspection of the distribution of the number of characters for each speech. After this procedure, we have eliminated five speeches: "Press Conference with Mikhail Gorbachev" (July 31, 1991), "Press Conference" (November 17, 1967), "Press Conference" (August 18, 1967), "Press Conference" (December 30, 1966), "Argument before the Supreme Court in the Case of United States v. Cinque" (February 24, 1841).

A further control for identifying speeches with the intervention of people different from the President has been next implemented. In particular, in some Press Conferences the question are introduced by the name of the journalist followed by the name of the newspaper. In order to capture such cases, one controls for the presence of the string "Press Conference" in the titles and for the absence of the mentioned markers of the questions ("Q" followed by blank or punctuation). This type of exception is met just with: "Press Conference in the East Room", July 20, 1966 [87]. This speech has been eliminated from the list. In fact, from a visual inspection, one can see that it is mainly made of words not provided by the President but rather by journalists.

The ninth phase is devoted to a last check of the outcomes of the previous steps. In particular, the string: "THE PRESIDENT" followed by punctuation or by the surname of the President (all upper or lower case) has been seek in the speeches. The presence of such a string points to talks in which there is the intervention of the public or other speakers; such interventions did not appear in the previous phases since they are placed in the first line of the speeches. As an example, refer to the Obama's speech: "Address to the United Nations" delivered in September 23, 2010 [88]. In other cases, the string is reported at the beginning of the speech just for indicating the point in which the president is starting to speak. This control was run over the entire dataset and not just on the modified transcripts as for the previous phases.

The tenth stage is called tokenization phase. A formal definition and some practical examples of tokenization are reported in [74]. Indeed, when the talks' transcripts are stored into the cells, long strings of characters are memorized without any particular distinction. Yet, in order to work with words' frequency, one needs to refine the speeches until they become a list of comparable units of analysis. Consequently, one needs to split these strings of characters (one for each speech) by making the procedure in R to be able to recognize single words in accord to the used tokens definition.

Specifically, with the R library "tokenizers", it is possible to use the command tokenize_words(). Such a command divides the text by using the blank space as a separator and without taking into consideration the punctuation except for the decimal and thousand numbers separators. Furthermore it does not consider the apostrophes between words as a separator, like in the case of contract form of verbs. As an outcome, we obtain that the variable containing the entire speech is transformed into a vector whose components are the words. Moreover, all the letters of the words are converted from upper case to lower case. In so doing, possible ambiguities due to the case sensitiveness of the words is forcefully removed.

In the eleventh step we have implemented a control for the speeches that are doubly reported into the same page section, hence leading to doubling the frequencies of the words. Sometimes it happens that those repeated transcripts do not have exactly the same words in common, but differ for few terms (for a maximum of about 20, according to our empirical experience). This can be noticed by observing that some speeches have very few words with frequency equal to one, the so-called hapax legomena. For this reason, the control for finding the double repeated transcripts is done by checking if the number of tokens that appear only once falls below a certain threshold. Thus, for each speech, we divided the number of words occurring just once, by the number of different words used, (see Figure 1); thanks to a visual inspection, one fixes the critical threshold at 20%.

The failure of this check pointed out to some technical inconsistencies of the website; one of the affected transcript is: "Remarks Honoring the Vietnam Warś Unknown Soldier" of May 28, 1984 [89]. To solve this bug, we divided the terms' frequencies by 2, each time the control on the threshold was failing. But, in so doing, the exceptional terms appearing once in the double reported speeches reached absolute frequencies equal to 0.5. For this reason one has to add a further control for eliminating the residual tokens with 0.5 absolute frequencies.

The twelfth phase concerns the creation of a table type variable that has a number of rows equal to the number of different words used in each single speech and two columns:

one for the tokens and another for the frequencies. Each couple is sorted out by decreasing order of frequencies. The tables of the speeches have been labeled by the title and date of the speech.

To make the dataset exportable and ready to be processed, one collects and save the data into a comma-separated values file (i.e. a csv file). A matrix with 951×2 columns and 3933 rows is then obtained. Each couple of columns is dedicated to the so sorted list of words and their respective frequencies for the individual speeches. In order to obtain a rectangular matrix, the number of rows is uniformed at the maximum number of different words used across speeches, which is 3931. Of course many speeches have a lower number of different words, and the empty cells are filled by NAs, which point to a missing value indicator. After this, one adds two rows at the top of the table: the first one is used to report the talks' speaker name; the second one is adopted to show the titles with the dates embedded.

As already preannounced above, the last phase concerns the export of that table into a .csv file. In so doing, one has a dataset which is easily analyzable, – also with different programming languages; this goes in the direction of making this study reproducible.

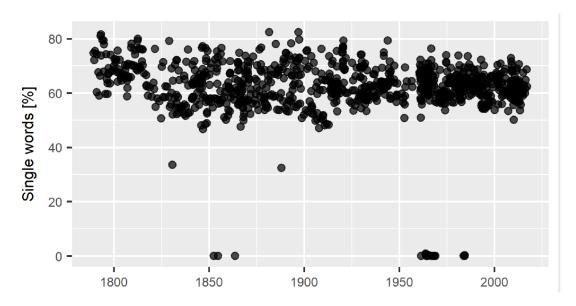


Figure 1: Percentage of words used just once on the number of different words used in each speech in a time varying representation.

4. Dataset Description

This section contains the description of the dataset.

At the end of the process described in the previous section, one has a dataset of 951 Presidential speeches stated by the 45 United States Presidents, from George Washington to Donald Trump. The dataset covers a wide period of time: from April 30, 1789 to February 28, 2017. Due to the Miller Center web site's content, the number of speeches per President

President	No.	President	No.
Lyndon B. Johnson	66	Jimmy Carter	18
Ronald Reagan	57	John Tyler	18
Barack Obama	50	Warren G. Harding	18
Franklin D. Roosevelt	49	Rutherford B. Hayes	16
John F. Kennedy	41	Abraham Lincoln	15
George W. Bush	39	Franklin Pierce	15
Bill Clinton	38	Gerald Ford	14
Woodrow Wilson	33	James Buchanan	14
Ulysses S. Grant	32	William McKinley	14
Andrew Johnson	31	Calvin Coolidge	12
Herbert Hoover	30	William Taft	12
Grover Cleveland	29	Chester A. Arthur	11
Andrew Jackson	26	James Monroe	10
James K. Polk	25	Martin Van Buren	10
Thomas Jefferson	24	John Adams	9
Richard Nixon	23	John Quincy Adams	8
James Madison	22	Millard Fillmore	7
Theodore Roosevelt	22	Dwight D. Eisenhower	6
George Washington	21	Zachary Taylor	4
George H. W. Bush	20	Donald Trump	2
Benjamin Harrison	19	James A. Garfield	1
Harry S. Truman	19	William Harrison	1

Table 4: Number of speeches (No.) per President. The list is for 44 presidents, instead of 45, because Grover Cleveland has had two non-consecutive mandates: the first from March 4, 1885 to March 4, 1889 and the second from March 4, 1893 to March 4, 1897. In order to count his number of speeches, the two mandates are grouped together.

are different and depends on criteria decided by the website owner. The number of the speeches per President is reported in Table 4.

The Miller Center website provides discourses stated in many occasions of United State Political life: for example there are 57 State of the Union Addresses, 142 Annual Messages, 58 Inaugural Addresses, 20 discourses state at universities or related to them, 18 speeches stated at National Conventions of Republicans or Democratic parties, 89 remarks pronounced by Presidents on salient topics and 567 other moments when the US Presidents have spoken to people. This information is summarized in Table 6.

All these declarations are collected as described in the previous section; furthermore, they are stored by organizing the words' distribution for each speech. In this way, it is easy to apply the rank-size analysis in order to investigate the different rhetoric structures, as shown in the next section. Table 5 presents a statistical description of the speeches' length in term of number of words per talk (second column) and in term of the different terms used in each speech (third column). The statistics will be commented upon in the result

section. The discourse with the minimum number of different words used is the "Message to Congress Requesting War Declarations with Germany and Italy" in December 11, 1941, by Franklin D. Roosevelt; the one with most words variety is the "Seventh Annual Message" of December 3, 1907, by Theodore Roosevelt. It is interesting to note that the impressive amount of different words used in the latter message is related to the fact that, during this talk, the President Roosevelt has read a part of another speech (Message to the Congress on December 5, 1905) and has mentioned events from the past, hence increasing the lexical richness of his speech.

Statistical indicators	Speech Length	Different words used
Max.	27551	3 931
Min.	132	76
Median (m)	2315	760
Mean (μ)	3533	916.23
RMS	5256.24	1 144.87
St. Dev. (σ)	3893.80	686.85
Variance	15145734.74	$471\ 265.19$
St. Error	126.27	22.27
Skewness	2.64	1.58
Kurtosis	12	6.02
μ/σ	0.91	1.33
$3(\mu-m)/\sigma$	0.94	0.68

Table 5: The numbers in column two offers a statistical summary of the speeches' lengths in term of total number of words per speech. The third column contains the statistics of the number of different words used in each speech.

Class of speeches	Number of speeches	
Speeches stated at National Conventions of the	10	
Republican or Democratic parties	18	
Discourses stated at Universities or related to them	20	
State of the Union Addresses	57	
Inaugural Addresses	58	
Remarks on salient topics	89	
Annual Messages	142	
Others	567	

Table 6: The pre-processed corpus scraped form the Miller Center web site has been dived in classes which contains the number of speeches here reported. See Section 3 for the details on the data collection and manipulation procedure.

5. Rank-Size: Zipf-Mandelbrot Law

The speeches' words frequencies are the size of the rank-size analysis. Specifically, each talk transcript is stored into a table with terms and respective frequencies, so the rank r=1 corresponds to the most repeated word of the speech. The tokens with lowest frequencies are stored in the positions corresponding to the highest ranks. The use of the plural is required because the terms with frequencies 1 and 2 represent generally the majority of the tokens in each discourse. Here, a best fit procedure to assess whenever the size-frequencies f and f_{rel} (absolute and relative respectively) might be view as a function of the ranks r is implemented. The considered fit function is the ZML:

$$f \sim g(r) = \alpha (r + \beta)^{-\gamma},\tag{1}$$

where α , β , γ must be calibrated individually for each one of the 951 speeches. All fits have been carried out through a Levenberg-Marquardt algorithm [72; 73; 75] with no restrictions on the parameters. The starting points used in each estimation are provided through a non linear regression run over the same function but with a brute-force algorithm also know as grid-based searches, to avoid the dependence on starting parameters or getting stuck in local solutions. The same procedure is applied over the following formula, hence on the relative frequencies of the speeches' words.

$$f_{rel} \sim \frac{g(r)}{N} = \tilde{\alpha}(r+\beta)^{-\gamma},$$
 (2)

where N is the length of the considered speech and $\tilde{\alpha}$, β , γ are the parameters to calibrate. The estimated parameters interpretations for the case of Eq. (1) are the following: α gives information on the the number N of words of the speech (see Figure 2), which is emphasized in Figure 3, and removed in the relative frequencies case of Eq. (2). This aspect is discussed more in detail in the next section.

The parameter β , contains information on the higher ranked words. In particular the model reduces the weights of tokens with high frequencies when β increases. Moreover, if one does not have the presence of outliers, β is small.

For what concerns γ , we expect that this parameter is close to 1, as it will be found; see results below. This parameter describes the concavity of the fitted models, whence is informative about the distribution of words frequencies thereby giving an idea about their density. Indeed, if the magnitudes of the frequencies in the medium-high ranked words are high, then the calibrated $\hat{\gamma}$ is to be found small. Furthermore, γ is affected by the number of hapax legomena. If their presence is bulky with respect to the rest of the tokens, the model concavity will increase in order to capture the transition point between high ranks and low ranks.

This interpretation of the parameters is coherent with [76], and will be also discussed later.

6. Results

The second column of the Table 5 contains the main statistical indicators of the speeches' length in term of total used words in each speech. When looking at such statistics we can

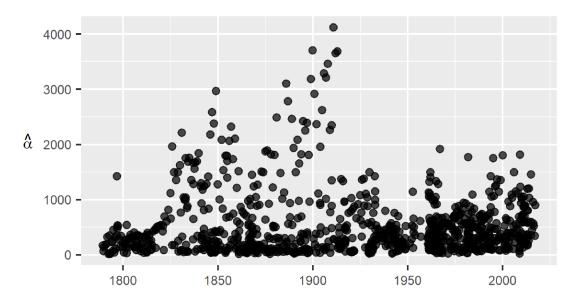


Figure 2: $\hat{\alpha}$ estimated on absolute frequencies with Eq. (1) for each speech.

notice that the length varies considerably and its mean and median do not coincide. The positive skewness suggests a right-tailed shape, and the kurtosis equal to 12 indicates a leptokurtic distribution with very heavy tails. A similar situations is presented in the third column of Table 5 where the main statistics of the number of different words used in each speeches are presented. The asymmetry is well identified by skewness value and confirmed by the different positional indicators. As for the previous case, we obtain a leptokurtic distribution but with a kurtosis equal to 6.02.

Figure 4 gives an idea of the talks lengths along the considered period of our sample and it is also informative about the years in which we have "grader masses" of speeches by observing the density of the points. The recent years denote a change of behavior, indeed after a small number of speeches in the 60's, there is a noticeable concentration of the speeches' in terms of their lengths. In a number of cases the speeches lengths and words variety are considerably high, due to the presence of very long speeches in which the President is reading some other documents or is quoting other talks.

In Figure 1, it is possible to note the pattern of the percentage of words used only once per talk. We can note also a slightly decreasing trend that could be associated to the need of reducing the number of different words used. In the light of the information available here, a possible explanation of that could be the recent communication need of repeating messages many times, with same terms, for a simpler and more aggressive media strategy. Anyway the speeches are mostly characterized by the presence of words pronounced only once. Such words populate the tails of each ZML distributions and provide a characterization of them.

The best fit procedure on Eq. (1) and Eq. (2) is performed for any individual speech, and a visual presentation of the goodness of fit measure for the second equation is reported in Figure 5. The main stats of R^2 s and regression standard errors for both equations are presented in Table 8. Furthermore we test the normality of the standardized regressions

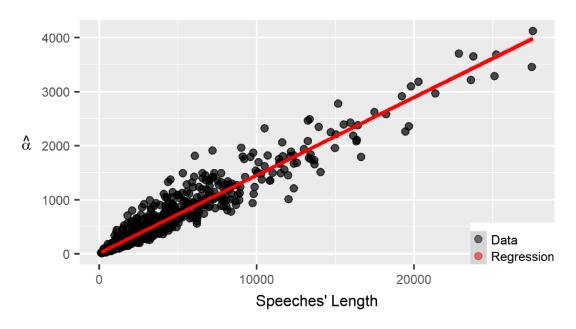


Figure 3: Relationship between $\hat{\alpha}$ and speech length in term of total number of words used per corpus

residuals by using the Shapiro-Wilk test (see [77]); the p-values are reported in Table 9. From the analysis of the absolute frequencies, some facts emerge.

A visual inspection of $\hat{\alpha}$ in Figure 2 shows a remarkable trend in the first years. Figure 3 evidences the positive correlation between $\hat{\alpha}$ and the speech length N over all the speeches, while Figure 6 shows that the correlation between $\hat{\alpha}$ and number of different tokens used in each talk is still present, but is less clear than the previous one. The dependence of $\hat{\alpha}$ on N represents a bias for the analysis of the results and a supportive argument for studying also relative frequencies. In fact, such a dependence disappears in calibrating the parameters with Eq. (2), i.e. by taking into consideration the relative frequencies of the words, as shown in Figure 7. The calibrated parameters on relative frequencies are reported in Figures 8,9,10. A statistical summary of the so obtained parameter values is reported in Table 7; the goodness of fit measures is reported in Table 8. The salient cases of $\hat{\alpha}$ are presented in Figure 11, 12. The last one is associated to Ronald Reagan's speech titled "Remarks on the Air Traffic Controllers Strike", which is very short, thus having such a small $\hat{\alpha}$. We need to say that the original transcript was longer; we remove them as discussed in the second section. Thus, the parameter $\tilde{\alpha}$ can be viewed still as an indicator of the highest relative frequency in each speech, even if its magnitude is mitigated by the relationships with the other parameters.

The calibrated β gives an indication on the differences among the frequencies within the various speeches. Given that, the biggest differences are between the low ranked words as well know by [42; 43]. The $\hat{\beta}$ s are showing the behaviour of the frequencies at the lowest ranks, thus on the most common words. An evidence of the feature of $\hat{\beta}$ is provided from establishing the differences between words' frequencies at consecutive ranks within each speech. Indeed, by summing for each speech the first 5 differences originated by the 6 most repeated words and comparing them to the $\hat{\beta}$ s, one has Figure 13. Such a Figure shows the

decay of $\hat{\beta}$ with respect to the differences in frequencies within each speech. The graph is confirming that high level of $\hat{\beta}$ is corresponding to tiny differences between top six repeated words within speeches (see also Figure 14) while the converse occurs for the low level of the parameters (see Figure 15). This occurrence can be interpreted also in terms of the rhetoric structure, by asserting that when $\hat{\beta}$ is large, the words of the speech have a more homogeneous distribution along the highest ranks, hence pointing to the presence of a "rich club" of outliers at the low ranks.

Another interesting feature that emerges by the best fit procedure on Eq. (2) is the correlation between $\hat{\alpha}$ and $\hat{\beta}$ (see Figure 16). The joint evaluation of such calibrated parameters gives then information on the magnitude of frequencies at low ranks.

The concavity of the fitted curve related to Eq. (2) is mainly affected by $\hat{\gamma}$. Therefore this parameter is informative about the decay of the model passing by the low ranks to the high ranks. The $\hat{\gamma}$ is peculiar of each speech, changing the focus of the hyperbolas in agreement with the features of the talks. Consequently, it is the parameter that mostly affects the areas under the fitted models, which is reduced when $\hat{\gamma}$ increases (see Figure 17 for a graphical representation of the relationship between areas and $\hat{\gamma}$). The areas have been calculated by computing the following integral over each model characterized by $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$, where the i is the indicator of the i^{th} speech, so that $i=1,\ldots,951$:

$$A_{i} = \int_{1}^{r_{Max,i}} \hat{\tilde{\alpha}}_{i} (r + \hat{\beta}_{i})^{-\hat{\gamma}_{i}} dr = \frac{\hat{\tilde{\alpha}}_{i}}{1 - \hat{\gamma}_{i}} \left[(r_{Max,i} + \hat{\beta}_{i})^{1 - \hat{\gamma}_{i}} - (1 + \hat{\beta}_{i})^{1 - \hat{\gamma}_{i}} \right]$$
(3)

 A_i is the i^{th} area corresponding to the model calibrated over the i^{th} talks, while $r_{Max,i}$ is the highest rank referred to the i^{th} transcript. Refer to Figure 18, where it is possible to notice the shape of the histograms defined in Eq. (2). Such a Figure gives a clear idea of the ZML capacity of being a density function in this specific case of the analysis of relative frequencies.

Notice also that the $\hat{\gamma}$ is low (high) when in the speech there is a more or less evident transition from low ranked words to high ranked words; see the two cases in Figures 19, 20.

Under an evolutive perspective, the most pronounced words for each speech have a constant decreasing rate along the years (see Figure 23). This result means that the repetition of a single word tends to be reduced with time. This occurrence goes hand in hand with a global reduction in the differences between the words' frequencies within the speeches, as shown in Figure 21. Considering that the most pronounced tokens are mainly conjunctions, articles and preposition, this phenomenon could be interpreted as the growing need in time of making simpler syntax of the sentences. Another useful hint of this fact is that the number of single words used (see Figure 1) is slightly decreasing along the years. So the global tendency is to use less single words and propose sentence structures which are not too complex.

7. Discussion and conclusive remarks

The Presidential speeches' sample analyzed in this paper is one of the most complete in the literature. It has been constructed under consistency criteria in a phase-wise form;

	\hat{lpha}	\hat{eta}	$\hat{\gamma}$	$\hat{ ilde{lpha}}$	\hat{eta}	$\hat{\gamma}$
Max	4117.92	6.13	1.23	0.40	6.13	1.23
Min	9.72	-0.57	0.54	0.05	-0.57	0.54
Median m	326.35	0.72	0.9	0.14	0.72	0.97
Mean μ	521.13	1.01	0.97	0.14	1.01	0.97
RMS	25.37	0.05	0.03	0.004	0.05	0.03
Standard Deviation	583.69	0.97	0.10	0.05	0.97	0.10
Variance	340339.10	0.94	0.01	0.002	0.94	0.01
Standard Error	18.93	0.03	0.003	0.001	0.03	0.003
Skewness	2.40	1.41	-0.45	1.55	1.41	-0.41
Kurtosis	10.40	5.35	3.65	6.90	5.36	3.53
μ/σ	0.89	1.03	9.53	3.04	1.04	9.53
$3(\mu-m)/\sigma$	-1.00	-0.89	0.20	-0.56	-0.89	0.20

Table 7: Statistical summary of the estimated parameters on relative and absolute frequencies in accordance with Eqs. (1) and (2) respectively.

	R^2	Std error	R_{rel}^2	Std $error_{rel}$
Max	1.00	5.68	1.00	0.0041
Min	0.91	0.26	0.91	0.0001
Median m	0.99	1.00	0.99	0.0004
Mean μ	0.98	1.25	0.98	0.0006
Standard Deviation	0.01	0.87	0.01	0.0005

Table 8: Statistical summary of R^2 s and non-linear regression standard errors calculated for each fit with Eqs. (1) and (2). They represent the models goodness of fit calibrated over each speech when considering absolute and relative frequencies.

finally it contains 951 talks over a span of about 228 years for all the US Presidents up to now. The source of the data is the Miller Center website; data have been retrieved at the end of June 2017.

The use of rank-size laws with sizes given by the word frequencies through the Zip-Mandelbrot law (see Eq. (1) and Eq. (2)) gives the opportunity of analyzing the rhetoric structure of the transcripts. More specifically, the method allows us to observe changes into the rhetoric frameworks without being drastically affected by the changes in usages and significance of terminologies that occurred during the years. Indeed such an objective analysis of the frequencies disregards the words' meanings and focuses on the exploration of the structures of the speeches when words are ranked. Undoubtedly, over the 228 years hereby considered, the Americans' language has changed a lot as well as the "society" at which the speeches are addressed; thus, the convergence toward a common scheme in producing talks could be even explored searching for "universal behavior". Thus, Eqs. (1) - (2) show a spectacular capacity of fitting the transcripts, but the considerations on regressions standard errors and values of R^2 always around 1 (see Table 8 and Figure 5) lead to the conclusion that Eq. (2) is more suitable to represent the talks, especially considering

	S.W. p-values of fits on Eq. (1)	S.W. p-values of fits on Eq. (2)
Max	0.04	0.04
Min	0.00	0.00
Median m	0.00	0.00
Mean μ	0.00	0.00
Standard Deviation	0.00	0.00

Table 9: Statistical summary of the Shapiro-Wilk test (S.W. in the Table) p-values resulting from the tests performed on standardized residuals of best fit run with Eqs. (1) and (2).

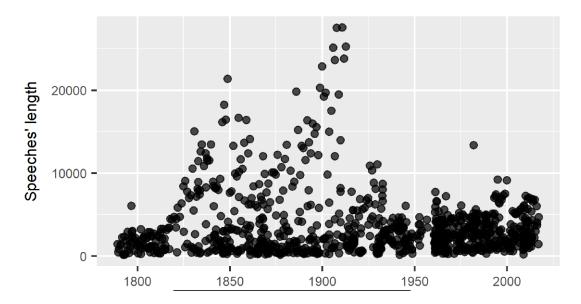


Figure 4: Speeches' length distribution in term of total number of words used per speech over the years

the bias on $\hat{\alpha}$. Further evidence of the impressive capacity of such a fitting can be noticed with a visual inspection of Figures 11,12,14,15,19,20. Therefore, jointly with the Shapiro-Wilk test p-values reported in Table 9, one can conclude that the selected approach does not reveal compromising weaknesses, can be considered rigorous, and recommended in further works.

The calibrated parameters on Eq. (2) are presented in Figures 8,9,10. It is possible to observe some changes along the years by visual inspecting $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$. Such calibrated parameters are used to resume some features of the speeches structures.

The $\hat{\alpha}$ of Eq. (2) has a small increment in volatility during the last years with a high concentration of outliers after the 1960s. Considering the fact that $\hat{\alpha}$ is giving an indication on the relative frequencies of the most often used words, the meaning of the related behavior along the years can be interpreted as an upcoming of irregularities in the use of words.

The analysis of $\hat{\alpha}$, so the parameter estimated with Eq. (1), whose behavior is reported in Figure 2, leads to the assessment of two remarkable trends of speeches' length during the years between 1800-1850 and 1850-1900. As we have said before, this outcome is grounded on the fact that the parameter α can be considered as a proxy for exploring the number of

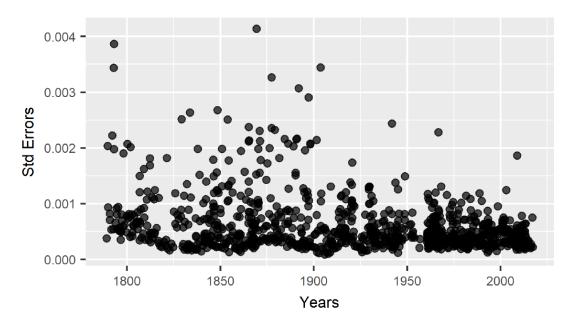


Figure 5: Regressions standard errors for each fitted speech on Eq. (2) over the years.

words employed in the speech.

The $\hat{\beta}$ has a similar behavior to that of $\hat{\alpha}$, as can be seen from Figure 16. Indeed, its points are quite homogeneously distributed between 0 and 1 until 1900 when $\hat{\beta}$ starts to rise with a contemporaneous increment of the volatility (see Figure 9).

The β 's increment when the differences between frequencies at low ranks are decreasing (see Figure 13) helps us to conclude that after 1900 the words frequencies distributions are converging toward more homogeneous distributions. A further confirmation of this is given by the areas delimited by the models and computed through Eq. (3). As it is possible to deduce from Figure 22, there is a feeble positive trend, combined with a reduction in variability. Furthermore, there is a clear decreasing trend in the most often used words' relative frequencies of each speech (see Figure 23), which reinforces the results and interpretations about $\hat{\beta}$.

From Figure 10, the calibrated $\hat{\gamma}$ appears to be quite stable in terms of trend and distance from 1. In the majority of the cases, such a parameter assumes a value around 1. When $\hat{\gamma} \geq 1$, then one can assert that there is a steeper decay of the data in the rank-size plot. Figure 24 assists in visualizing that the distribution of the $\hat{\gamma}$'s is asymmetric and the left tail is a bit longer than the right tail, giving a further indication of the tendency toward President producing "more homogeneous talks", in our sense.

Finally we can assert that the speeches' structures exhibit in general a sort of common framework, with a specific proportion of words. Consequently, this means that the typology of the rhetoric involved in the political public speaking is identifiable. This can be considered as a supporting argument of the generic structural system that lies behind the rhetoric of American Presidents' political speeches. Moreover, patterns in the parameters behaviors can be viewed as an hint about a common generative womb of the speeches rhetoric. The

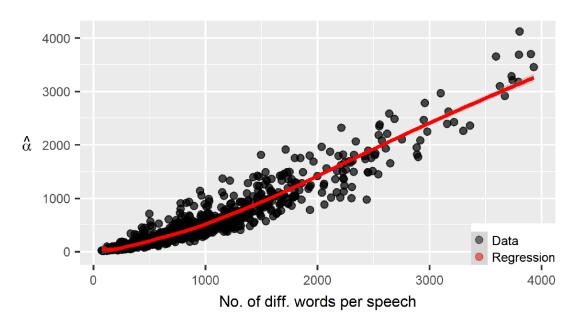


Figure 6: Relationship between $\hat{\alpha}$ and number of different words used in each speech (each word considered just once per corpus)

method here applied is informative and the robust capacity of fitting provides a certain confidence in reaching a conclusion.

The obtained results suggest the presence of common linguistics thoughts of the Presidents when delivering a talk. One can argue that Presidents imitate their predecessors and look at the speeches already delivered to get inspiration for their own talks. The historical discrepancies point to the presence of periodical modes of the structure of the talks and on the linguistic productivity. Such an outcome should be correctly interpreted not only in terms of individual Presidents, but also accordingly to the evolution of the language. The regularity over all the Presidents of the decay of the best-fit curve from the high ranks to the low ones can be viewed as an evidence of the similarities among the speeches. Such an outcome suggests that the Presidents' speeches share the same way to use words and their frequencies, hence pointing to a sort of coded communication channel when delivering talks.

All these elements impose rigorous and deep investigations, and are opening the door for further studies.

One of the most prominent proposal for future research concerns the assessment of the stochastic properties of the rhetoric of the speeches for performing some forecast of the structure of future President speeches. A further research theme is associated to the way Presidents' speeches might be clustered. In this respect, it is possible to propose different concepts of distance between couples of speeches bringing different typologies of information, and identify accordingly the closest speeches. Results might be interpreted at the level of individual Presidents or in the specific historical context. This proposal suggests to analyze also subsets of speeches delivered in specific situations (like war periods or times of economic distress) or by specific Presidents to identify sources of regularities and guess the presence

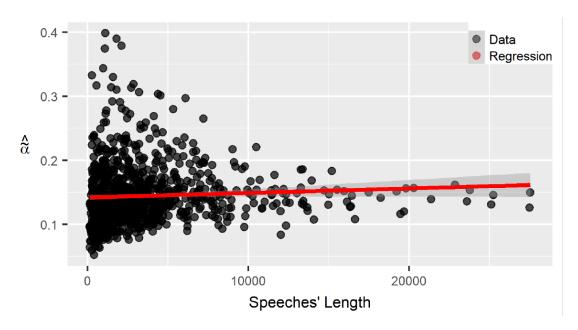


Figure 7: Relationship between $\hat{\tilde{\alpha}}$ calibrated for relative frequencies and speeches' length in term of total number of words used.

of imitative behaviors or coded communication rules.

Under a pure methodological perspective, the proposed combination of text-mining and rank-size approaches exhibits characteristics of flexibility and generality, hence suggesting the possibility of an effective reproduction of the analysis in other relevant expert systems contexts. In this respect, the analysis of the speeches of the Governors of the Central Banks might be quite informative on the way in which the different phases of business cycles and remarkable economic situations are treated by leading economists with a so relevant institutional role.

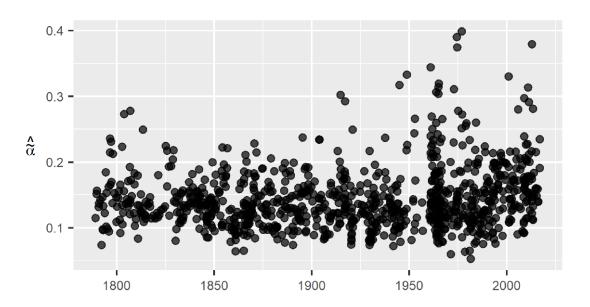


Figure 8: Estimated $\hat{\tilde{\alpha}}$ on relative frequencies for each speech over years (Eq. (2)).

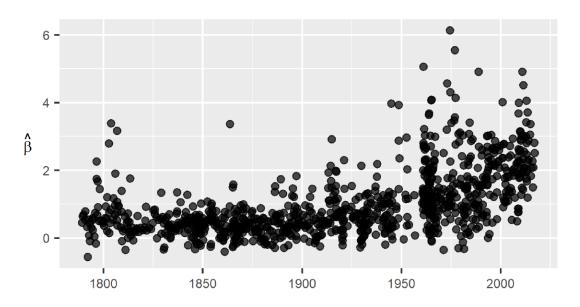


Figure 9: Estimated $\hat{\beta}$ on relative frequencies for each speech over years (Eq. (2)).

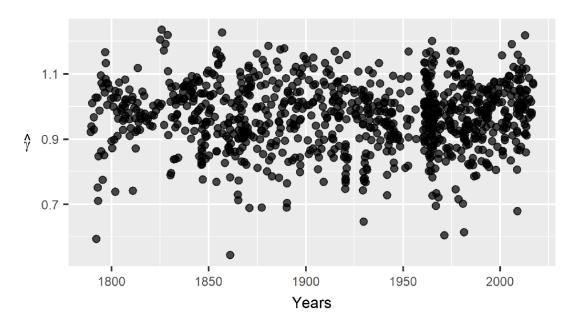


Figure 10: Estimated $\hat{\gamma}$ on relative frequencies for each speech over years (Eq. (2)).

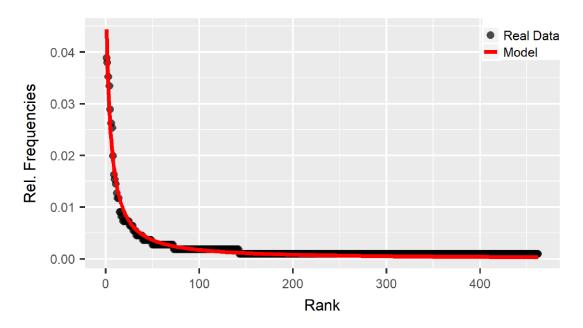


Figure 11: January 20, 1977 - Inaugural Address - Jimmy Carter. Comparisons between real data and fitted models over the speech's words relative frequencies for the case of the highest $\hat{\tilde{\alpha}}=0.39;\,\hat{\beta}=5.54;\,\hat{\gamma}=1.16;\,$ N = 1107; $R^2=0.97.$

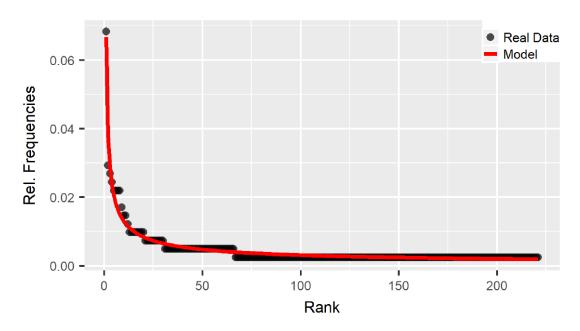


Figure 12: August 3, 1981 - Remarks on the Air Traffic Controllers Strike - Ronald Reagan. Comparisons between real data and fitted models over the speech's words relative frequencies for the case of the lowest $\hat{\alpha}=0.05; \, \hat{\beta}=-0.32; \, \hat{\gamma}=0.61; \, N=410 \, R^2=0.97.$

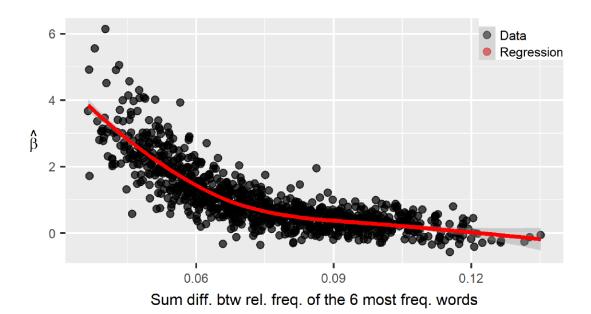


Figure 13: $\hat{\beta}$ against the summed differences in relative frequencies of the top 6 repeated words within each speech.

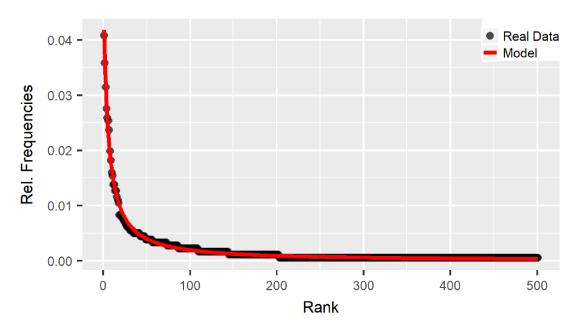


Figure 14: August 9, 1974 - Remarks on Departure From the White House - Richard Nixon. Comparisons between real data and fitted models over the speech's words relative frequencies for the case of the highest $\hat{\beta} = 6.12$; $\hat{\alpha} = 0.38$; $\hat{\gamma} = 1.13$; N = 1815; $R^2 = 0.99$.

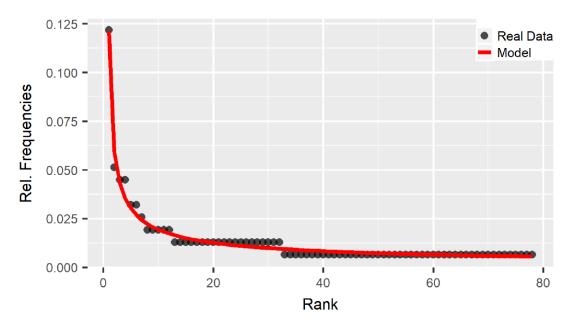


Figure 15: April 5, 1792 - Veto Message on Congressional Redistricting - George Washington. Comparisons between real data and fitted models over the speech's words relative frequencies for the case of the lowest $\hat{\beta} = -0.56$; $\hat{\alpha} = 0.07$; $\hat{\gamma} = 0.59$; N = 156; $R^2 = 0.98$.

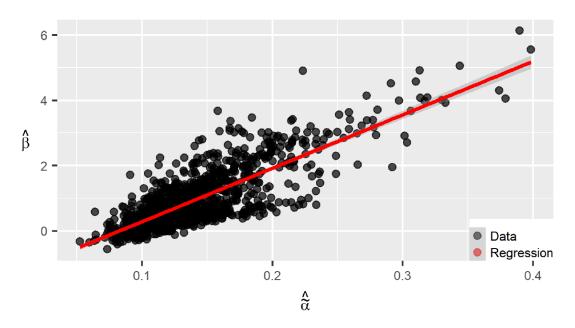


Figure 16: Graphical insight of the relationship between $\hat{\alpha}$ and $\hat{\beta}$ in the estimation run using Eq. (2).

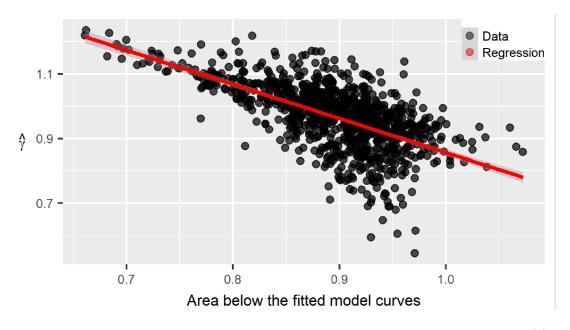


Figure 17: The $\hat{\gamma}$ against the areas underlined by each fitted model computed with Eq. (3).

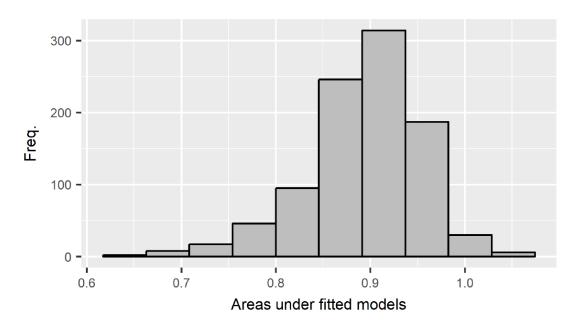


Figure 18: Areas under the fitted models computed with Eq. (3).

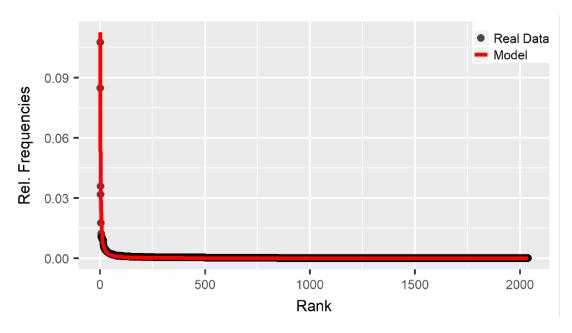


Figure 19: December 6, 1825 - First Annual Message - John Quincy Adams. Comparisons between real data and fitted models over the speech's words relative frequencies for the case of the highest $\hat{\gamma}=1.23;\,\hat{\hat{\alpha}}=0.21;\,\hat{\beta}=0.70;\,\mathrm{N}=9023;\,R^2=0.96.$

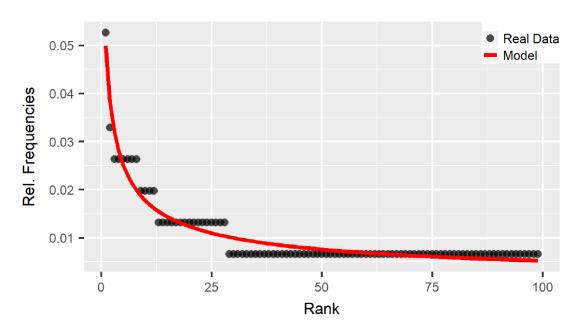


Figure 20: "February 11, 1861 - Farewell Address - Abraham Lincoln. Comparisons between real data and fitted models over the speech's words relative frequencies for the case of the lowest $\hat{\gamma}=0.54;~\hat{\alpha}=0.06;~\hat{\beta}=0.58;~N=152;~R^2=0.93.$

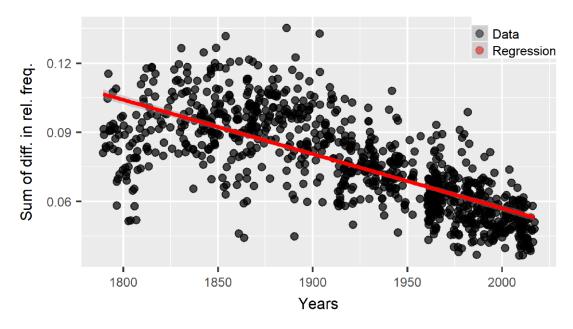


Figure 21: Sum of the differences computed between all the words' relative frequencies within each speech along the years.

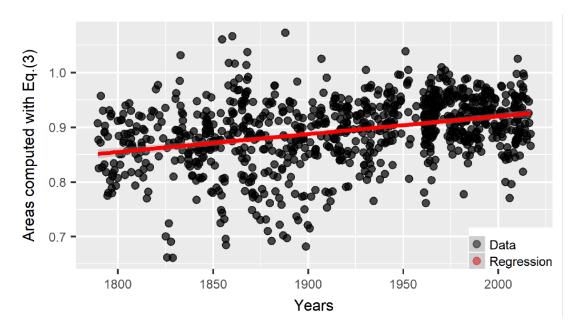


Figure 22: Each point represents an area under the respective fitted model computed with Eq. (3).

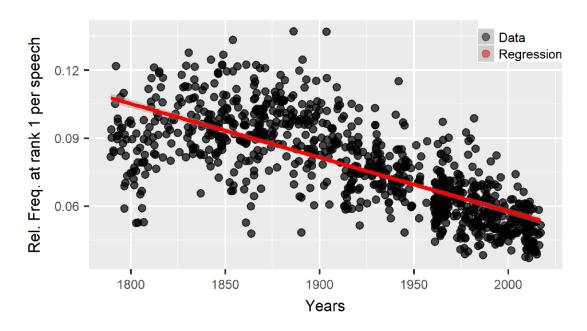


Figure 23: The relative frequencies of the most used word in each speech along the years.

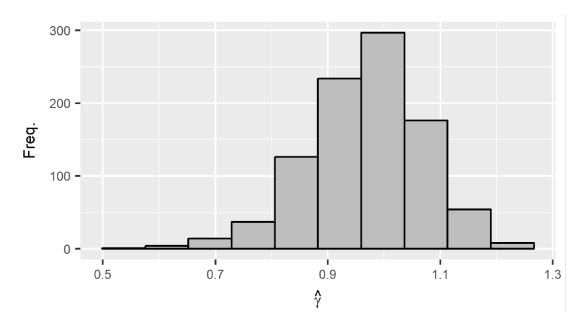


Figure 24: Histogram of the $\hat{\gamma}^{\circ}$ values.

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